A Diverging Lens Has A Focal Length Of Magnitude 13 Cm . At What Object Distance Will The Magnification

A Diverging Lens Has A Focal Length Of Magnitude 13 Cm . At What Object Distance Will The Magnification is a question that delves into the fundamental principles of optics, specifically concerning diverging lenses. Understanding the relationship between focal length, object distance, and magnification is essential for students, educators, and professionals working with optical devices. In this article, we will explore the concepts behind diverging lenses, derive the relevant formulas, and work through the problem step-by-step to determine the object distance for a given magnification.

Understanding Diverging Lenses

What Is a Diverging Lens?

A diverging lens, also known as a concave lens, is a type of optical device that causes parallel rays of light to spread apart or diverge after passing through it. These lenses are thinner at the center than at the edges and are commonly used in applications like eyeglasses for myopia correction, optical instruments, and laser beam expanders.

Properties of Diverging Lenses

- Focal length (f): Negative for diverging lenses, indicating the direction of the focus.
- Image formation: Produces virtual, erect, and diminished images for objects placed beyond the focal point.
- Refraction: The bending of light occurs inward, causing divergence.

Fundamental Concepts and Formulas

Lens Formula

The relationship between the object distance (u), the image distance (v), and the focal length (f) is given by the lens formula:

$$
\frac{1}{f} = \frac{1}{v} + \frac{1}{u}
$$

Where:
- f is the focal length of the lens.

- v is the image distance from the lens.
- u is the object distance from the lens (measured from the lens to the object).

Note: Sign conventions are vital:
- For diverging lenses, f is negative.
- Object and image distances are positive if they are real and on the same side as the incoming light.

Magnification Formula

Magnification (M) describes how much larger or smaller the image appears compared to the object:

$$
M = \frac{v}{u}
$$

- Positive magnification indicates an erect image.
- Negative magnification indicates an inverted image.

Given Data and Objective

The problem states:
- The focal length magnitude of the diverging lens is 13 cm, so $f = -13\,cm$.
- The goal is to find the object distance u when the magnification M is known or specified.

However, in the original problem statement, the specific value of magnification is not provided, implying that the task involves understanding how to find u for a given M. For the purpose of this explanation, assume we are asked to determine the object distance for a specific magnification value, say $M = 0.5$, but the process applies to any value.

Deriving the Relationship Between Object Distance, Focal Length, and Magnification

Step 1: Express v in terms of u and M

From the magnification formula:

$$
v = M \times u
$$

Since the lens formula involves v, substitute $v = M u$:

$$
\frac{1}{f} = \frac{1}{v} + \frac{1}{u} = \frac{1}{M u} + \frac{1}{u}
$$

Step 2: Simplify to find u

Combine the right side over a common denominator:

$$
\frac{1}{f} = \frac{1 + M}{M u}
$$

Rearranged:

$$
u = \frac{(1 + M)}{(1/f) \times M}
$$

Given that $f = -13\,cm$:

$$
u = \frac{(1 + M)}{(1/ -13) \times M} = \frac{(1 + M) \times (-13)}{M}
$$

Simplify:

$$
u = -13 \times \frac{(1 + M)}{M}
$$

This formula allows us to compute the object distance u for any given magnification M.

Calculating Object Distance for a Specific Magnification

Suppose we want to find the object distance when the magnification $M = 0.5$.

$$
u = -13 \times \frac{1 + 0.5}{0.5} = -13 \times \frac{1.5}{0.5} = -13 \times 3 = -39\,cm
$$

The negative sign indicates that the object is on the same side of the lens as the incoming light, which is the usual convention for real objects.

Interpretation:
- The object is located 39 cm in front of the diverging lens.
- The image formed will be virtual, upright, and diminished, consistent with the properties of diverging lenses.

General Approach to Find Object Distance for Any Magnification

To determine the object distance for a diverging lens with a known focal length and desired magnification, follow these steps:

1. Identify the focal length f (negative for diverging lenses).

2. Determine the desired magnification M.

3. Use the formula:
   $$
   u = -f \times \frac{1 + M}{M}
   $$

4. Calculate u to find the object distance.

Note: The sign conventions are critical. Negative u indicates the object is in front of the lens, consistent with real object placement.

Practical Applications and Examples

Example 1: Magnification of 1 (Equal Size Image)

Suppose the magnification is 1, meaning the image is the same size as the object:

$$
u = -13 \times \frac{1 + 1}{1} = -13 \times 2 = -26\,cm
$$

The object should be placed 26 cm in front of the diverging lens to produce a virtual, upright, same-sized image.

Example 2: Magnification of 2 (Enlarged Image)

For a magnification of 2:

$$
u = -13 \times \frac{1 + 2}{2} = -13 \times \frac{3}{2} = -19.5\,cm
$$

The object must be approximately 19.5 cm from the lens.

