

A Random Sample Of Size 36 Is Taken From A Normal Population With A Mean Of 50 And A Standard Deviation

Introduction

A Random Sample Of Size 36 Is Taken From A Normal Population With A Mean Of 50 And A Standard Deviation is a fundamental concept in statistics, especially in the context of inferential statistics where we aim to draw conclusions about a population based on sample data. Understanding how to analyze such samples is crucial for researchers, data analysts, and students who wish to make informed decisions based on statistical evidence.

In this article, we will explore the statistical principles involved in analyzing a sample of size 36 drawn from a normal population with a known mean and standard deviation. We will discuss key concepts such as sampling distribution, standard error, confidence intervals, hypothesis testing, and the significance of sample size. Additionally, we will illustrate practical applications of these concepts through examples and detailed explanations to enhance your comprehension.

This comprehensive guide aims to equip you with the knowledge required to perform statistical analysis confidently and accurately, ensuring your interpretations and conclusions are both valid and reliable.

Understanding the Population and Sampling Framework

Population Characteristics

In our scenario:

- The population follows a normal distribution.
- The population mean (μ) is 50.
- The population standard deviation (σ) is known (though the exact value isn't specified here, typically it would be provided).

Knowing that the population is normally distributed simplifies many aspects of statistical analysis, as the properties of the normal distribution are well-understood and mathematically tractable.

Sample Size and Its Importance

The sample size (n) in this case is 36. Sample size plays a critical role in statistical inference:

- Larger samples tend to produce more accurate estimates of the population parameters.
- The Central Limit Theorem states that, regardless of the population distribution, the sampling distribution of the sample mean will tend to be normal if the sample size is sufficiently large. For

normally distributed populations, even smaller samples like 36 are adequate.

In particular, a sample size of 36 is generally considered sufficiently large to apply many normal approximation techniques, including z-tests and confidence intervals based on the normal distribution.

Sampling Distribution of the Sample Mean

The Concept of Sampling Distribution

When we take multiple samples of size 36 from the population, the sample means will vary from sample to sample. The distribution of these sample means is called the sampling distribution.

Key properties:

- The mean of the sampling distribution ($\mu_{\bar{x}}$) equals the population mean (μ).
- The standard deviation of the sampling distribution, called the standard error (SE), measures the variability of the sample mean.

Calculating the Standard Error

Given:

- Population standard deviation (σ)
- Sample size ($n = 36$)

The standard error of the mean is calculated as:

$$SE = \frac{\sigma}{\sqrt{n}}$$

Since σ is known, this value allows us to determine how much the sample mean is expected to fluctuate around the true population mean.

Implications of the Standard Error

- A smaller SE indicates that the sample mean is a more precise estimate of the population mean.
- As n increases, SE decreases, improving the accuracy of the sample mean as an estimator.

Estimating Population Parameters Using Sample Data

Constructing Confidence Intervals

A confidence interval provides a range of plausible values for the population mean based on the sample data.

For known population standard deviation (σ), the confidence interval is:

$$\bar{x} \pm Z_{\alpha/2} \times \frac{\sigma}{\sqrt{n}}$$

Where:

- $\bar{x}$ is the sample mean.
- $Z_{\alpha/2}$ is the critical z-value corresponding to the desired confidence level (e.g., 1.96 for 95%).

Example:

Suppose the sample mean ($\bar{x}$) is 50.5. For a 95% confidence level:

$$\text{Margin of Error} = 1.96 \times \frac{\sigma}{\sqrt{36}} = 1.96 \times \frac{\sigma}{6}$$

The confidence interval is then:

$$(50.5 - \text{Margin of Error}, 50.5 + \text{Margin of Error})$$

This interval estimates the range where the true population mean is likely to lie with 95% confidence.

Hypothesis Testing About the Population Mean

Suppose we want to test a claim about the population mean, such as whether it is equal to a specific value (say, 50).

Null hypothesis (H_0): $\mu = 50$

Alternative hypothesis (H_A): $\mu \neq 50$

Since the population standard deviation is known, we can use a z-test:

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

where:

- μ_0 is the hypothesized population mean (here, 50).
- $\bar{x}$ is the sample mean.

