A Particle Is Moving With The Given Data. Find The Position Of The Particle.a) A(t) = T2 - 9t + 5, S(0)

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Understanding the motion of a particle involves analyzing its displacement, velocity, and acceleration over time. Given the acceleration function, $A(t) = t^2 - 9t + 5$, along with the initial position $S(0)$, the task is to determine the position function $S(t)$ for the particle. This process involves integrating the acceleration function to obtain velocity, then integrating velocity to find displacement or position, while applying initial conditions for specific constants of integration.

In this article, we will explore step-by-step how to determine the position $S(t)$ of the particle from the given acceleration data. We will review the fundamental relationships between acceleration, velocity, and displacement, perform necessary integrations, and interpret the results to understand the particle's motion comprehensively.

Understanding the Relationship Between Acceleration, Velocity, and Position

Fundamental Concepts

Before delving into calculations, it is essential to understand the basic kinematic relationships:

- Acceleration ($A(t)$): The rate of change of velocity with respect to time.

- Velocity ($V(t)$): The rate of change of position with respect to time.

- Position ($S(t)$): The location of the particle at any given time.

Mathematically, these relationships are expressed as:

1. $A(t) = \frac{dV(t)}{dt}$
2. $V(t) = \frac{dS(t)}{dt}$

From these, it follows that:

- To find $V(t)$ when $A(t)$ is known, integrate $A(t)$ with respect to t.
- To find $S(t)$ when $V(t)$ is known, integrate $V(t)$ with respect to t.

Initial Conditions and Their Role

To fully determine the functions $V(t)$ and $S(t)$, initial conditions are necessary. The problem provides $S(0)$, the initial position at time $t = 0$. Usually, the initial velocity $V(0)$ is also needed, but in this scenario, it is not explicitly given. If $V(0)$ is known, it would serve as the constant of integration during the integration process.

In the absence of $V(0)$, we can proceed with a general solution that includes an arbitrary constant, which can later be specified if additional data becomes available.

Calculating the Velocity Function $V(t)$

Given Data and Integration

The acceleration function is:

$$A(t) = t^2 - 9t + 5$$

Since $A(t) = \frac{dV(t)}{dt}$, integrating both sides with respect to t:

$$
V(t) = \int A(t) dt + C_v
$$

where C_v is the constant of integration, representing the initial velocity $V(0)$.

Performing the Integration

Let’s integrate each term separately:

$$
V(t) = \int (t^2 - 9t + 5) dt + C_v
$$

$$
V(t) = \int t^2 dt - 9 \int t dt + 5 \int dt + C_v
$$

Calculating each integral:

- $\int t^2 dt = \frac{t^3}{3}$
- $\int t dt = \frac{t^2}{2}$
- $\int dt = t$

Putting it all together:

$$
V(t) = \frac{t^3}{3} - 9 \times \frac{t^2}{2} + 5t + C_v
$$

Simplify:

$$
V(t) = \frac{t^3}{3} - \frac{9t^2}{2} + 5t + C_v
$$

This expression gives the velocity at any time t, with C_v being an unknown constant that depends on initial velocity.

Determining the Position Function $S(t)$

Integration of Velocity

Since $V(t) = \frac{dS(t)}{dt}$, integrating $V(t)$ with respect to t:

$$
S(t) = \int V(t) dt + C_s
$$

where C_s is the constant of integration, which can be determined from initial position $S(0)$.

Substituting the expression for $V(t)$:

$$
S(t) = \int \left(\frac{t^3}{3} - \frac{9t^2}{2} + 5t + C_v \right) dt + C_s
$$

Break into parts:

$$
S(t) = \int \frac{t^3}{3} dt - \int \frac{9t^2}{2} dt + \int 5t dt + \int C_v dt + C_s
$$

Calculate each integral:

- $\int \frac{t^3}{3} dt = \frac{1}{3} \times \frac{t^4}{4} = \frac{t^4}{12}$
- $\int \frac{9t^2}{2} dt = \frac{9}{2} \times \frac{t^3}{3} = \frac{9}{2} \times \frac{t^3}{3} = \frac{9 t^3}{6} = \frac{3 t^3}{2}$
- $\int 5t dt = \frac{5 t^2}{2}$
- $\int C_v dt = C_v t$

Putting everything together:

$$
S(t) = \frac{t^4}{12} - \frac{3 t^3}{2} + \frac{5 t^2}{2} + C_v t + C_s
$$

Applying Initial Conditions

At $t=0$:

$$
S(0) = \text{initial position}
$$

Substitute $t=0$:

$$
S(0) = 0 - 0 + 0 + C_v \times 0 + C_s = C_s
$$

Thus:

$$
C_s = S(0)
$$

Similarly, if $V(0)$ is known, we can find C_v by evaluating $V(0)$:

$$
V(0) = \frac{0^3}{3} - \frac{9 \times 0^2}{2} + 5 \times 0 + C_v = C_v
$$

Therefore, $C_v = V(0)$.

In the absence of $V(0)$, the velocity and position functions contain these constants, which can be specified once initial velocity data is available.

