A Plum Is Located At Coordinates (2.0 M,0,4.0 M). In Unit-vector Notation, What Is The Torque About The

A Plum Is Located At Coordinates (2.0 M,0,4.0 M). In Unit-vector Notation, What Is The Torque About The is a compelling question that bridges the concepts of physics, vector calculus, and mechanical analysis. Understanding how to determine torque in unit-vector notation is fundamental in physics and engineering, especially when analyzing rotational forces acting on objects in three-dimensional space. This article provides a comprehensive exploration of how to compute torque given a specific position vector, emphasizing clarity, detailed explanations, and practical applications.

Understanding the Concept of Torque

What Is Torque?

Torque, often denoted by the Greek letter τ (tau), is a measure of the rotational force applied to an object. It determines how effectively a force can cause an object to rotate about a specific axis or point. In simple terms, torque indicates the tendency of a force to produce rotational motion around a pivot point or axis.

Mathematically, torque is expressed as:

$$
\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}
$$

where:
- $\boldsymbol{\tau}$ is the torque vector.
- $\mathbf{r}$ is the position vector from the pivot point to the point where the force is applied.
- $\mathbf{F}$ is the force vector.
- $\times$ indicates the cross product.

Relevance in Physics and Engineering

Torque plays a vital role in:
- Mechanical systems (e.g., engines, gearboxes).
- Structural analysis (e.g., stress and strain in beams).
- Robotics and automation.
- Rotational dynamics and angular momentum studies.

Understanding how to compute torque using vector operations allows engineers and physicists to predict rotational behaviors accurately.

Position Vector and Coordinates

Given Coordinates of the Plum

The problem states that the plum is located at coordinates:

$$
(2.0\, \text{m}, 0, 4.0\, \text{m})
$$

This point can be represented as a position vector in three-dimensional space:

$$
\mathbf{r} = 2.0\, \hat{i} + 0\, \hat{j} + 4.0\, \hat{k}
$$

or simply:

$$
\mathbf{r} = 2.0\, \hat{i} + 4.0\, \hat{k}
$$

since the j component is zero.

Unit-Vector Notation

Unit-vector notation allows us to express vectors succinctly and conveniently:

$$
\mathbf{r} = 2.0\, \hat{i} + 4.0\, \hat{k}
$$

Here, $\hat{i}$, $\hat{j}$, and $\hat{k}$ are the unit vectors along the x, y, and z axes respectively.

Calculating the Torque in Unit-Vector Notation

Step 1: Identifying the Force Vector

To find the torque, we need the force vector $\mathbf{F}$. The problem statement, as given, does not specify the force. For a comprehensive understanding, we will consider a general force $\mathbf{F}$, which can be expressed in unit-vector notation as:

$$
\mathbf{F} = F_x\, \hat{i} + F_y\, \hat{j} + F_z\, \hat{k}
$$

where F_x, F_y, F_z are the components of the force.

Note: For specific calculations, you need the magnitude and direction of the force applied at the point. If not given, you can proceed with a general force or an example.

Step 2: Applying the Cross Product

The torque vector is calculated via the cross product:

$$
\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}
$$

Using the determinant method:

$$
\boldsymbol{\tau} =
\begin{vmatrix}
\hat{i} & \hat{j} & \hat{k} \\
r_x & r_y & r_z \\
F_x & F_y & F_z
\end{vmatrix}
$$

Given $\mathbf{r} = 2.0\, \hat{i} + 0\, \hat{j} + 4.0\, \hat{k}$, the components are:

$$
r_x = 2.0\, \text{m}, \quad r_y = 0, \quad r_z = 4.0\, \text{m}
$$

The force vector components are as previously defined:

$$
F_x, F_y, F_z
$$

The determinant expands as:

$$
\boldsymbol{\tau} = \hat{i} (r_y F_z - r_z F_y) - \hat{j} (r_x F_z - r_z F_x) + \hat{k} (r_x F_y - r_y F_x)
$$

Substituting the known components:

$$
\boldsymbol{\tau} = \hat{i} (0 \times F_z - 4.0 \times F_y) - \hat{j} (2.0 \times F_z - 4.0 \times F_x) + \hat{k} (2.0 \times F_y - 0 \times F_x)
$$

which simplifies to:

$$
\boldsymbol{\tau} = -4.0\, F_y\, \hat{i} - (2.0\, F_z - 4.0\, F_x)\, \hat{j} + 2.0\, F_y\, \hat{k}
$$

Step 3: Expressing the Final Torque Vector

The final expression of torque depends on the force components:

$$
\boxed{
\boldsymbol{\tau} = (-4.0\, F_y) \hat{i} - (2.0\, F_z - 4.0\, F_x) \hat{j} + (2.0\, F_y) \hat{k}
}
$$

Note: To obtain specific numerical values for torque, you must provide the force components \(F_x, F_y, F_z\). Otherwise, the expression remains in general form.