Conclusion

Understanding the relationships between focal length, object distance, and magnification is essential in optical physics, especially when working with diverging lenses. By applying the lens formula alongside the magnification equation, one can precisely determine the object distance for any desired magnification. These calculations are crucial not only in theoretical physics but also in designing optical devices, correcting vision problems, and various scientific applications.

Remember, sign conventions and careful substitution are vital to arriving at correct solutions. Whether you're an educator guiding students or an engineer designing optical systems, mastering these fundamental concepts ensures accurate and effective use of diverging lenses in a wide range of contexts.

Frequently Asked Questions

What is the focal length of the diverging lens in the problem?

The diverging lens has a focal length of magnitude 13 cm, which means its focal length is −13 cm.

What is the formula relating object distance, image distance, and focal length for a diverging lens?

The lens formula is 1/f = 1/do + 1/di, where f is the focal length, do is the object distance, and di is the image distance.

How is magnification (m) related to object and image distances in a diverging lens?

Magnification is given by m = −di / do, where di is the image distance and do is the object distance.

If the magnification is known, how can we find the object distance for a diverging lens?

Using m = −di / do and the lens formula, you can solve for do once you have m and the image distance di.

What assumptions are made in calculating object distance for a diverging lens?

Assumptions include the lens being thin, the sign conventions are followed (f negative for diverging), and the given data are accurate.

Is the focal length negative or positive for a

diverging lens?

The focal length is negative for a diverging lens, so in this case, f = -13 cm.

Can we determine the object distance without knowing the image distance?

No, we need either the image distance or the magnification value to determine the object distance directly.

What additional information is needed to find the magnification in this problem?

You need either the image distance or the specific magnification value to calculate the object distance.

How does the sign of magnification relate to the nature of the image in a diverging lens?

A negative magnification indicates an erect virtual image, which is typical for diverging lenses.

What is the general approach to solve for object distance given focal length and magnification?

Use the magnification formula m = -di / do and the lens formula to express di in terms of do, then solve for do.

Additional Resources

A Diverging Lens Has A Focal Length Of Magnitude 13 Cm: At What Object Distance Will The Magnification?

Introduction

Optical phenomena involving diverging lenses are fundamental in understanding how images are formed and manipulated in various scientific and technological applications. Among the key parameters in lens optics are the focal length, object distance, image distance, and magnification. When dealing with a diverging lens, these parameters interrelate in a manner that often requires careful analytical consideration. Specifically, the question of at what object distance will the magnification be a certain value hinges on understanding the principles governing lens behavior.

In this detailed exploration, we focus on a diverging lens with a magnitude of 13 cm focal length—meaning its focal length is -13 cm—and analyze how to determine the object distance corresponding to a specified magnification. This investigation is not only academically interesting but also practically relevant across fields such as optical device design, microscopy, and vision correction.

Understanding Diverging Lenses and Focal Length

The Nature of Diverging Lenses

Diverging lenses, often concave in shape, spread incident light rays outward. Their primary characteristic is that they produce virtual, upright, and reduced images of objects located in front of them. These lenses are commonly used in eyeglasses for nearsightedness, optical devices requiring divergence, and in complex optical systems.

Sign Convention and Focal Length

- Focal Length (f): The distance from the lens to the focal point, where parallel rays converge or appear to diverge from.
- Sign Convention: In optics, the sign of the focal length depends on the lens type:
- Concave (Diverging) Lens: Focal length f is negative.

Given the problem states magnitude 13 cm, the focal length is:

$$ f = -13 \text{ cm} $$

This negative value confirms the diverging nature of the lens.

Fundamental Lens Formula and Magnification

Lens Equation

The relationship between object distance (u), image distance (v), and focal length (f) is given by the thin lens equation:

$$
\frac{1}{f} = \frac{1}{v} + \frac{1}{u}
$$

Where:

- u = object distance (positive if object is on the same side as incoming light)
- v = image distance (positive if real image, negative if virtual)

Magnification

Magnification (M) relates the size of the image to the size of the object:

$$
M = \frac{\text{height of image}}{\text{height of object}} = - \frac{v}{u}
$$

The negative sign indicates that real images are inverted relative to the object, while virtual images are upright.

The Core Problem: Determining Object Distance for a Given Magnification

Suppose we are asked to find the object distance (u) at which the magnification (M) attains a specific value. For the purposes of this analysis, let's consider an example where the magnification is $M = 2$. That is, the image is twice as large as the object, and upright (since for diverging lenses, virtual images are upright).

This choice simplifies calculations and demonstrates the general procedure. However, the methodology applies broadly to any specified magnification.

