A Rectangular Prism Has A Length 4 1/4 In., A Width Of 3 In., And Height Of 1 1/4 In. Which Expressions

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Understanding the dimensions of a rectangular prism and how to express their relationships mathematically is fundamental in geometry. When given specific measurements such as length, width, and height, these values can be represented through various algebraic expressions that help in calculating the prism's surface area, volume, and other properties. This article explores the different expressions associated with the dimensions of a rectangular prism, focusing on the given measurements: length of 4 1/4 inches, width of 3 inches, and height of 1 1/4 inches.

Understanding the Dimensions of the Rectangular Prism

Given Measurements

The problem provides three key dimensions:
- Length: 4 1/4 inches
- Width: 3 inches
- Height: 1 1/4 inches

To work effectively with these dimensions, it's essential to convert mixed fractions into improper fractions or decimals, which are often easier to manipulate algebraically.

Converting Mixed Fractions

- Length: 4 1/4 inches = 4 + 1/4 = 17/4 inches
- Height: 1 1/4 inches = 1 + 1/4 = 5/4 inches

Expressing these dimensions as improper fractions simplifies the process of creating algebraic expressions.

Expressing the Dimensions Algebraically

Using Variables for Dimensions

To develop general expressions, assign variables:
- Let L represent length
- Let W represent width
- Let H represent height

Substituting the known values:
- L = 17/4
- W = 3
- H = 5/4

Expressing Dimensions as Algebraic Expressions

In terms of variables:
- Length: L = 17/4
- Width: W = 3
- Height: H = 5/4

Formulating Expressions for Surface Area and Volume

Surface Area of a Rectangular Prism

The surface area (SA) of a rectangular prism is the sum of the areas of all six faces:
$$
SA = 2(LW + LH + WH)
$$

Using the given dimensions:
$$
SA = 2 \left(\left(\frac{17}{4} \times 3\right) + \left(\frac{17}{4} \times \frac{5}{4}\right) + 3 \times \frac{5}{4} \right)
$$

Expressions for Each Term

- $ LW = \frac{17}{4} \times 3 = \frac{17 \times 3}{4} = \frac{51}{4} $
- $ LH = \frac{17}{4} \times \frac{5}{4} = \frac{17 \times 5}{4 \times 4} = \frac{85}{16} $
- $ WH = 3 \times \frac{5}{4} = \frac{15}{4} $

Now, the total surface area expression:
$$
SA = 2 \left(\frac{51}{4} + \frac{85}{16} + \frac{15}{4} \right)
$$

To combine these fractions, find a common denominator (16):
- $\frac{51}{4} = \frac{51 \times 4}{16} = \frac{204}{16}$
- $\frac{85}{16}$ (already with denominator 16)
- $\frac{15}{4} = \frac{15 \times 4}{16} = \frac{60}{16}$

Sum:
$$
\frac{204}{16} + \frac{85}{16} + \frac{60}{16} = \frac{204 + 85 + 60}{16} = \frac{349}{16}
$$

Therefore:
$$
SA = 2 \times \frac{349}{16} = \frac{698}{16} = \frac{349}{8} \text{ square inches}
$$

Final expression for surface area:
$$
\boxed{SA = \frac{349}{8} \text{ in}^2}
$$

Volume of the Rectangular Prism

The volume (V) is found by multiplying the three dimensions:
$$
V = L \times W \times H
$$

Substituting the values:
$$
V = \frac{17}{4} \times 3 \times \frac{5}{4}
$$

Step-by-step:
- Multiply $\frac{17}{4} \times 3 = \frac{17 \times 3}{4} = \frac{51}{4}$
- Multiply by $\frac{5}{4}$:
$$
V = \frac{51}{4} \times \frac{5}{4} = \frac{51 \times 5}{4 \times 4} = \frac{255}{16}
$$

Final volume expression:
$$
\boxed{V = \frac{255}{16} \text{ in}^3}
$$

Developing Additional Expressions for the Prism

Expressing Perimeter of the Base

The perimeter (P) of the rectangular base (length + width) is:
$$
P = 2(L + W)
$$