Special Cases and Practical Examples

Example 1: Force Along the x-direction

Suppose the force is purely along the x-axis:
\[
\mathbf{F} = 10\, \hat{i}\, \text{N}
\]
Then:
\[
F_x=10, \quad F_y=0, \quad F_z=0
\]
Substituting into the torque expression:
\[
\boldsymbol{\tau} = -4.0 \times 0\, \hat{i} - (2.0 \times 0 - 4.0 \times 10)\, \hat{j} + 2.0 \times 0\, \hat{k}
\]
\[
\boldsymbol{\tau} = 0\, \hat{i} - (-40)\, \hat{j} + 0\, \hat{k} = 40\, \hat{j}\, \text{Nm}
\]

This indicates a torque of 40 Nm about the y-axis, generated by a force along the x-direction at the given position.

Example 2: Force Along the y-direction

Suppose:
\[
\mathbf{F} = 15\, \hat{j}\, \text{N}
\]
then:
\[
F_x=0,\quad F_y=15,\quad F_z=0
\]
Substituting:
\[
\boldsymbol{\tau} = -4.0 \times 15\, \hat{i} - (2.0 \times 0 - 4.0 \times 0)\, \hat{j} + 2.0 \times 15\, \hat{k}
\]
\[
\boldsymbol{\tau} = -60\, \hat{i} - 0\, \hat{j} + 30\, \hat{k}
\]
This yields a torque vector:
\[

\boldsymbol{\tau} = -60\, \hat{i} + 0\, \hat{j} + 30\, \hat{k}
\]
which indicates torques about the x and z axes.

Applications of Calculating Torque in Engineering and Physics

Designing Mechanical Systems

Engineers use torque calculations to:
- Determine the required force to rotate machinery.
- Analyze the stress and load distribution on rotating parts.
- Design gears and pulleys for optimal torque transfer.

Robotics and Automation

Robotics engineers analyze torque vectors to:
- Control joint movements.
- Ensure stability and precision in robotic arms.
- Optimize energy consumption.

Structural Analysis

Structural engineers evaluate torque to:
- Assess the stability of beams and columns.
- Prevent structural failure due to rotational stresses.

Summary and Key Takeaways

- Torque is a vector quantity that depends on the position vector and the applied force.
- The position vector in unit-vector notation simplifies calculations in three dimensions.
- The cross product operation provides the direction and magnitude of torque.
- The

Frequently Asked Questions

What is the position vector of the plum located at coordinates (2.0 m, 0, 4.0 m) in unit-vector notation?

The position vector is $r = 2.0\,\hat{i} + 0\,\hat{j} + 4.0\,\hat{k}$ meters.

How do you compute the torque about a point given the position vector and a force?

Torque τ is calculated as $\tau = r \times F$, where r is the position vector from the point to the point of force application, and F is the force vector.

What additional information is needed to determine the torque about the point for the plum?

The force acting on the plum must be specified, including its magnitude and direction, to compute the torque.

If a force of 10 N acts vertically downward at the plum's location, what is the torque about the origin?

Assuming force $F = -10\,\hat{j}$ N, the torque $\tau = r \times F = (2.0\,\hat{i} + 0\,\hat{j} + 4.0\,\hat{k}) \times (-10\,\hat{j}) = (2.0\,\hat{i} + 0\,\hat{j} + 4.0\,\hat{k}) \times (-10\,\hat{j}) = (0\,\hat{i} + 20\,\hat{k} - 40\,\hat{i})$ N·m $= (-40\,\hat{i} + 20\,\hat{k})$ N·m.

How do unit vectors facilitate the calculation of torque in three-dimensional space?

Unit vectors $\hat{i}$, $\hat{j}$, and $\hat{k}$ provide a clear framework to express vectors in 3D space, allowing straightforward computation of cross products when calculating torque.

What is the significance of choosing the point about which torque is calculated?

The chosen point (origin or any other point) affects the position vector r and thus the calculated torque. The torque depends on the moment arm relative to that point.

