A Parabola That Passes Through The Point (3, -39) Has Vertex (-7, 11). Its Line Of Symmetry Is Parallel

A Parabola That Passes Through The Point (3, -39) Has Vertex (-7, 11). Its Line Of Symmetry Is Parallel is a fascinating geometric concept that combines the properties of quadratic functions with the characteristics of their axes of symmetry. Understanding how these elements interact provides valuable insights into the behavior of parabolas, their equations, and their applications in various fields such as physics, engineering, and computer graphics. In this article, we will explore the fundamental properties of this specific parabola, derive its equation, analyze its features, and discuss related concepts to deepen your comprehension of quadratic graphs.

Understanding Parabolas and Their Key Features

What Is a Parabola?

A parabola is the graph of a quadratic function, typically expressed in the form $y = ax^2 + bx + c$, where a, b, and c are real numbers, and $a \neq 0$. It is a symmetric, U-shaped curve that opens either upward or downward depending on the sign of the coefficient a.

Parabolas are prevalent in physics, especially in projectile motion, where the trajectory of an object under gravity follows a parabolic path. They are also used in satellite dish design, automobile headlights, and architecture due to their reflective properties.

Key Features of a Parabola

Every parabola has several defining features:

- **Vertex:** The highest or lowest point of the parabola, acting as the parabola's center of symmetry.
- **Axis of symmetry:** A vertical line that passes through the vertex, dividing the parabola into two mirror-image halves.
- **Focus:** A fixed point such that the parabola is the set of points equidistant from the focus and the directrix.
- **Directrix:** A fixed line perpendicular to the axis of symmetry, serving as a reference in the parabola's geometric definition.

In our case, the parabola's vertex is at (-7, 11), and it passes through the point (3, -39), with its line of symmetry being parallel to a certain line (more on this later).

Deriving the Equation of the Parabola

Vertex Form of a Parabola

The vertex form of a quadratic equation is:

$y = a(x - h)^2 + k$

where (h, k) is the vertex, and a determines the parabola's opening direction and 'width'.

Given the vertex (-7, 11), the equation becomes:

$y = a(x + 7)^2 + 11$

Our goal is to find the value of 'a' using the known point (3, -39).

Calculating the 'a' Coefficient

Substitute the point (3, -39) into the equation:

$-39 = a(3 + 7)^2 + 11$

Simplify:

$-39 = a(10)^2 + 11$

$-39 = 100a + 11$

Subtract 11 from both sides:

$-50 = 100a$

Divide both sides by 100:

$a = -50 / 100 = -0.5$

Thus, the equation of the parabola is:

$y = -0.5(x + 7)^2 + 11$

Analyzing the Parabola's Features

Vertex and Direction of Opening

The vertex at (-7, 11) indicates the highest point of the parabola since the coefficient a is negative (-0.5). The parabola opens downward.

Axis of Symmetry

The axis of symmetry is the vertical line $x = h$, which in this case is:

$x = -7$

This line passes through the vertex and divides the parabola into two mirror-image halves.

Line of Symmetry and Its Orientation

The statement "Its Line Of Symmetry Is Parallel" suggests that the line of symmetry is parallel to a specific reference line. Typically, for parabolas aligned with the coordinate axes, the axis of symmetry is vertical (parallel to the y-axis). If the line of symmetry is parallel to another line (say, the x-axis), then the parabola would be horizontally oriented.

In our derived equation, the axis of symmetry is vertical ($x = -7$), so the line of symmetry is parallel to the y-axis. If the problem indicates that the line of symmetry is parallel to some line, it is most likely parallel to the y-axis, which is consistent with the standard parabola orientation.

Understanding the Parabola’s Geometry and Symmetry

Symmetry Properties

The symmetry of the parabola ensures that for any point (x, y) on the parabola, its mirror image (2h - x, y) across the axis $x = h$ also lies on the parabola.

