A Pencil At A Stationery Store Costs $1, And A Pen Costs $1.50. Shawn Spent $12 At The Store. He Bought

A Pencil At A Stationery Store Costs $1, And A Pen Costs $1.50. Shawn Spent $12 At The Store. He Bought a combination of pencils and pens. This scenario is a classic example of a linear algebra problem often encountered in mathematics and problem-solving exercises. Understanding how to determine the number of pencils and pens Shawn purchased requires analyzing the relationship between their prices and the total amount spent.

In this comprehensive guide, we will explore various methods to solve such problems, including setting up equations, using algebraic techniques, and applying logical reasoning. Whether you're a student preparing for exams or someone interested in practical applications of math, this article aims to provide clear explanations and step-by-step solutions.

Understanding the Problem

Before diving into calculations, let’s break down the problem:

- Price of one pencil = $1
- Price of one pen = $1.50
- Total amount spent by Shawn = $12
- Total number of items purchased = ? (unknown)
- Number of pencils purchased = ? (unknown)
- Number of pens purchased = ? (unknown)

The goal is to find the number of pencils and pens Shawn bought, given the total spent and the prices.

Setting Up the Equations

To solve this problem systematically, we can use algebra by defining variables:

- Let x = number of pencils Shawn bought
- Let y = number of pens Shawn bought

Based on the problem:

1. The total cost equation:

$1 \times x + 1.50 \times y = 12$

2. The total items purchased:

$x + y = \text{Total items}$

However, since the total number of items isn’t directly given, the primary focus is on the total cost equation, which relates x and y.

Formulating the Mathematical Equation

The key equation based on prices and total amount:

$$x + 1.5y = 12$$

We need to find integer solutions for x and y that satisfy this equation.

Solving the Equation

There are multiple approaches to solving this linear equation:

1. Algebraic Method

- Express x in terms of y:

$$x = 12 - 1.5y$$

Since x and y represent quantities of items purchased, both should be non-negative integers:

$$x \geq 0, \quad y \geq 0$$

Because x must be an integer, and $12 - 1.5y$ must be an integer, y must be chosen accordingly.

2. Finding Integer Solutions

- Note that $1.5y$ must be an integer; thus, y must be a multiple of 2 (since 1.5 times an even number yields an integer):

$$y = 0, 2, 4, 6, 8, 10, 12$$

- Now, substitute these values into the equation for x:

y	$x = 12 - 1.5y$	Is x an integer?	Valid?
0	$12 - 0 = 12$	Yes	Yes
2	$12 - 3 = 9$	Yes	Yes
4	$12 - 6 = 6$	Yes	Yes
6	$12 - 9 = 3$	Yes	Yes
8	$12 - 12 = 0$	Yes	Yes
10	$12 - 15 = -3$	No	No
12	$12 - 18 = -6$	No	No

- Since negative quantities aren't possible, the valid solutions are when y is 0, 2, 4, 6, or 8.

3. Valid Solutions Summary

y	x	Total items	Total spent	Explanation
0	12	12 pencils	$12 \times \$1 = \$12$	All pencils
2	9	9 pencils + 2 pens	$(9 \times \$1) + (2 \times \$1.50) = \$12$	Mix of pencils and pens
4	6	6 pencils + 4 pens	$\$6 + \$6 = \$12$	Mix of pencils and pens
6	3	3 pencils + 6 pens	$\$3 + \$9 = \$12$	Mix of pencils and pens
8	0	0 pencils + 8 pens	$0 + (8 \times \$1.50) = \$12$	All pens

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Interpreting the Results

From the solutions above, Shawn could have bought any of the following combinations:

- 12 pencils and 0 pens
- 9 pencils and 2 pens
- 6 pencils and 4 pens
- 3 pencils and 6 pens
- 0 pencils and 8 pens

Each of these combinations sums to a total of \$12, respecting the item count constraints.

Additional Considerations

1. Practical Scenario

In real-world shopping, Shawn might have purchased a mix of pencils and pens, not necessarily the maximum or minimum quantities. The problem allows for multiple solutions, and without further constraints, all these solutions are valid.

2. Choosing the Most Likely Scenario

If the problem asks for the most balanced or realistic purchase, you might consider the combination where the number of pencils and pens are close, or where the total number of items purchased is maximized or minimized.

3. Constraints and Limitations

- Non-negative integers: The number of items purchased cannot be negative or fractional.
- Budget constraint: Total cost must be exactly $12.
- Item availability: The store may have limited stock, influencing the possible combinations.

Practical Applications of the Problem

This problem isn't just a math exercise; it has real-world applications in areas such as:

- Budget planning
- Inventory management
- Cost analysis
- Purchase optimization

Understanding how to set up and solve such equations helps in making informed purchasing decisions and managing budgets effectively.

Conclusion

To summarize, Shawn could have bought several different combinations of pencils and pens, all totaling $12. The key steps involved:

- Recognizing the problem as a linear equation
- Defining variables for unknown quantities
- Formulating the equation based on prices
- Solving for integer solutions within constraints

By following this systematic approach, you can confidently solve similar problems involving budgets and item prices. Whether you're dealing with stationery, groceries, or any other shopping scenario, understanding how to set up and solve these equations is an essential skill.