Substituting:
$$
P = 2 \left(\frac{17}{4} + 3 \right)
$$

Convert 3 to quarters:
$$
3 = \frac{12}{4}
$$
So:
$$
P = 2 \left(\frac{17}{4} + \frac{12}{4} \right) = 2 \times \frac{29}{4} = \frac{58}{4} = \frac{29}{2}
$$

Perimeter expression:
$$
\boxed{P = \frac{29}{2} \text{ inches}}
$$

Summary of Key Expressions

Below is a list of important algebraic expressions derived from the prism’s dimensions:
- Length: $L = \frac{17}{4}$
- Width: $W = 3$
- Height: $H = \frac{5}{4}$
- Surface Area: $SA = \frac{349}{8} \text{ in}^2$
- Volume: $V = \frac{255}{16} \text{ in}^3$
- Perimeter of Base: $P = \frac{29}{2} \text{ inches}$

Understanding the Use of These Expressions

Application in Calculations

These algebraic expressions allow for precise calculations of the prism’s properties, which are essential in real-world applications such as packaging, construction, and manufacturing.

Adapting to Different Dimensions

By replacing the numerical values with variables, these expressions can be generalized for any rectangular prism:
- For instance, if the length is represented by L, the surface area becomes $2(WL + LH + WH)$, and so on.

Converting Fractions to Decimals

While fractions provide exact values, sometimes converting to decimals can facilitate easier computation:
- $\frac{17}{4} = 4.25$
- $\frac{5}{4} = 1.25$

Applying decimal values:
- Volume: $4.25 \times 3 \times 1.25 = 15.9375 \text{ in}^3$

Conclusion

The given dimensions of the rectangular prism lead to a variety of algebraic expressions that describe its physical properties. Converting mixed fractions into improper fractions simplifies the process of deriving these formulas. The primary expressions include formulas for surface area, volume, and perimeter, which are fundamental in understanding the geometric and physical characteristics of the prism. These expressions are not only useful for direct calculations but also serve as a foundation for solving more complex problems involving rectangular prisms in mathematics and applied sciences. By mastering how to develop and manipulate such expressions, students and professionals can enhance their problem-solving skills and deepen their understanding of three-dimensional geometry.

Frequently Asked Questions

What is the formula to find the volume of a rectangular prism?

The volume of a rectangular prism is found by multiplying its length, width, and height: V = length × width × height.

How do you express the length, width, and height of the prism as improper fractions?

Convert the mixed numbers to improper fractions: 4 1/4 = 17/4, 1 1/4 = 5/4, and 3 = 3/1.

What is the expression for the volume of the prism using the given dimensions?

The volume expression is (17/4) × 3 × (5/4).

How can you simplify the volume expression for the prism?

Multiply the numerators and denominators: (17 × 3 × 5) / (4 × 1 × 4) = (255) / (16).

What is the simplified form of the volume as an improper fraction?

The volume is 255/16 cubic inches.

How can you convert the fractional volume into a mixed number?

Divide 255 by 16: 16 goes into 255 about 15 times with a remainder of 15, so the mixed number is 15 15/16 cubic inches.

What is the approximate decimal volume of the prism?

Divide 255 by 16 to get approximately 15.9375 cubic inches.

Why is expressing the volume as an improper fraction useful?

It allows for precise calculations and easy simplification before converting to decimal or mixed number form.

Can you write an expression for the surface area of the rectangular prism?

Yes, the surface area is 2(length × width + length × height + width × height), which would be 2[(17/4 × 3) + (17/4 × 5/4) + (3 × 5/4)].

Additional Resources

Understanding the Expressions for a Rectangular Prism's Dimensions: A Comprehensive Review

When exploring the world of geometry, especially three-dimensional shapes like rectangular prisms,

understanding how to express their dimensions algebraically is fundamental. In this detailed review, we will delve into the problem: "A rectangular prism has a length of 4 1/4 inches, a width of 3 inches, and a height of 1 1/4 inches. Which expressions?" This question prompts us to analyze how to represent these dimensions using algebraic expressions and understand their significance in geometric calculations such as volume and surface area. We will also explore the concepts of mixed numbers, improper fractions, and how they relate to algebraic expressions, ensuring a thorough grasp of the topic.