Summary of the Position Equation

The general position function, incorporating initial position $S(0)$ and initial velocity $V(0)$, is:

$$
S(t) = \frac{t^4}{12} - \frac{3 t^3}{2} + \frac{5 t^2}{2} + V(0) t + S(0)
$$

This formula describes the particle's position at any time t, given the initial conditions.

Analyzing the Particle’s Motion

Impact of the Acceleration Function

The quadratic acceleration function $A(t) = t^2 - 9t + 5$ suggests that the particle's acceleration varies with time, initially possibly negative or positive depending on t, and changing direction at certain points.

To understand when the particle accelerates or decelerates, analyze the acceleration function:

- Find critical points where $A(t) = 0$:

$$
t^2 - 9t + 5 = 0
$$

Solve this quadratic:

$$
t = \frac{9 \pm \sqrt{81 - 20}}{2} = \frac{9 \pm \sqrt{61}}{2}
$$

which gives two roots. Between these roots, the acceleration's sign changes, indicating periods of acceleration and deceleration.

Velocity and Displacement Behavior

- The velocity function $V(t)$ is a cubic polynomial, with its shape influenced by the initial velocity $V(0)$. Its behavior (increasing or decreasing) depends on the sign and magnitude of the derivative $A(t)$.

- The position $S(t)$ reflects the combined effects of acceleration and velocity over time, revealing the particle's trajectory.

Conclusion

By integrating the given acceleration function and applying initial conditions, we derive the general expressions for the velocity and position of the particle:

$$
V(t) = \frac{t^3}{3} - \frac{9 t^2}{2} + 5 t + V(0)
$$

\[

S(t) = \frac{t^4}{12

Frequently Asked Questions

What is the given acceleration function of the particle?

The acceleration function is $A(t) = t^2 - 9t + 5$.

How do you find the velocity of the particle from the acceleration function?

Integrate the acceleration function with respect to time t to obtain the velocity function, often adding a constant of integration.

Given S(0), how can we determine the position of the particle at any time t?

First find the velocity function by integrating acceleration, then integrate the velocity function to get the position function S(t), using the initial position S(0) to find the constant of integration.

If initial position S(0) is given as a specific value, how does it influence the calculation of S(t)?

It provides the initial condition necessary to determine the constant of integration when integrating the velocity function to find the position function.

How do you compute the velocity function from the acceleration function $A(t) = t^2 - 9t + 5$?

Integrate A(t) with respect to t: $v(t) = \int(t^2 - 9t + 5)\, dt = (1/3)t^3 - (9/2)t^2 + 5t + C$, where C is the constant of integration.

What is the process to find the position function S(t) after obtaining the velocity v(t)?

Integrate v(t) with respect to t: $S(t) = \int v(t)\, dt$, then use the initial position S(0) to find the constant of integration.

How can we determine the constant of integration in the velocity and position functions?

Use initial conditions such as S(0) and v(0). For example, if S(0) is known, plug in t=0 to solve for the constants.

What is the significance of the initial position S(0) in solving this problem?

It sets the baseline from which the particle's position at any time t can be calculated, ensuring the solution matches initial conditions.

Once the position function S(t) is found, how do you interpret the particle's movement?

Analyze S(t) to understand how the particle's position changes over time, including points of maximum, minimum, or specific positions.

What are the key steps to find the position of the particle given A(t) and S(0)?

1. Integrate A(t) to find v(t) and determine C using initial velocity. 2. Integrate v(t) to find S(t) and use S(0) to find the constant. 3. Write the complete S(t) to describe the particle's position over time.

Additional Resources

Particle Motion Analysis: Determining the Position from Given Acceleration Data

Understanding the motion of particles is fundamental in physics and engineering, providing insights into how objects behave over time under various forces. When given specific data such as acceleration, the challenge lies in deducing the position of the particle at any given moment. In this comprehensive analysis, we will explore the process of finding the position of a particle, given its acceleration function $A(t) = t^2 - 9t + 5$, along with initial position data $S(0)$.

This article adopts an expert, detailed approach, breaking down each step into manageable parts, illustrating the mathematical principles involved, and providing a clear route from acceleration to position. Whether you're a student, educator, or enthusiast, this guide aims to deepen your understanding of particle kinematics and the calculus techniques employed.

Understanding the Given Data: Acceleration Function and Initial Position

The Acceleration Function $A(t) = t^2 - 9t + 5$

The core data provided is the acceleration of the particle as a function of time:

\[

A(t) = t^2 - 9t + 5
\]

This quadratic expression indicates that the particle's acceleration varies with time in a non-uniform manner. The key to understanding the particle's motion lies in integrating this acceleration to find velocity and position.

Initial Position \(S(0)\)

The problem states that the initial position \(S(0)\) is known, although its explicit value isn't provided in the snippet. Typically, initial position \(S(0) = s_0\) is crucial for determining the complete position function \(S(t)\).

Note: For the sake of completeness, we'll denote \(S(0) = s_0\), and the final position function will be expressed in terms of this initial data.

From Acceleration to Velocity: The Integration Process

Why Integrate Acceleration?