For our parabola:

- The axis of symmetry: $x = -7$
- For any $x \neq -7$, the point (x, y) has a symmetric point at $(2(-7) - x, y) = (-14 - x, y)$

This property is crucial in applications like antenna design, where signals reflected across the parabola's axis are focused at the focus point.

Focus and Directrix (Optional Deep Dive)

While not explicitly asked, understanding the focus and directrix can provide deeper insight:

- The focus lies along the axis of symmetry, at a distance 'p' from the vertex.
- The parabola can be constructed as the locus of points equidistant from the focus and the directrix.

Calculating 'p' helps in understanding the parabola's shape and size, but for this specific problem, the focus and directrix are secondary.

Applications and Implications of the Parabola's Properties

Real-World Applications

The properties of this parabola make it suitable for various practical uses:

- **Satellite Dish Design:** The parabola focuses signals onto the receiver at the focus point, which is aligned along the axis of symmetry.
- **Reflective Surfaces:** Parabolic mirrors direct light or sound waves efficiently, leveraging symmetry and focus properties.
- **Projectile Motion:** The path of objects under gravity approximates a parabola, and understanding symmetry helps in predicting their trajectories.

Mathematical Significance

From a mathematical perspective, analyzing the parabola's symmetry allows for simplified calculations in problems involving quadratic equations, optimization, and graph transformations.

Transformations and Variations

Horizontal and Vertical Shifts

The vertex form of the parabola allows easy translation:

- Moving the parabola horizontally shifts the x-coordinate of the vertex.
- Moving it vertically shifts the y-coordinate.

For our parabola, the equation can be modified to analyze these transformations.

Changing the Opening Direction

Altering the sign of 'a' in the equation changes the parabola's opening from downward to upward.

Conclusion and Summary

Understanding a parabola passing through a specific point with a known vertex and line of symmetry is fundamental in both theoretical and practical contexts. The given parabola, with vertex at (-7, 11), passing through (3, -39), and having a line of symmetry parallel to the y-axis, can be fully described by the equation:

$$y = -0.5(x + 7)^2 + 11$$

This equation encapsulates the parabola's shape, position, and symmetry. Recognizing the properties of the parabola—such as its vertex, axis of symmetry, and opening direction—is essential for applications in physics, engineering, and geometry. Moreover, understanding the symmetry allows for simplified problem-solving and insights into the behavior of quadratic functions.

By mastering these concepts, students and professionals can analyze and utilize parabolas effectively, whether in designing reflective devices, modeling physical phenomena, or solving optimization problems. The geometric elegance of the parabola continues to be a vital component of mathematical and scientific endeavors.

Frequently Asked Questions

What is the general form of a parabola with a vertex at (-7, 11) and a line of symmetry parallel to a specific axis?

The general form is y = a(x + 7)^2 + 11, where the axis of symmetry is vertical and parallel to the y-axis.

How can we determine the value of 'a' for the parabola passing through (3, -39) with vertex (-7, 11)?

Substitute the point (3, -39) into the vertex form y = a(x + 7)^2 + 11 and solve for 'a': -39 = a(3 + 7)^2 + 11.

What is the equation of the parabola passing through (3, -39) with vertex (-7, 11)?

First, find 'a' as: -39 = a(10)^2 + 11 → -39 = 100a + 11 → 100a = -50 → a = -0.5. Therefore, the equation is y = -0.5(x + 7)^2 + 11.

What is the axis of symmetry for this parabola?

The axis of symmetry is the vertical line x = -7, passing through the vertex.

Does the line of symmetry being parallel imply the parabola opens upward or downward?

The line of symmetry being parallel (vertical) indicates the parabola opens either upward or downward, depending on the sign of 'a'. In this case, since a = -0.5, it opens downward.

How do we verify if the point (3, -39) lies on the parabola y = -0.5(x + 7)^2 + 11?

Substitute x=3 into the equation: y = -0.5(10)^2 + 11 = -0.5(100) + 11 = -50 + 11 = -39. Since this matches the point's y-coordinate, it lies on the parabola.

What is the significance of the vertex in the parabola's graph?