Can the torque be zero at the given coordinates? Under what conditions?

Yes, if the force acts through the point's line of action passing through the pivot point, resulting in no perpendicular component, the torque can be zero.

How would the torque change if the force acting on the plum is applied at a different angle?

The magnitude and direction of the torque depend on the force vector's orientation; changing the angle alters the cross product, thus changing the torque.

In what practical scenarios might calculating torque about a

point be essential when dealing with objects like a plum?

Calculating torque helps determine stability, rotational effects, and potential tipping points of objects in mechanical and structural systems.

How do you convert a force vector into unit-vector notation for torque calculations?

Express the force in terms of its components along î, ĵ, and k̂, such as F = Fx î + Fy ĵ + Fz k̂, then use this in the cross product with the position vector to find torque.

Additional Resources

A Plum Is Located At Coordinates (2.0 M, 0, 4.0 M). In Unit-Vector Notation, What Is The Torque About The?

In the realm of physics, understanding the concept of torque is fundamental to analyzing rotational dynamics. The question, "A plum is located at coordinates (2.0 M, 0, 4.0 M). In unit-vector notation, what is the torque about the...", captures the essence of applying vector calculus to real-world problems. This investigation delves into the intricacies of calculating torque in a three-dimensional coordinate system, emphasizing the importance of precise notation, vector operations, and contextual interpretation.

Introduction to Torque and Its Significance

What is Torque?

Torque, often represented by the Greek letter τ, is a measure of the tendency of a force to rotate an object about a specific axis or point. Mathematically, torque is defined as the cross product of the position vector (r) and the force vector (F):

$$
\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}
$$

This vector quantity encapsulates both the magnitude and the direction of the rotational effect produced by a force.

Why Is Torque Important?

Understanding torque is crucial across various fields—from mechanical engineering to biomechanics. Whether analyzing the turning of a wrench, the rotation of celestial bodies, or the forces acting on a hanging object, torque provides insight into rotational equilibrium, stability, and motion.

Contextualizing the Problem

The question specifies that a "plum" (likely a point mass or object) is located at coordinates $(2.0\, \mathrm{m}, 0, 4.0\, \mathrm{m})$. To analyze the torque exerted about a particular point or axis, we need to:

1. Define the point about which the torque is calculated.
2. Determine the force acting on the object.
3. Express both the position vector and the force vector in unit-vector notation.

While the prompt doesn't specify the force acting on the plum or the axis about which the torque is calculated, typical scenarios involve gravitational force or a force applied at the point. For the sake of thoroughness, this investigation considers both the gravitational force and an arbitrary force to demonstrate the process.

Coordinate System and Vector Representation

The Position Vector

Given the coordinates:

$$
\mathbf{r} = (2.0\, \mathrm{m}, 0, 4.0\, \mathrm{m})
$$

Expressed in unit-vector notation:

$$
\mathbf{r} = 2.0\, \hat{\imath} + 0\, \hat{\jmath} + 4.0\, \hat{k}
$$

This vector points from the origin (or the chosen reference point) to the location of the plum.

The Force Vector

The force acting on the plum depends on the scenario:

- Gravitational Force: $\mathbf{F}_g = m \mathbf{g}$
- Applied Force: Could be any force vector $\mathbf{F}$

Assuming the mass of the plum is m, and gravity acts downward:

$$
\mathbf{g} = -9.8\, \hat{k}\, \mathrm{m/s^2}
$$

Hence,

$$
\mathbf{F}_g = m \times (-9.8\, \hat{k}) = -9.8 m\, \hat{k}
$$

Alternatively, if a generic force $\mathbf{F}$ is specified, it must be expressed similarly.

Calculating Torque in Unit-Vector Notation

The General Formula

The torque about the origin or any point O is given by:

$$
\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}
$$

where:

- $\mathbf{r}$ is the position vector from the reference point to the point of application
- $\mathbf{F}$ is the force vector

Example: Torque Due to Gravity

Assuming the force is gravity:

$$
\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}_g
$$

Substituting:

$$
\boldsymbol{\tau} = (2.0\, \hat{\imath} + 0\, \hat{\jmath} + 4.0\, \hat{k}) \times (-9.8 m\, \hat{k})
$$

Note that the force vector is along the $\hat{k}$ direction.

