A Line In The Coordinate Plane Has A Slope Of $4,$ And A Distance Of 1 Unit From The Origin. Find The

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Introduction

In coordinate geometry, understanding the properties of lines—such as their slope, position relative to the origin, and distance from specific points—is fundamental. These concepts are not only essential for solving geometric problems but also form the basis for more advanced topics like conic sections, transformations, and analytic geometry applications.

Today, we focus on a particular problem involving a line with a given slope, a specified distance from the origin, and the task of determining certain characteristics of that line. Specifically, the problem states:

"A line in the coordinate plane has a slope of 4, and its shortest distance from the origin is 1 unit. Find the..."

While the statement ends abruptly, typical problems of this nature often ask for:

- The equation of the line
- The coordinates of the points where the line intersects axes
- The perpendicular distance from a point to the line
- The equations of lines parallel or perpendicular to this line at certain distances

In this comprehensive article, we'll explore the methods to find the complete equation of such a line, interpret the geometric meaning of the given data, and demonstrate step-by-step solutions with detailed explanations.

Understanding the Problem Components

Before delving into calculations, it's critical to understand each element of the problem:

1. The Slope of the Line ($m=4$)

- The slope indicates the steepness and direction of the line.
- A slope of 4 means that for every 1 unit increase in x, the y value increases by 4 units.
- The line is inclined at an angle whose tangent is 4.

2. Distance from the Origin (1 Unit)

- The shortest distance from a point (here, the origin $(0,0)$) to a line is measured along a perpendicular segment.
- The problem states that this minimum distance is 1 unit.
- This implies the line is located at a specific position in the plane, offset from the origin but with the same slope.

Mathematical Foundations

Equation of a Line in Slope-Intercept Form

The general form of a line with slope m is:

\[

Y = MX + B
\]

WHERE:

- M IS THE SLOPE,
- B IS THE Y-INTERCEPT.

DISTANCE FROM A POINT TO A LINE

FOR A LINE IN THE FORM:

\[
AX + BY + C = 0
\]

THE PERPENDICULAR DISTANCE FROM A POINT (X_0, Y_0) TO THE LINE IS GIVEN BY:

\[
D = \FRAC{|AX_0 + BY_0 + C|}{\SQRT{A^2 + B^2}}
\]

IN OUR CASE, SINCE THE DISTANCE FROM THE ORIGIN $(0,0)$ TO THE LINE IS 1, AND THE LINE PASSES THROUGH SOME POSITION WITH SLOPE 4, THIS RELATION WILL BE KEY TO FINDING THE PRECISE LINE EQUATION.

DERIVING THE EQUATION OF THE LINE

STEP 1: EXPRESS THE LINE IN STANDARD FORM

BECAUSE THE LINE HAS A SLOPE OF 4, ITS EQUATION IN SLOPE-INTERCEPT FORM IS:

\[
Y = 4X + B
\]

REARRANGED INTO STANDARD FORM:

\[
4X - Y + B = 0
\]

OR EQUIVALENTLY:

\[
A X + B Y + C = 0
\]

WITH:

\[
A = 4, \QUAD B = -1, \QUAD C = B
\]

NOTE: TO AVOID CONFUSION, LET'S USE CONSISTENT NOTATION. LET'S DENOTE THE INTERCEPT AS B, SO THE LINE'S STANDARD FORM IS:

\[

$$
4x - y + c = 0
$$

Step 2: Use the distance condition

The perpendicular distance from the origin $(0,0)$ to the line is:

$$
d = \frac{|c|}{\sqrt{4^2 + (-1)^2}} = \frac{|c|}{\sqrt{16 + 1}} = \frac{|c|}{\sqrt{17}}
$$

Given that $d = 1$, we have:

$$
\frac{|c|}{\sqrt{17}} = 1 \implies |c| = \sqrt{17}
$$

Thus, the possible values of c are:

$$
c = \pm \sqrt{17}
$$

Step 3: Write the equations of the lines

Therefore, the two possible lines are:

$$
\boxed{
\text{Line 1: } 4x - y + \sqrt{17} = 0
}
$$
and
$$
\boxed{
\text{Line 2: } 4x - y - \sqrt{17} = 0
}
$$

These lines each have a slope of 4 and are located at a distance of 1 unit from the origin, positioned on either side of the origin.

Visualizing and Interpreting the Results

Geometric interpretation:

- The two lines are parallel, both with slope 4.
- They are separated by a perpendicular distance of 2 units (since the distance between two parallel lines $ax + by + c_1=0$ and $ax + by + c_2=0$ is:

$$
\frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}
$$

which here is:

\[

$$
\frac{|\sqrt{17} - (-\sqrt{17})|}{\sqrt{17}} = \frac{2\sqrt{17}}{\sqrt{17}} = 2
$$

- The distance from the origin $(0,0)$) to each line is 1 unit, consistent with the problem statement.

Additional properties:

- Both lines pass through points that can be obtained by substituting values of x or y, but since the problem might ask for the explicit equations, the two lines are fully characterized now.