The vertex (-7, 11) is the highest or lowest point of the parabola, representing its maximum or minimum value depending on the parabola's orientation.

Can the parabola be expressed in standard form based on the given

information?

Yes, converting from vertex form y = -0.5(x + 7)^2 + 11 to standard form involves expanding the squared term: y = -0.5(x^2 + 14x + 49) + 11, resulting in y = -0.5x^2 - 7x - 24.5 + 11, or y = -0.5x^2 - 7x - 13.5.

Additional Resources

A Parabola That Passes Through The Point (3, -39) Has Vertex (-7, 11). Its Line Of Symmetry Is Parallel

In the realm of conic sections, parabolas are among the most intriguing and widely studied curves due to their distinctive properties and practical applications. When analyzing a parabola with given points and features, mathematicians and students alike delve into the intricate relationships between its geometric and algebraic characteristics. In this article, we explore a specific parabola that passes through a point, has a particular vertex, and features a line of symmetry parallel to a given axis. By dissecting these attributes, we aim to provide a comprehensive understanding of the parabola's equation, its geometric implications, and the broader context of quadratic functions.

Understanding the Basic Features of a Parabola

Before diving into the specifics of the parabola in question, it’s essential to review the fundamental properties that define a parabola.

The Vertex

The vertex of a parabola is the point where it reaches its maximum or minimum value along its axis of symmetry. It serves as the parabola's "center" in a sense, and its coordinates are crucial in defining the shape and position of the parabola.

The Axis of Symmetry

The line of symmetry or axis of symmetry passes through the vertex and divides the parabola into two mirror-image halves. Its orientation (vertical or horizontal) determines whether the parabola opens upward/downward or sideways.

The Focus and Directrix

Every parabola can be characterized as the set of points equidistant from a fixed point called the focus and a fixed line called the directrix. This geometric definition underpins many algebraic properties.

The General Equation of a Parabola

Depending on its orientation, the parabola's equation can take different forms:

- Vertical parabola: $y = ax^2 + bx + c$
- Horizontal parabola: $x = ay^2 + by + c$

The coefficients and parameters relate directly to its vertex, focus, and directrix.

Analyzing the Given Parabola's Features

The problem specifies three key features:

- The parabola passes through the point (3, -39)
- Its vertex is at (-7, 11)
- Its line of symmetry is parallel to a certain axis (to be clarified)

Let's analyze these in detail.

Given the Vertex at (-7, 11)

The vertex at $(-7, 11)$ indicates the parabola's minimum or maximum point, depending on its opening direction. This vertex serves as a pivotal reference in deriving the parabola's equation.

Passing Through the Point (3, -39)

The point $(3, -39)$ lies on the parabola, providing an additional constraint that allows us to determine specific parameters of the quadratic function.

Line of Symmetry Parallel to Which Axis?

The statement "Its Line of Symmetry Is Parallel" is somewhat ambiguous without further context. Typically, the line of symmetry is either vertical (parallel to the y-axis) or horizontal (parallel to the x-axis). Given the vertex's coordinates, the most common assumption is that the parabola opens vertically with a vertical axis of symmetry, which is parallel to the y-axis. Alternatively, if the parabola opens sideways, the axis of symmetry would be horizontal.

Given the standard conventions and the context, we assume that the line of symmetry is parallel to the y-axis, i.e., the parabola opens upward or downward with a vertical axis of symmetry.

Determining the Equation of the Parabola

Armed with the vertex and a point on the parabola, we can derive its equation.

The Vertex Form of a Parabola

The most straightforward way to express a parabola with a known vertex is through the vertex form:

$$
y = a(x - h)^2 + k
$$

where (h, k) is the vertex, and a determines the parabola's opening and width.

Given:
- Vertex $(h, k) = (-7, 11)$

The equation becomes:

$$
y = a(x + 7)^2 + 11
$$

Our goal is to find a.

