A Disk Of Radius 25 Cm Spinning At A Rate Of 30 Rpm Slows To A Stop Over 3 Seconds. What Is The Angular

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Understanding rotational motion is fundamental in physics, especially when analyzing the behavior of spinning objects such as disks, wheels, or gears. In this context, a disk with a radius of 25 cm initially spinning at 30 revolutions per minute (rpm) gradually comes to rest over a period of 3 seconds. The critical question is: what is the angular deceleration involved, and what are the angular displacement and initial angular velocity? This comprehensive guide will explore the problem in detail, providing step-by-step calculations, explanations of key concepts, and real-world applications.

Introduction to Rotational Motion and Key Concepts

What is Angular Velocity?

Angular velocity (ω) describes how fast an object rotates around a fixed axis. It is measured in radians per second (rad/s).

- Conversion from rpm to rad/s:

$\omega = \text{rpm} \times \frac{2\pi}{60}$

- Initial Angular Velocity (ω_0):

For the disk, initial rpm = 30 rpm.

What is Angular Acceleration?

Angular acceleration (α) quantifies how quickly the angular velocity changes over time, measured in rad/s^2.

- Negative angular acceleration indicates the disk is decelerating (slowing down).

Relationship Between Angular Quantities

The fundamental equations of rotational kinematics, analogous to linear motion, are:

\[

$$
\omega = \omega_0 + \alpha t
$$

$$
\theta = \omega_0 t + \frac{1}{2} \alpha t^2
$$

$$
\omega^2 = \omega_0^2 + 2 \alpha \theta
$$

where:
- ω is the final angular velocity,
- ω_0 is the initial angular velocity,
- α is angular acceleration,
- θ is the angular displacement,
- t is time.

Step-by-Step Solution to the Problem

Step 1: Convert Initial RPM to Radians per Second

Given:
- Initial RPM = 30 rpm

Conversion:

$$
\omega_0 = 30 \times \frac{2\pi}{60} = 30 \times \frac{\pi}{30} = \pi \text{ rad/sec}
$$

Therefore:

$$
\boxed{\omega_0 = \pi \text{ rad/sec} \approx 3.1416 \text{ rad/sec}}
$$

Step 2: Determine Final Angular Velocity (ω_f)

Since the disk comes to a complete stop:

$$
\omega_f = 0 \text{ rad/sec}
$$

Step 3: Calculate Angular Acceleration (α)

Using the angular kinematic equation:

$$
\omega_f = \omega_0 + \alpha t
$$

Rearranged to solve for α:

$$
\alpha = \frac{\omega_f - \omega_0}{t}
$$

Plugging in the values:

$$
\alpha = \frac{0 - \pi}{3} = -\frac{\pi}{3} \text{ rad/sec}^2
$$

Numerically:

$$
\boxed{\alpha \approx -1.0472 \text{ rad/sec}^2}
$$

The negative sign indicates deceleration.

Step 4: Find the Total Angular Displacement (θ) During Deceleration

Using:

$$
\theta = \omega_0 t + \frac{1}{2} \alpha t^2
$$

Calculations:

$$
\theta = \pi \times 3 + \frac{1}{2} \times (-\pi/3) \times 3^2
$$

$$
\theta = 3\pi + \frac{1}{2} \times (-\pi/3) \times 9
$$

$$
\theta = 3\pi - \frac{1}{2} \times \pi \times 3
$$

$$
\theta = 3\pi - \frac{3\pi}{2} = \frac{6\pi}{2} - \frac{3\pi}{2} = \frac{3\pi}{2}
$$

Numerically:

$$
\theta \approx \frac{3 \times 3.1416}{2} \approx 4.7124 \text{ radians}
$$

Understanding the Physical Significance of Results

Angular Displacement and its Meaning

- The disk rotates through approximately 4.71 radians during the 3 seconds of deceleration.
- To relate this to revolutions:

$$
\text{Revolutions} = \frac{\theta}{2\pi} \approx \frac{4.7124}{6.2832} \approx 0.75
$$

- The disk completes roughly 0.75 revolutions (or 3/4 of a full turn) before coming to rest.

Implications of Angular Deceleration

- The angular acceleration of approximately $-1.0472 \text{ rad/sec}^2$ indicates a uniform deceleration.
- The initial angular velocity was about 3.14 rad/sec, which is a moderate spin.
- The stopping time of 3 seconds is relatively quick, reflecting a sizable deceleration.

Additional Calculations and Applications

Calculating the Torque Required to Stop the Disk

Assuming the disk has a moment of inertia I, the torque τ needed for deceleration is:

$$
\tau = I \alpha
$$

- For a solid disk:

$$
I = \frac{1}{2} m r^2

\]

Where:
- m is the mass of the disk,
- $r = 25\text{ cm} = 0.25\text{ m}$.