Extending the Problem: Finding Specific Points on the Line

Suppose you are asked to find:

- The points where the line intersects the axes,
- Or the equations of the perpendicular lines passing through the origin at certain distances.

Let's explore how to find the intercepts for each line.

Intercepts of the lines:

For line 1: $4x - y + \sqrt{17} = 0$

- x-intercept: set $y=0$:

$$
4x + \sqrt{17} = 0 \Rightarrow x = -\frac{\sqrt{17}}{4}
$$

- y-intercept: set $x=0$:

$$
- y + \sqrt{17} = 0 \Rightarrow y = \sqrt{17}
$$

Similarly, for line 2: $4x - y - \sqrt{17} = 0$

- x-intercept:

$$
4x - 0 - \sqrt{17} = 0 \Rightarrow x = \frac{\sqrt{17}}{4}
$$

- y-intercept:

$$
- y - \sqrt{17} = 0 \Rightarrow y = -\sqrt{17}
$$

These points give the exact locations where each line crosses the axes, providing a complete geometric picture.

Additional Considerations and Related Problems

1. Finding the Equation of the Line Given the Distance and Slope

We've shown that, given the slope and distance, the line's equation can be derived systematically. This process involves:

- Converting the slope-intercept form to standard form.
- Using the distance formula to find the constant term (c).

2. Distance Between Two Parallel Lines

Understanding the separation between the two lines with equations:

$$
4x - y + \sqrt{17} = 0
$$
and
$$
4x - y - \sqrt{17} = 0
$$

is important in applications involving parallel lines, such as in construction, navigation, or design.

3. Perpendicular and Parallel Lines

- The line perpendicular to the given line has slope $ -\frac{1}{4} $.
- Equations of lines parallel or perpendicular can be derived similarly using the provided methods.

4. Application in Real-World Contexts

Problems like these are foundational in fields such as:

- Engineering design, where distances from a point to a line are critical.
- Computer graphics, for determining object placement.
- Navigation and pathfinding algorithms, involving shortest distances.

Summary of Key Results

- The lines with slope 4 and a distance of 1 from the origin are given by:

$$
\boxed{
4x - y \pm \sqrt{17} = 0
}
$$

- These lines are parallel, separated by a distance of 2 units, and both are exactly 1 unit from the origin.

- The intercepts are:

- For the line $4x - y + \sqrt{17} = 0$:

$$
x = -\frac{\sqrt{17}}{4}, \quad y = \sqrt{17}
$$

- For the line $4x - y - \sqrt{17} = 0$:

$$
x = \frac{\sqrt{17}}{4}, \quad y = -\sqrt{17}
$$

\]

Final Remarks

Understanding how to determine a line

Frequently Asked Questions

A line in the coordinate plane has a slope of 4 and is exactly 1 unit away from the origin. How can we find the equations of such lines?

Use the point-to-line distance formula combined with the line equation y = 4x. Set up the distance formula and solve for the line's intercept to find the equations satisfying the distance condition.

What is the general form of the line equations with slope 4 that are 1 unit away from the origin?

The lines can be expressed as y = 4x + c, where c satisfies the condition that the perpendicular distance from the origin to the line is 1. Solving for c using the distance formula yields c = ±⍰ (1 + 16).

How do you derive the specific equations of lines with slope 4 at a distance of 1 from the origin?

Start with y = 4x + c and apply the point-to-line distance formula: distance = |c| / ⍰ (1 + m²). Set this equal to 1 and solve for c: |c| / ⍰ 17 = 1, giving c = ±⍰ 17.

What are the equations of the two lines with slope 4 that are 1 unit away from the origin?

They are y = 4x + ⍰ 17 and y = 4x − ⍰ 17.

If a line has slope 4 and is 1 unit from the origin, what is the perpendicular distance formula applied to find its equations?

The perpendicular distance from the origin to the line y = 4x + c is |c| / ⍰ (1 + (4)²) = |c| / ⍰ 17. Setting this equal to 1 allows us to solve for c.

What is the significance of the value ⍰ 17 in determining the lines' equations?

⍰ 17 is the value of c when the line's perpendicular distance from the origin is exactly 1, derived from the distance formula with slope 4.

Are the two lines symmetric with respect to the origin or y-axis?

They are symmetric with respect to the origin because their equations differ only by the sign of c, indicating they are equidistant from the origin on opposite sides.

How does changing the distance from the origin affect the value of c in the line equations?

The absolute value of c is directly proportional to the distance from the origin, specifically $c = \pm$distance ⍰ (1 + m^2). Increasing the distance increases |c| accordingly.

Additional Resources

A Line In The Coordinate Plane Has A Slope Of 4, And A Distance Of 1 Unit From The Origin. Find The — this intriguing problem combines fundamental concepts of coordinate geometry, including the equation of a line, the distance from a point to a line, and the properties of slopes. It offers a comprehensive exploration of how these elements interact and how to approach such problems systematically. In this review, we will delve into the underlying concepts, step-by-step solutions, and broader implications of solving this type of problem, providing clarity and insight for students and enthusiasts alike.