Cross Product Calculation

Recall the cross product rules:

$$
\hat{\imath} \times \hat{k} = -\hat{\jmath}
$$
$$
\hat{\jmath} \times \hat{k} = \hat{\imath}
$$
$$
\hat{k} \times \hat{k} = \mathbf{0}
$$

Calculating:

$$
\boldsymbol{\tau} = 2.0\, \hat{\imath} \times (-9.8 m\, \hat{k}) + 0\, \hat{\jmath} \times (-9.8 m\, \hat{k}) + 4.0\, \hat{k} \times (-9.8 m\, \hat{k})
$$

Simplifies to:

$$
\boldsymbol{\tau} = 2.0 \times -9.8 m (\hat{\imath} \times \hat{k}) + 0 + 0
$$

$$
\boldsymbol{\tau} = -19.6 m (\hat{\imath} \times \hat{k})
$$

$$
\boldsymbol{\tau} = -19.6 m (-\hat{\jmath}) = 19.6 m\, \hat{\jmath}
$$

Result:

$$
\boxed{
\boldsymbol{\tau} = 19.6 m\, \hat{\jmath}
}
$$

This indicates the torque vector points in the positive $\hat{\jmath}$ direction with magnitude proportional to the mass m.

Extending to General Forces and Points

Calculating Torque About Different Points

If torque is to be calculated about a point other than the origin—say, a pivot point $O = (x_0, y_0, z_0)$—the position vector becomes:

$$
\mathbf{r} = \mathbf{r}_{\text{point}} - \mathbf{r}_{\text{pivot}}
$$

Expressed as:

$$
\mathbf{r} = (x - x_0)\, \hat{\imath} + (y - y_0)\, \hat{\jmath} + (z - z_0)\, \hat{k}
$$

The subsequent calculation for torque proceeds similarly, with the adjusted position vector.

Considering Other Forces

Suppose an applied force $\mathbf{F} = F_x\, \hat{\imath} + F_y\, \hat{\jmath} + F_z\, \hat{k}$, the torque becomes:

$$
\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}
$$

Calculating the cross product involves the determinant:

$$
\boldsymbol{\tau} =
\begin{vmatrix}
\hat{\imath} & \hat{\jmath} & \hat{k} \\
x & y & z \\
F_x & F_y & F_z
\end{vmatrix}
$$

which expands to:

$$
\boldsymbol{\tau} = \left(y F_z - z F_y \right) \hat{\imath} - \left(x F_z - z F_x \right) \hat{\jmath} + \left(x F_y - y F_x \right) \hat{k}
$$

Practical Implications and Applications

Engineering Design

Understanding how to compute torque in vector notation aids in designing mechanical systems, from simple levers to complex robotic arms. Precise calculations ensure safety, efficiency, and functionality.

Physics Education

Explicitly expressing vectors in unit-vector notation enhances comprehension of spatial relationships and vector operations, foundational skills for physics students.

Real-World Problem Solving

In applied physics, such as analyzing forces on structures or celestial mechanics, the ability to manipulate vectors accurately determines the success of predictive models.

Limitations and Assumptions

- The analysis assumes the force acts at the specified point; in reality, forces may be distributed or vary with position.
- The calculation presumes a static scenario; dynamic effects like angular acceleration require additional considerations.
- The force vector's magnitude and direction are crucial; in absence of explicit information, assumptions are made based on typical scenarios like gravity.

Conclusion

Calculating the torque of an object located at a specific coordinate in three-dimensional space demands a rigorous understanding of vector operations and notation. By expressing both position and force vectors in unit-vector notation, the problem becomes a systematic exercise in vector calculus. In the case of the plum at \((2.0\, \mathrm{m}, 0, 4.0\, \mathrm{m})\) under the influence of gravity, the resulting torque vector points along the \(\hat{\jmath}\) axis with a magnitude scaled by the object's mass. Such analyses are not only academically engaging but also practically indispensable across numerous scientific and engineering disciplines.

References

- Serway, R. A., & Jewett, J. W. (2018). Physics for

[A Plum Is Located At Coordinates 2 0 M 0 4 0 M In Unit Vector Notation What Is The Torque About The]()

A Plum Is Located At Coordinates 2 0 M 0 4 0 M In Unit Vector Notation What Is The Torque About The

Related Articles

- As The Analysis Of Leadership Characteristics As Evolved, The Concept Of What Makes A Strong Leader Has
- A Federal Reserve Decision To Purchase Dollars By Selling Foreign Assets In The Foreign Exchange Market
- An Ant Needs To Travel Along A 20cm 20cm Cube To Get From Point A To Point B. What Is The Shortest Path

Back to Home