FAQs

Q1: Can Shawn buy fractional quantities of pencils or pens?

A1: No, since you cannot buy a fraction of an individual item in a typical store, solutions must involve whole numbers.

Q2: What if the total amount spent was different, say $15?

A2: You would set up a similar equation with the new total and solve for integer solutions accordingly.

Q3: Are there other methods to solve such problems?

A3: Yes, besides algebra, methods like graphing, using systems of equations, or logical reasoning can be applied.

By mastering these techniques, you can approach and solve a variety of budget and item-count problems efficiently and accurately.

Frequently Asked Questions

How many pencils did Shawn buy if he only bought pencils and pens for $12?

Assuming Shawn bought only pencils and pens, let x be the number of pencils and y be the number of pens. The total cost equation is: 1x + 1.5y = 12. To find the maximum number of pencils, if he bought no pens (y=0), then x=12. Alternatively, if he bought only pens (x=0), then y=8. So, he could have bought any combination where 1x + 1.5y = 12 with x and y non-negative integers.

What is the minimum number of pens Shawn could have bought?

Since each pen costs $1.50, the minimum number of pens Shawn could buy is 0, which would mean he spent $12 on pencils only. The maximum number of pens he could buy with $12 is 8 (since 8 x $1.50 = $12), meaning he could have bought up to 8 pens, possibly with some pencils.

How many pencils and pens could Shawn have bought in total to spend exactly $12?

Shawn could have bought any combination where 1pencils + 1.5pen = 12, with non-negative integers for both. For example, 0 pencils and 8 pens, or 6 pencils and 4 pens, or 12 pencils and 0 pens, as long as the total cost sums to $12.

If Shawn bought 4 pens, how many pencils did he buy?

Cost of 4 pens = 4 x $1.50 = $6. Subtracting this from $12 leaves $6 for pencils. Since each pencil costs $1, he bought 6 pencils. So, total pencils = 6.

Is it possible for Shawn to have bought 10 pencils and no pens with $12?

Yes, since 10 pencils at $1 each total $10, which is less than $12. He could have spent $10 on pencils and still have $2 remaining, but since the question states he spent exactly $12, buying only 10 pencils is not possible unless he bought additional items or changed the total amount spent.

What is the maximum number of pencils Shawn could have bought if he bought only pencils and pens totaling $12?

The maximum number of pencils occurs when Shawn buys no pens. Since each pencil costs $1, he could have bought up to 12 pencils, spending exactly $12.

If Shawn bought 2 pens, how many pencils did he purchase?

Cost of 2 pens = 2 x $1.50 = $3. Subtracting from $12 leaves $9 for pencils. Since each pencil costs $1, he bought 9 pencils. So, total pencils = 9.

Can Shawn buy fractional quantities of pencils or pens?

No, since pencils and pens are typically sold in whole units, Shawn would buy whole numbers of each item. Therefore, the combinations must satisfy the equation with integer values for pencils and pens.

Additional Resources

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In the bustling aisles of a local stationery store, every item tells a story—be it the simple pencil or the sophisticated pen. Recently, Shawn stepped into such a store with a specific budget in mind. With $12 in his pocket, he aimed to purchase a combination of pencils and pens to meet his writing needs. His shopping trip, seemingly straightforward, actually presents a classic problem in algebra and problem-solving that can be unpacked to reveal the underlying mathematical relationships. In this article, we will explore the details of Shawn’s purchases, analyze the possible combinations of items he could have bought, and understand the broader implications of such shopping scenarios.

Understanding the Basic Pricing and Constraints

Before delving into the possible combinations Shawn could have purchased, it's essential to understand the basic parameters:

- Cost of a pencil: $1
- Cost of a pen: $1.50
- Total money spent: $12

From these, we can formulate the primary question: How many pencils and pens did Shawn buy to spend exactly $12? The problem hinges on two key variables:

- Let x be the number of pencils Shawn purchased.
- Let y be the number of pens Shawn purchased.

Given these, the total cost can be expressed mathematically as:

$$1 \times x + 1.5 \times y = 12$$

This equation forms the foundation of our analysis. Our goal is to find all integer solutions that satisfy this equation, considering that Shawn cannot buy a negative number of items.

Formulating the Mathematical Model

The core of the problem is a linear Diophantine equation:

$$x + 1.5y = 12$$

Since the number of items bought must be whole numbers (you can't purchase half a pencil or half a pen), both x and y should be non-negative integers.

To simplify calculations, multiply both sides of the equation by 2 to eliminate the decimal:

$$2x + 3y = 24$$

This transformed equation is easier to analyze because it involves only integers:

$$2x + 3y = 24$$

Our task is to find all non-negative integer pairs (x, y) satisfying this equation.

Finding All Possible Combinations

Let's explore the solutions systematically.