Note: Without the mass, the exact torque cannot be computed, but this formula illustrates the relationship.

Real-World Applications of Rotational Deceleration

Mechanical and Automotive Engineering

- Braking systems in vehicles rely on controlled deceleration, akin to the disk slowing down.
- Engineers must calculate the torque and angular deceleration to design effective brakes.

Industrial Machinery

- Rotating equipment often requires controlled stopping to prevent damage or accidents.
- Understanding angular deceleration helps optimize safety and efficiency.

Sports Equipment

- Rotational dynamics are crucial in designing sports gear, such as spinning discs, wheels, or machinery involved in sports.

Summary of Key Points

1. Initial angular velocity (ω_0) is approximately 3.1416 rad/sec.

2. The disk decelerates uniformly at about -1.0472 rad/sec^2.

3. The total angular displacement during deceleration is approximately 4.71 radians, or about 0.75 revolutions.

4. The calculations demonstrate the application of rotational kinematic equations to real-world physics problems.

5. Understanding these principles is essential in engineering, physics, and various technological applications involving rotational motion.

Conclusion

Analyzing the deceleration of a spinning disk involves understanding angular velocity, angular acceleration, and displacement. Starting from the initial rpm, converting to radians per second, and applying kinematic equations allows us to determine the angular deceleration and the total rotation during the stopping process. Such analyses are fundamental in designing mechanical systems, safety mechanisms, and understanding natural and engineered rotational phenomena. Mastery of these concepts provides a solid foundation in rotational dynamics, bridging theoretical physics with practical engineering solutions.

Frequently Asked Questions

What is the initial angular velocity of the disk in radians per second?

The initial angular velocity ω_0 = (30 revolutions per minute) × (2π radians / 1 revolution) × (1 minute / 60 seconds) = π radians per second.

How do you calculate the angular deceleration of the spinning disk?

Using the formula $\alpha = (\omega_{final} - \omega_{initial}) / time$, where $\omega_{final} = 0$ (since it comes to a stop), so $\alpha = (0 - \pi) / 3$ seconds = $-\pi/3$ rad/sec².

What is the initial angular velocity of the disk in degrees per second?

Initial angular velocity in degrees per second = (30 rpm) × (360° / 1 revolution) × (1 minute / 60 seconds) = 180° per second.

How can we determine the initial tangential speed at the rim of the disk?

Tangential speed $v = r \times \omega$, where r = 25 cm = 0.25 m and ω = π rad/sec, so v = 0.25 m × π rad/sec ≈ 0.785 m/sec.

What is the total angular displacement of the disk during the 3 seconds it takes to stop?

Using $\theta = \omega_{initial} \times t + 0.5 \times \alpha \times t^2$, $\theta = \pi \times 3 + 0.5 \times (-\pi/3) \times 9 = 3\pi - (0.5 \times \pi/3 \times 9) = 3\pi - (1.5\pi) = 1.5\pi$ radians, approximately 4.712 radians.

Why is understanding angular deceleration important in rotational dynamics?

Angular deceleration helps in analyzing how quickly a rotating object slows down, which is essential in designing brakes, rotors, and understanding energy dissipation in mechanical systems.

Additional Resources

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Introduction

Imagine a spinning disk, perhaps part of a high-precision machinery or a fan blade in an industrial setting. It begins its motion at a lively pace, rotating at 30 revolutions per minute (rpm), and then, due to some external influence—like friction or braking—it gradually comes to a complete stop over a span of just three seconds. From a physics perspective, this scenario offers a fascinating glimpse into angular motion, rotational dynamics, and the forces involved in bringing a rotating object to rest.

In this article, we'll explore how to analyze such a situation by calculating key quantities such as angular acceleration, initial and final angular velocities, and the total angular displacement during deceleration. These calculations not only deepen our understanding of rotational physics but also have practical implications in engineering, design, and safety systems.

Let's embark on this analytical journey by first understanding the fundamental concepts involved.

Understanding the Scenario

Before delving into calculations, it’s essential to interpret the given data accurately:

- Radius of the disk (r): 25 centimeters (cm), which converts to 0.25 meters (m).
- Initial rotational speed (ω_0): 30 revolutions per minute (rpm).
- Time to stop (t): 3 seconds.
- Final rotational speed (ω): 0 rpm (since it comes to rest).

Our goal is to find the angular acceleration (α) during the deceleration phase, and possibly the total angular displacement in radians or revolutions.

Converting Units and Understanding Angular Quantities

Since the initial rotational speed is given in revolutions per minute, and the time in seconds, we must convert rpm to radians per second for consistency and clarity in calculations.

Revolutions per minute to radians per second:

- 1 revolution = 2π radians.
- 1 rpm = (2π radians) / 60 seconds.