Analytical Approach

Step 1: Express v in Terms of u

From the magnification formula:

$$
M = - \frac{v}{u}
$$

Rearranged:

$$
v = - M u
$$

For $M = 2$:

$$
v = - 2 u
$$

The negative sign indicates the virtual nature of the image.

Step 2: Apply the Lens Equation

Substituting $v = - 2 u$ into the lens equation:

$$
\frac{1}{f} = \frac{1}{v} + \frac{1}{u}
$$

$$
\frac{1}{-13} = \frac{1}{-2 u} + \frac{1}{u}
$$

Simplify the right side:

$$
\frac{1}{-13} = - \frac{1}{2 u} + \frac{1}{u}
$$

Expressing the right side over a common denominator:

$$
- \frac{1}{2 u} + \frac{1}{u} = - \frac{1}{2 u} + \frac{2}{2 u} = \frac{1}{2
$$

u}
\]

So,

\[
\frac{1}{-13} = \frac{1}{2 u}
\]

Cross-multiplied:

\[
2 u = -13
\]

\[
u = - \frac{13}{2} = -6.5 \text{ cm}
\]

The negative object distance indicates that the object must be located on the same side as the incoming light and virtual image formation, consistent with the properties of a diverging lens.

Generalized Solution for Any Magnification

The above procedure can be generalized for any magnification \(M \):

\[
v = - M u
\]

Plug into the lens equation:

\[
\frac{1}{f} = \frac{1}{v} + \frac{1}{u} = - \frac{1}{M u} + \frac{1}{u} = \frac{1}{u} \left(1 - \frac{1}{M} \right)
\]

Expressed as:

\[
\frac{1}{f} = \frac{1}{u} \left(\frac{M - 1}{M} \right)
\]

Rearranged:

\[
u = \frac{(M - 1)}{M} \times f
\]

Substituting \(f = -13 \text{ cm} \):

\[
u = \frac{(M - 1)}{M} \times (-13)
\]

This formula allows calculation of the object distance for any desired magnification M.

Practical Implications and Examples

Desired Magnification	Calculation of u	Object Distance u (cm)	Nature of the Image
$M = 2$	$u = \frac{(2 - 1)}{2} \times -13$	$u = -6.5$	Virtual, upright, magnified
$M = 0.5$	$u = \frac{(0.5 - 1)}{0.5} \times -13$	$u = -13$	Virtual, upright, reduced
$M = 1$	$u = 0$	Zero (not physically meaningful)	No image formation

Note: The physical significance depends on the specific lens setup; negative u indicates the object is positioned on the same side as the incoming light, which is typical for virtual images in diverging lenses.

Broader Context and Significance

Understanding the relationship between focal length, object distance, and magnification in diverging lenses is critical in multiple domains:

- Optical Instrument Design: Precise control over image size and position enables the development of microscopes, telescopes, and corrective eyewear.

- Medical Imaging: Virtual images formed by diverging lenses are exploited in devices like endoscopes.

- Educational Demonstrations: Visualizing how divergence affects magnification enhances comprehension of fundamental optics principles.

Furthermore, the methodology exemplified in this analysis underscores the importance of sign conventions and theoretical models in predicting real-world optical behavior.

Limitations and Assumptions

While the calculations presented are accurate within the framework of the thin lens approximation, real-world factors such as lens thickness, aberrations, and material imperfections can influence actual image formation. Moreover, the assumption of ideal point objects and monochromatic light simplifies the analysis but may not fully reflect practical scenarios.

Conclusion

In the context of a diverging lens with a focal length of magnitude 13 cm, determining the object distance for a given magnification involves a systematic application of the lens formula and magnification relation. The key takeaway is that the object distance can be directly calculated using the derived formula:

$$
u = \frac{(M - 1)}{M} \times f
$$

with appropriate sign conventions applied. For example, to achieve a magnification of 2, the object must be approximately 6.5 cm on the same side as the incoming light, resulting in a virtual, upright, and magnified image. Such insights are vital for both theoretical understanding and practical implementation in optical systems.

In summary, the interplay of focal length, object distance, and magnification in diverging lenses reveals nuanced behaviors essential for advancing optical science and technology. Proper comprehension of these relationships empowers scientists, engineers, and educators to harness the power of light in innovative and effective ways.

A Diverging Lens Has A Focal Length Of Magnitude 13 Cm At What Object Distance Will The Magnification

A Diverging Lens Has A Focal Length Of Magnitude 13 Cm At What Object Distance Will The Magnification

Related Articles

- A Project Requires An Initial Investment Of $200,000 And Is Expected To Generate The Following Net Cash
- Begin By Graphing The Square Root Function,f(x)=x. Then, Use Transformations Of This Graph To Graph The
- Because The P-value Is ____ Than The Significance Level 0.05, There ____ Sufficient Evidence To Support

Back to Home