Decision rule:

- Compare the calculated z-value to critical z-values for the chosen significance level (e.g., 0.05).
- If $|z|$ exceeds the critical value, reject H_0 .

Example:

If the sample mean is 50.5, and σ is known to be 10:

$$z = \frac{50.5 - 50}{10 / 6} = \frac{0.5}{1.6667} \approx 0.3$$

Since 0.3 is less than 1.96, we fail to reject (H_0) , indicating insufficient evidence to conclude the population mean differs from 50 at the 5% significance level.

Significance of Sample Size and Known Standard Deviation

Why Sample Size Matters

- Larger samples reduce the standard error, which increases the precision of estimates.
- They lead to narrower confidence intervals, providing more specific information about the population.
- Sample size impacts the power of hypothesis tests; larger samples are more likely to detect true effects.

Known vs. Unknown Standard Deviation

- When (σ) is known, z-tests are used.
- When (σ) is unknown, t-tests are appropriate, especially with smaller samples.
- In our scenario, knowing (σ) simplifies analysis and allows straightforward application of the normal distribution.

Practical Applications and Examples

Example 1: Calculating a Confidence Interval

Suppose you collect a sample of size 36 from the population, and the sample mean is 50.8 with a known standard deviation of 12.

Calculate the 95% confidence interval for the population mean.

Solution:

- $(\bar{x} = 50.8)$
- $(\sigma = 12)$
- $(n = 36)$
- $(Z_{0.025} = 1.96)$

Calculate standard error:

$$SE = \frac{12}{\sqrt{36}} = 2$$

Margin of error:

$$ME = 1.96 \times 2 = 3.92$$

$$1.96 \times 2 = 3.92$$

\]

Confidence interval:

\[

$$(50.8 - 3.92, 50.8 + 3.92) = (46.88, 54.72)$$

\]

This means we are 95% confident that the true population mean lies between 46.88 and 54.72.

Example 2: Hypothesis Testing

Test whether the population mean is 50 at a 5% significance level, given:

- Sample mean ($\bar{x}$) = 50.8

- Known ($\sigma = 12$)

- Sample size (n) = 36

Solution:

Calculate z:

\[

$$z = \frac{50.8 - 50}{12/\sqrt{36}} = \frac{0.8}{2} = 0.4$$

\]

Critical z-value at 5% significance level (two-tailed test):

\[

$$\pm 1.96$$

\]

Since $|0.4| < 1.96$, we fail to reject the null hypothesis, concluding there is insufficient evidence to suggest the population mean differs from 50.

Conclusion

Analyzing a random sample of size 36 from a normal population with a known mean and standard deviation involves understanding the properties of sampling distributions, calculating standard error, constructing confidence intervals, and performing hypothesis tests. The sample size plays a pivotal role in the accuracy and reliability of the estimates, with larger samples providing more precise insights.

Knowing the population parameters allows for the application of z-tests and confidence intervals based on the normal distribution, simplifying analysis and interpretation. These statistical tools are essential for making informed decisions and deriving meaningful conclusions from data.

In real-world scenarios, whether in quality control, market research, or scientific studies, these principles enable practitioners to evaluate hypotheses, measure uncertainty, and infer population characteristics with confidence. Mastery of these concepts ensures robust and valid statistical analysis, ultimately leading to better decision-making.

Keywords: random sample, normal distribution,

Frequently Asked Questions

What is the sampling distribution of the sample mean when taken from a normal population with a known standard deviation?

The sampling distribution of the sample mean is normally distributed with mean equal to the population mean (50) and standard deviation equal to the population standard deviation divided by the square root of the sample size, i.e., $\sigma/\sqrt{n}$.

Given a sample size of 36 from a normal population with a mean of 50 and a standard deviation of σ , what is the standard error of the mean?

The standard error of the mean is $\sigma/\sqrt{36} = \sigma/6$.

How do you construct a 95% confidence interval for the population mean in this scenario?

Use the formula: sample mean $\pm Z(\sigma/\sqrt{n})$, where Z is the critical value for 95% confidence (approximately 1.96).