Understanding the Problem

The Core Elements

The problem states:

- The line has a slope of 4.
- The distance from the line to the origin (0,0) is 1 unit.
- The goal is to find the equation of the line.

At first glance, the problem appears straightforward, but it involves multiple layers of understanding:

- The geometric interpretation of the slope.
- The algebraic form of the line.
- The concept of the perpendicular distance from a point to a line.

This combination makes it an excellent example for illustrating the process of translating geometric conditions into algebraic equations.

Why This Problem Is Important

Understanding how to determine a line given certain conditions is fundamental in coordinate geometry. Such problems are common in exams and real-world applications like navigation, computer graphics, and physics. They reinforce skills in:

- Deriving line equations from slope and point conditions.
- Using the distance formula in the context of lines.
- Combining algebra and geometry to solve complex conditions.

The Equation of a Line with a Given Slope

General Form

The slope-intercept form of a line is:

$$ y = mx + c $$

where:

- m is the slope (given as 4 in this case),
- c is the y-intercept, which we need to determine.

Point-Slope Form

Alternatively, since the slope is known, the line can be represented as:

$$y - y_1 = m(x - x_1)$$

for any point (x_1, y_1) on the line. But because the line's exact position isn't specified beyond the distance from the origin, it's more practical to express the line as a general form:

$$y = 4x + c$$

which allows us to incorporate the distance condition directly.

The Perpendicular Distance From a Point to a Line

The Distance Formula

The perpendicular distance d from a point (x_0, y_0) to a line $Ax + By + C = 0$ is given by:

$$d = \frac{|A x_0 + B y_0 + C|}{\sqrt{A^2 + B^2}}$$

In our problem:

- The point is the origin: $(0,0)$.
- The line is in slope-intercept form, but to use the distance formula conveniently, we convert it to standard form.

Converting the Line Equation to Standard Form

Given $y = 4x + c$, rearranged as:

$$4x - y + c = 0$$

Here:

- $A = 4$,
- $B = -1$,
- $C = c$.

Applying the Distance Formula

Since the distance from the origin to the line is 1:

$$1 = \frac{|A \cdot 0 + B \cdot 0 + C|}{\sqrt{A^2 + B^2}} = \frac{|c|}{\sqrt{4^2 + (-1)^2}} = \frac{|c|}{\sqrt{16 + 1}} = \frac{|c|}{\sqrt{17}}$$

Thus,

$$|c| = \sqrt{17}$$

which implies:

$$c = \pm \sqrt{17}$$

Final Equations of the Line

Two Possible Lines

Given the two values of c, the equations are:

1. $y = 4x + \sqrt{17}$
2. $y = 4x - \sqrt{17}$

Both lines are at a perpendicular distance of 1 from the origin, and both have slope 4.

Geometric Interpretation

- The lines are symmetric with respect to the origin.
- They are parallel, each at a distance of 1 unit from the origin.
- The positive c line lies above the origin, while the negative c line lies below.

Broader Insights and Applications

Features of the Solution

- The problem demonstrates how algebraic and geometric perspectives combine seamlessly.
- It illustrates the importance of converting between forms of line equations to facilitate different calculations.
- Highlights the significance of the distance formula in geometric problems.

Pros and Cons

Pros	Cons
Reinforces understanding of line equations and distance	Requires multiple steps, which can be error-prone
Demonstrates symmetry in geometric configurations	Assumes familiarity with converting forms and formulas
Applicable to various geometric and real-world problems	The problem's simplicity may not capture complex scenarios

Variations and Extensions

- Find the line with a different distance from the origin.
- Consider a different slope and determine possible lines.
- Extend to three dimensions, finding planes at a certain distance from a point.

Step-by-Step Summary

1. Identify the line's slope: $m = 4$.
2. Express the line in standard form: $4x - y + c = 0$.
3. Apply the distance formula for the origin to find c:

$$
1 = \frac{|c|}{\sqrt{17}} \Rightarrow c = \pm \sqrt{17}
$$

4. Write the final equations:

\[
y = 4x + \sqrt{17}
\]

and

\[
y = 4x - \sqrt{17}
\]

Conclusion

This problem exemplifies the power of algebraic methods in solving geometric configurations in the coordinate plane. By understanding the relationship between the slope, the standard form of a line, and the distance from a point to a line, students can approach similar problems with confidence. The key takeaways include mastering the conversion between line forms, applying the distance formula correctly, and interpreting the geometric significance of algebraic solutions. With these techniques, one can tackle a wide array of problems involving lines, distances, and slopes, forming a solid foundation in coordinate geometry.

Final Remarks

Coordinate geometry remains an essential part of mathematics education, with broad applications beyond the classroom. Problems like "A line in the coordinate plane has a slope of 4, and a distance of 1 unit from the origin" foster critical thinking, combining algebraic manipulation with geometric intuition. Mastery of these concepts not only prepares students for exams but also enhances spatial reasoning skills applicable in engineering, physics, computer graphics, and many other fields.

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