Calculating:

$$ \omega_0 = 30\, \text{rpm} \times \frac{2\pi \, \text{radians}}{60\, \text{seconds}} = 30 \times \frac{\pi}{30} = \pi \, \text{rad/sec} $$

Thus,

$$ \omega_0 = \pi \, \text{rad/sec} \approx 3.1416\, \text{rad/sec} $$

The final angular velocity:

$$ \omega = 0\, \text{rad/sec} $$

Calculating Angular Acceleration

Since the disk slows down uniformly from its initial angular velocity to zero over 3 seconds, we can assume a constant angular acceleration (or deceleration). The basic kinematic equation for angular motion:

$$ \omega = \omega_0 + \alpha t $$

Given:

$$ \omega = 0 $$
$$ \omega_0 = \pi $$
$$ t = 3\, \text{sec} $$

Rearranged:

$$ 0 = \pi + \alpha \times 3 $$
$$ \Rightarrow \alpha = - \frac{\pi}{3} \, \text{rad/sec}^2 $$

The negative sign indicates deceleration (opposite the direction of initial motion).

Final result:

$$ \boxed{
\alpha = -1.0472\, \text{rad/sec}^2
} $$

This constant angular acceleration tells us how quickly the disk's rotation slows down.

Determining Angular Displacement

Next, we might be interested in the total angular distance the disk travels during deceleration. The angular displacement (θ) in radians can be found via:

$$ \theta = \omega_0 t + \frac{1}{2} \alpha t^2 $$

Plugging in the known values:

$$ \theta = \pi \times 3 + \frac{1}{2} \times (-\frac{\pi}{3}) \times 3^2 $$

Calculations:

- First term:

$$ \pi \times 3 = 3\pi \approx 9.4248\, \text{radians} $$

- Second term:

$$ \frac{1}{2} \times (-\frac{\pi}{3}) \times 9 = -\frac{\pi}{6} \times 9 = -\frac{9\pi}{6} = -\frac{3\pi}{2} \approx -4.7124\, \text{radians} $$

Adding:

$$ \theta \approx 9.4248 - 4.7124 = 4.7124\, \text{radians} $$

Expressed in revolutions, since 1 revolution = 2π radians:

$$ \text{Revolutions} = \frac{4.7124}{2\pi} \approx \frac{4.7124}{6.2832} \approx 0.75 $$

Summary:

- The disk rotates approximately 0.75 revolutions during the 3 seconds of deceleration.

Interpreting the Physical Meaning

The analysis reveals that:

- The disk slows down from about 3.1416 rad/sec to zero in 3 seconds.
- It experiences a constant angular deceleration of approximately -1.0472 rad/sec^2.
- During this deceleration, it covers about 4.71 radians, or roughly three-quarters of a revolution.

This understanding can be critical in designing braking systems or safety mechanisms that rely on predictable deceleration patterns.

Practical Applications and Engineering Significance

Understanding the angular dynamics of a spinning disk has broad applications:

- Brake System Design: Engineers can determine the required braking torque to stop rotating machinery within a specified time, ensuring safety and efficiency.
- Rotational Kinematics in Robotics: Precise calculations allow for controlled deceleration in robotic joints or rotating platforms.
- Material Stress Analysis: Knowing the angular acceleration helps predict stresses on the disk, critical for material selection and durability analysis.
- Gyroscopic Devices: Stability and control depend on understanding how angular velocities change over time.

Additional Considerations

While the calculations here assume ideal conditions—constant angular acceleration and no external forces beyond braking—real-world scenarios can involve additional factors:

- Friction and Air Resistance: These forces can cause non-uniform deceleration.
- Variable Braking Torque: In some systems, braking force may change over time.
- Material Deformation: Rapid deceleration could induce stresses leading to deformation or failure.

Designers and engineers must weigh these factors when translating theoretical calculations into practical applications.

Conclusion

The case of a 25 cm radius disk spinning at 30 rpm and coming to a complete stop over 3 seconds exemplifies the beauty of rotational physics. By converting units, applying fundamental kinematic equations, and interpreting results, we gain a comprehensive understanding of the angular deceleration involved.

Such analyses are not just academic exercises; they underpin the safety, efficiency, and reliability of countless mechanical systems in our daily lives—from industrial machinery to consumer electronics. As rotational dynamics continue to be a critical aspect of engineering, mastering these concepts remains essential for innovation and safety in a rotating world.

Final Thoughts

In essence, the key takeaway is that even seemingly simple scenarios, like a spinning disk slowing down, involve rich physics that can be precisely quantified. Whether designing a braking system or understanding the motion of celestial bodies, the principles demonstrated here serve as foundational tools for scientists and engineers alike.

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