If the sample mean is 52, what is the z-score for this sample mean?

The z-score is $(52 - 50) / (\sigma/6) = 2 / (\sigma/6) = 12 / \sigma$.

What is the probability that the sample mean exceeds 52, assuming σ is known?

Calculate the z-score for 52 and find the corresponding probability from the standard normal distribution table.

How does increasing the sample size affect the standard error and confidence intervals?

Increasing the sample size decreases the standard error (since $\sigma/\sqrt{n}$ decreases), resulting in narrower confidence intervals.

If the population standard deviation is unknown, what alternative method should be used?

Use the t-distribution with the sample standard deviation and degrees of freedom $n-1$ to construct confidence intervals.

What assumptions are made when sampling from a normal population with known standard deviation?

It is assumed that the population is normally distributed and the standard deviation is known; the sample is randomly drawn.

How would the calculation change if the population standard deviation was unknown?

You would estimate the standard deviation with the sample standard deviation and use the t-distribution instead of the normal distribution.

What is the significance of the sample size being 36 in this context?

A sample size of 36 is generally considered sufficiently large for the Central Limit Theorem to ensure the sampling distribution of the mean is approximately normal, especially if the population is normal.

Additional Resources

A Random Sample Of Size 36 Is Taken From A Normal Population With A Mean Of 50 And A Standard Deviation — this scenario sets the stage for a classic statistical analysis rooted in the principles of probability, sampling distributions, and inferential statistics. Whether you're a student, a data analyst, or a researcher, understanding the implications of such a sample is essential for making informed decisions about the population parameter, estimating probabilities, and conducting hypothesis tests.

In this comprehensive guide, we will explore the key concepts, calculations, and interpretations that are involved when dealing with a sample of size 36 from a normally distributed population with known parameters. We will walk through the fundamentals of sampling distributions, confidence intervals, and hypothesis testing, providing clarity and practical insights along the way.

Understanding the Basics: Population, Sample, and Parameters

Before diving into calculations, it's crucial to clarify some foundational definitions:

- Population: The entire group you are interested in. In this case, the population has a mean (μ) of 50 and a standard deviation (σ) unknown but assumed to be known from the population data.
- Sample: A subset of the population, here of size $n = 36$.
- Sample Mean ($\bar{x}$): The average value of the observations in the sample.

- Parameters:
- Population mean (μ): 50
- Population standard deviation (σ): Known (not specified numerically in the prompt, but assumed to be known for the analysis)

The Significance of the Sample Size: Why 36?

Sample size plays a pivotal role in statistical inference. A sample size of 36 is quite common in statistical analysis because:

- It is large enough to invoke the Central Limit Theorem (CLT), which states that the distribution of the sample mean tends toward normality as the sample size increases, regardless of the population distribution, provided the population distribution is normal or approximately normal.
- It allows for more precise estimates of the population parameters.
- It facilitates hypothesis testing with well-understood distributions.

Given that the population is normal, even smaller samples would produce normally distributed sample means, but $n=36$ is sufficiently large to approximate the sampling distribution with high confidence.

Sampling Distribution of the Sample Mean

The key concept here is the sampling distribution of the sample mean ($\bar{x}$).

What is the sampling distribution?

- The distribution of the sample mean $\bar{x}$ across many repeated samples.
- For samples of size n drawn from a population with mean μ and standard deviation σ , the sampling distribution of $\bar{x}$ is normally distributed (by the CLT and given the population is normal) with:
 - Mean: $\mu_{\bar{x}} = \mu = 50$
 - Standard deviation (standard error): $\sigma_{\bar{x}} = \sigma / \sqrt{n}$

Calculating the Standard Error

If the population standard deviation σ is known, the standard error (SE) of the mean is:

$$SE = \sigma / \sqrt{n}$$

Given $n = 36$, the standard error simplifies to:

$$SE = \sigma / 6$$

(Note: Without a specific value for σ , the analysis remains in terms of σ .)

Probabilistic Questions and Applications

1. What is the probability that the sample mean is less than a certain value?

Suppose we want to find the probability that $\bar{x} < x_0$, for some value x_0 .