Step 1: Express x in terms of y:

$$
2x = 24 - 3y \implies x = \frac{24 - 3y}{2}
$$

For x to be an integer, $\frac{24 - 3y}{2}$ must be an integer, meaning the numerator must be even.

Step 2: Determine the parity condition:

Since 24 is even, $3y$ must also be even for the numerator to be even:

$$
3y \text{ is even} \implies y \text{ must be even}
$$

Because 3 times an even number is even, and 3 times an odd number is odd, the only way for $3y$ to be even is if y is even.

Step 3: List possible values of y:

- $y \geq 0$
- y is even
- $x \geq 0$

Now, determine the maximum value of y where x remains non-negative:

$$
x = \frac{24 - 3y}{2} \geq 0 \implies 24 - 3y \geq 0 \implies 3y \leq 24 \implies y \leq 8
$$

Since y is even and non-negative, possible values are:

$$
y = 0, 2, 4, 6, 8
$$

Step 4: Calculate corresponding x for each y:

- For $y=0$:

$$
x = \frac{24 - 3 \times 0}{2} = \frac{24}{2} = 12
$$

- For $y=2$:

$$
x = \frac{24 - 3 \times 2}{2} = \frac{24 - 6}{2} = \frac{18}{2} = 9
$$

- For $y=4$:

$$
x = \frac{24 - 3 \times 4}{2} = \frac{24 - 12}{2} = \frac{12}{2} = 6
$$

- For $y=6$:

$$
x = \frac{24 - 3 \times 6}{2} = \frac{24 - 18}{2} = \frac{6}{2} = 3
$$

- For $y=8$:

$$
x = \frac{24 - 3 \times 8}{2} = \frac{24 - 24}{2} = 0
$$

Step 5: Summarize the solutions:

Number of Pens (y)	Number of Pencils (x)	Total Cost Calculation
0	12	$12 \times \$1 = \$12$
2	9	$9 \times \$1 + 2 \times \$1.50 = \$9 + \$3 = \$12$
4	6	$6 + 4 \times 1.50 = \$6 + \$6 = \$12$
6	3	$3 + 6 \times 1.50 = \$3 + \$9 = \$12$
8	0	$0 + 8 \times 1.50 = \$12$

All these combinations satisfy Shawn's total expenditure precisely.

Interpreting the Results and Practical Implications

The mathematical analysis reveals multiple feasible shopping options for Shawn, each with its own practical considerations.

Option 1: Purchase 12 pencils only

- Items: 12 pencils
- Total cost: $12
- Use case: Ideal if Shawn needs only basic writing tools and prefers pencils for erasability.

Option 2: 9 pencils and 2 pens

- Items: 9 pencils, 2 pens
- Total cost: $9 \times \$1 + 2 \times \$1.50 = \$9 + \$3 = \$12$
- Use case: Suitable if Shawn desires a mix of tools—perhaps pens for formal writing and pencils for sketches or notes.

Option 3: 6 pencils and 4 pens

- Items: 6 pencils, 4 pens
- Total cost: \(6 + 4 \times \$1.50 = \$6 + \$6 = \$12\)
- Use case: A balanced choice for someone who prefers more pens but still values pencils.

Option 4: 3 pencils and 6 pens

- Items: 3 pencils, 6 pens
- Total cost: \(3 + 6 \times \$1.50 = \$3 + \$9 = \$12\)
- Use case: For individuals who favor pens over pencils, perhaps for professional notes or signatures.

Option 5: No pencils, 8 pens

- Items: 8 pens
- Total cost: \(8 \times \$1.50 = \$12\)
- Use case: For someone who exclusively prefers pens, maybe for a specific project or preference.

Broader Implications:

- Flexibility in shopping: The problem demonstrates that multiple combinations can satisfy the same budget constraint, highlighting the importance of flexibility in purchasing decisions.
- Mathematical modeling in real life: Such problems are common in budgeting, resource allocation, and decision-making scenarios, where understanding the relationships between quantities and costs can lead to optimal choices.
- Educational value: For students and learners, this problem serves as an excellent example of applying algebra to practical situations, reinforcing concepts like linear equations, integer solutions, and problem-solving strategies.

Additional Considerations and Variations

While the problem as posed is straightforward, several factors could alter the analysis:

- Quantity restrictions: If the store limited the maximum number of pens or pencils per customer, the solution set would need adjustment.
- Discounts or sales: Promotions could change the unit prices, adding complexity to the calculations.
- Budget flexibility: If Shawn had slightly more or less than $12, the range of possible combinations would expand or contract accordingly.
- Different item types: Introducing other stationery items like markers, erasers, or notebooks could create more complex systems of equations.

By understanding the core relationship between cost and quantity, shoppers and store managers alike can make informed decisions, optimize purchases, and plan budgets effectively.

Conclusion: The Power of Mathematics in Everyday Shopping

Shawn’s simple purchase scenario exemplifies the practical application of algebra and problem-solving in everyday life. By translating a real-world situation into a mathematical model, we can explore all the possible purchase options

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