In kinematics, acceleration is the rate of change of velocity:

\[
A(t) = \frac{dV}{dt}
\]

Thus, to find velocity \(V(t)\), we integrate \(A(t)\) with respect to time:

\[
V(t) = \int A(t) \, dt + C_v
\]

where \(C_v\) is the constant of integration, determined by initial velocity \(V(0)\).

Calculating Velocity \(V(t)\)

Given:

\[

$$
A(t) = t^2 - 9t + 5
$$

Integrate term-by-term:

$$
V(t) = \int (t^2 - 9t + 5) \, dt + C_v
$$

$$
V(t) = \frac{t^3}{3} - \frac{9t^2}{2} + 5t + C_v
$$

The constant C_v is found using the initial velocity $V(0)$:

$$
V(0) = \left. \frac{t^3}{3} - \frac{9t^2}{2} + 5t + C_v \right|_{t=0} = 0 - 0 + 0 + C_v = C_v
$$

Hence:

$$
C_v = V(0)
$$

Summary:

$$
V(t) = \frac{t^3}{3} - \frac{9t^2}{2} + 5t + V(0)
$$

From Velocity to Position: The Integration Process

Why Integrate Velocity?

Position $S(t)$ is the integral of velocity:

$$
V(t) = \frac{dS}{dt}
$$

Therefore:

$$
S(t) = \int V(t) \, dt + C_s
$$

\]

where C_s is the constant of integration, determined by the initial position $S(0) = s_0$.

Calculating Position $S(t)$

Substituting the expression for $V(t)$:

$$
S(t) = \int \left(\frac{t^3}{3} - \frac{9t^2}{2} + 5t + V(0) \right) dt + C_s
$$

Break down the integral:

$$
S(t) = \int \frac{t^3}{3} \, dt - \int \frac{9t^2}{2} \, dt + \int 5t \, dt + \int V(0) \, dt + C_s
$$

Calculate each term:

1. $\int \frac{t^3}{3} \, dt = \frac{1}{3} \times \frac{t^4}{4} = \frac{t^4}{12}$

2. $\int \frac{9t^2}{2} \, dt = \frac{9}{2} \times \frac{t^3}{3} = \frac{9}{2} \times \frac{t^3}{3} = \frac{3}{2} t^3$

3. $\int 5t \, dt = \frac{5 t^2}{2}$

4. $\int V(0) \, dt = V(0) t$

Putting it all together:

$$
S(t) = \frac{t^4}{12} - \frac{3}{2} t^3 + \frac{5 t^2}{2} + V(0) t + C_s
$$

The constant C_s is determined by the initial position:

$$
S(0) = s_0 = \left. S(t) \right|_{t=0} = 0 - 0 + 0 + 0 + C_s \Rightarrow C_s = s_0
$$

Final Expression for Position:

$$
\boxed{
S(t) = \frac{t^4}{12} - \frac{3}{2} t^3 + \frac{5 t^2}{2} + V(0) t + s_0
}
$$

Additional Considerations and Practical Applications

Determining Initial Conditions

In real-world problems, initial velocity $V(0)$ can be given or measured. If not specified, assumptions may be made based on context, such as starting from rest ($V(0) = 0$). The initial position s_0 is often known or set as a reference point (e.g., $s_0 = 0$).

Analyzing the Motion

Once the position function $S(t)$ is established, various analyses can follow:

- Displacement over time: $S(t) - s_0$
- Velocity at any time: using the derived $V(t)$
- Acceleration profile: understanding how acceleration varies, identifying points where it changes direction or magnitude
- Critical points: solving for t where velocity or acceleration is zero to find turning points or points of inflection

Graphical Interpretation

Plotting the functions:

- Acceleration $A(t)$ as a quadratic
- Velocity $V(t)$ as a cubic
- Position $S(t)$ as a quartic

provides visual insight into the particle's motion, such as when it speeds up, slows down, or changes direction.

Summary and Final Remarks

In this detailed examination, we've demonstrated how to derive the position function $S(t)$ of a particle moving under a given acceleration profile $A(t) = t^2 - 9t + 5$, starting from known initial conditions.

Key steps include:

1. Integrating acceleration to find velocity:

$$
V(t) = \int A(t) \, dt + V(0) = \frac{t^3}{3} - \frac{9 t^2}{2} + 5 t + V(0)
$$

2. Integrating velocity to find position:

$$
S(t) = \int V(t) \, dt + s_0 = \frac{t^4}{12} - \frac{3}{2} t^3 + \frac{5 t^2}{2} + V(0) t + s_0
$$

3. Applying initial conditions to determine constants of integration.

This approach exemplifies how calculus serves as a powerful tool in analyzing motion, bridging the gap between instantaneous rates and overall displacement. With the explicit formula for $S(t)$, predictions about the particle’s position at any time become straightforward, enabling further analysis or practical application in physics, engineering, or related fields.

In conclusion, the process of determining a particle's position from acceleration data underscores the elegance of calculus in physics. Starting from the acceleration function, we systematically integrated to uncover velocity and, finally, position—each step illuminating the dynamic journey of the particle through space over time.

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