Standardize the value:

$$Z = \frac{x_0 - \mu}{\sigma / \sqrt{n}}$$

Then, using the standard normal distribution:

$$P(\bar{x} < x_0) = P(Z < \frac{x_0 - \mu}{\sigma / \sqrt{n}})$$

2. Constructing Confidence Intervals

A common goal is to estimate the population mean μ using the sample mean $\bar{x}$, with a specified confidence level (e.g., 95%).

Confidence interval formula:

$$\left(\bar{x} - z_{\alpha/2} \times \frac{\sigma}{\sqrt{n}}, \quad \bar{x} + z_{\alpha/2} \times \frac{\sigma}{\sqrt{n}} \right)$$

where:

- $z_{\alpha/2}$ is the z-score corresponding to the desired confidence level (for 95%, $z \approx 1.96$).
- The margin of error is $z_{\alpha/2} \times SE$.

3. Hypothesis Testing

Suppose we want to test:

Null hypothesis (H_0): $\mu = 50$

Alternative hypothesis (H_1): $\mu \neq 50$

Given a sample mean $\bar{x}$, the test statistic is:

$$Z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

Compare this Z-value to critical values to determine if H_0 can be rejected.

Practical Example: Hypothetical Calculations

Let's assume the population standard deviation $\sigma = 5$. With this assumption, we can perform specific calculations.

Calculating the Standard Error

$$SE = \frac{5}{\sqrt{36}} = \frac{5}{6} \approx 0.8333$$

Finding the Probability that Sample Mean is Less Than 49

Calculate Z:

$$Z = \frac{49 - 50}{0.8333} \approx -1.2$$

From standard normal tables, $P(Z < -1.2) \approx 0.1151$.

Interpretation: There is approximately an 11.51% chance that the sample mean will be less than 49 in repeated samples.

Constructing a 95% Confidence Interval for μ

Using $z = 1.96$:

$$\text{Margin of error} = 1.96 \times 0.8333 \approx 1.633$$

Suppose the observed sample mean $\bar{x} = 50$:

$$CI = (50 - 1.633, 50 + 1.633) = (48.367, 51.633)$$

Interpretation: We are 95% confident that the true population mean μ lies between approximately 48.37 and 51.63.

Considerations When Population Standard Deviation Is Unknown

In many real-world scenarios, σ is unknown. In such cases, the sample standard deviation s is used, and the t-distribution replaces the z-distribution for inference.

Adjusted formulas:

- Standard Error: $s / \sqrt{n}$
- Critical value: $t_{\{\alpha/2, n-1\}}$

Example:

Suppose $s = 5$, and $n = 36$.

The degrees of freedom:

$$df = n - 1 = 35$$

For a 95% confidence level:

$$t_{\{0.025, 35\}} \approx 2.03$$

Margin of error:

$$2.03 \times \frac{5}{6} \approx 2.03 \times 0.8333 \approx 1.69$$

Sample mean $\bar{x} = 50$:

$$[CI = (50 - 1.69, \quad 50 + 1.69) = (48.31, \quad 51.69)]$$

Summary of Key Takeaways

- Sample size ($n=36$) ensures the sampling distribution of the mean is approximately normal, especially given the population is normal.
- The standard error decreases with larger sample sizes, increasing the precision of estimates.
- Using the known population standard deviation simplifies calculations, allowing the use of the normal distribution.
- Probabilities regarding the sample mean can be calculated using the Z-score formula.
- Confidence intervals provide a range within which the true population mean likely falls, with an associated confidence level.
- When σ is unknown, the t-distribution is used, and the sample standard deviation s replaces σ .

Final Thoughts

Understanding the dynamics of sampling from a normal population with known parameters is foundational in statistics. A sample size of 36 strikes a balance between practical feasibility and statistical robustness, enabling precise estimation and hypothesis testing. Whether estimating the population mean or evaluating hypotheses, the principles outlined here serve as essential tools for rigorous analysis, informed decision-making, and scientific discovery.

Always remember to verify assumptions (such as normality and known standard deviation), interpret results within context, and consider potential sources of bias or variability in real-world applications.

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