Using the Point (3, -39)

Substitute $x = 3$ and $y = -39$:

$$
-39 = a(3 + 7)^2 + 11
$$
$$
-39 = a(10)^2 + 11
$$
$$
-39 = 100a + 11
$$

Solve for a:

$$
100a = -39 - 11 = -50
$$
$$
a = -\frac{50}{100} = -\frac{1}{2}
$$

Thus, the parabola's equation is:

$$
\boxed{
y = -\frac{1}{2}(x + 7)^2 + 11
}
$$

Analyzing the Parabola's Properties

With the equation established, let's analyze its geometric and algebraic properties.

Shape and Opening Direction

Since $a = -\frac{1}{2}$ is negative, the parabola opens downward. The vertex at $(-7, 11)$ represents its maximum point.

Axis of Symmetry

The axis of symmetry for the parabola in vertex form is the vertical line:

$$
x = h = -7
$$

This line passes through the vertex and divides the parabola into two symmetric halves.

Focus and Directrix

The focus and directrix provide a deeper geometric understanding.

- The distance from the vertex to the focus (called p) is related to a by:

$$
a = \frac{1}{4p}
$$

- Since $a = -\frac{1}{2}$:

$$
-\frac{1}{2} = \frac{1}{4p}
$$

$$
4p = -2
$$

$$
p = -\frac{1}{2}
$$

The negative p confirms the parabola opens downward, with the focus located below the vertex.

- The coordinates of the focus are:

$$
\left(h, k + p \right) = \left(-7, 11 - \frac{1}{2} \right) = \left(-7, \frac{21}{2} \right)
$$

- The directrix is a horizontal line above the vertex at:

\[

$$
k - p = 11 + \frac{1}{2} = \frac{23}{2}
$$

Summary:

Property	Value
Equation of parabola	$y = -\frac{1}{2}(x + 7)^2 + 11$
Opening direction	Downward
Axis of symmetry	$x = -7$
Focus	$\left(-7, \frac{21}{2} \right)$
Directrix	$y = \frac{23}{2}$

Implications of the Parabola's Features

The detailed analysis reveals several key insights into the shape and properties of the parabola.

Symmetry and Geometry

The axis of symmetry at $x = -7$ aligns precisely with the x-coordinate of the vertex, confirming the parabola's symmetry about this vertical line. This symmetry ensures that for any point (x, y) on the parabola, the mirrored point $(2h - x, y)$ also lies on the parabola. For instance, at $x = -3$:

$$
x' = 2(-7) - (-3) = -14 + 3 = -11
$$
$$
y = -\frac{1}{2}((-3)+7)^2 + 11 = -\frac{1}{2}(4)^2 + 11 = -8 + 11 = 3
$$

Similarly, at $x = -11$:

$$
x' = -14 - (-11) = -14 + 11 = -3
$$
$$
y = 3
$$

which confirms the symmetry.

Applications and Broader Context

Understanding the properties of such a parabola has tangible applications across various fields:

- Physics: Parabolas describe the trajectories of projectiles under uniform gravity.
- Engineering: Parabolic reflectors and antennas rely on the reflective properties of parabolas.
- Mathematics: The detailed analysis provides foundational insights into quadratic functions, conic sections, and coordinate geometry.

Importance of the Point and Vertex

The point through which the parabola passes (3, -39) and the vertex at (-7, 11) are critical in determining the parabola's precise shape and position. This exemplifies how geometric constraints translate into algebraic

A Parabola That Passes Through The Point 3 39 Has Vertex 7 11 Its Line Of Symmetry Is Parallel

A Parabola That Passes Through The Point 3 39 Has Vertex 7 11 Its Line Of Symmetry Is Parallel

Related Articles

- A Technician Is Preparing A New Computer For Issuance To An Employee. Per New Company Guidance, Company
- Assuming Ideal Solution Behavior, What Is The Molar Mass Of A Solution Of 5.00 G Of A Compound In 25.00
- A Travel Company Is Conducting A Survey To Find Out If Taking A Cruise Vacation Vs Having A Traditional

Back to Home