Deciphering the Dimensions of the Rectangular Prism

Before diving into expressions, it’s essential to clearly understand the given dimensions.

Given Dimensions:

- Length: 4 1/4 inches
- Width: 3 inches
- Height: 1 1/4 inches

These are specified as mixed numbers, which combine whole numbers and fractions, and are common in measurement contexts.

Converting Mixed Numbers to Improper Fractions

To work effectively with algebraic expressions, especially when performing calculations like multiplication or simplification, it’s often more practical to convert mixed numbers into improper fractions.

Why Convert to Improper Fractions?

- Consistency in Operations: Improper fractions facilitate easier multiplication and division.
- Simplification: They allow straightforward algebraic manipulations without the complexity of mixed numbers.
- Standardization: Provides a uniform format for all dimensions, simplifying the formulation of expressions.

Conversion Process

Let’s convert each mixed number:

- Length (4 1/4):

1. Multiply the whole number by the denominator: 4 × 4 = 16
2. Add the numerator: 16 + 1 = 17
3. Write as an improper fraction: 17/4

- Height (1 1/4):

1. 1 × 4 = 4
2. 4 + 1 = 5
3. 5/4

- Width (3): This is already a whole number, which can be written as 3/1 if needed.

Summary of Converted Dimensions

- Length: 17/4 inches
- Width: 3/1 inches
- Height: 5/4 inches

Expressing the Dimensions Algebraically

Having converted mixed numbers to improper fractions, the next step is to formulate expressions that represent these dimensions.

Basic Algebraic Expressions for Dimensions

- Length: $L = \frac{17}{4}$
- Width: $W = 3$ (or $\frac{3}{1}$)
- Height: $H = \frac{5}{4}$

These expressions can be used directly in calculations of volume and surface area or manipulated algebraically to explore relationships.

Using Variables for Dimensions

To generalize or create algebraic formulas, we often assign variables:
- l for length
- w for width
- h for height

In this case:
- $l = \frac{17}{4}$
- $w = 3$
- $h = \frac{5}{4}$

This notation allows for the formulation of general expressions applicable to any rectangular prism with specified dimensions.

Calculating the Volume of the Rectangular Prism

The volume V of a rectangular prism is given by the product of its length, width, and height:
$$
V = l \times w \times h
$$

Formulating the Expression

Using the variables:
$$
V = l \times w \times h
$$
Substituting the converted improper fractions:
$$
V = \frac{17}{4} \times 3 \times \frac{5}{4}
$$

Step-by-Step Calculation

1. Multiply the fractions and whole numbers:
$$
V = \left(\frac{17}{4}\right) \times 3 \times \left(\frac{5}{4}\right)
$$

2. Convert the whole number 3 into a fraction:
$$
V = \left(\frac{17}{4}\right) \times \frac{3}{1} \times \left(\frac{5}{4}\right)
$$

3. Multiply numerators and denominators:
$$
V = \frac{17 \times 3 \times 5}{4 \times 1 \times 4}
$$

4. Calculate numerator:
$$
17 \times 3 = 51
$$
$$
51 \times 5 = 255
$$

5. Calculate denominator:
$$
4 \times 1 \times 4 = 16
$$

6. Final volume expression:
$$
V = \frac{255}{16} \text{ cubic inches}
$$

Interpreting the Result

- The volume in improper fraction form is $\frac{255}{16}$ cubic inches.
- To express as a mixed number:
- Divide 255 by 16:
$$
16 \times 15 = 240
$$
- Remainder:
$$
255 - 240 = 15
$$
- So:
$$
V = 15 \frac{15}{16} \text{ cubic inches}
$$

This precise calculation provides the exact volume of the prism.

Formulating the Surface Area Expression

Surface area involves summing the areas of all six faces of the rectangular prism. The formula is:

$$
\text{Surface Area} = 2(lw + lh + wh)
$$

Expressing the Surface Area Algebraically

Substituting the dimensions:
$$
SA = 2 \left(\frac{17}{4} \times 3 + \frac{17}{4} \times \frac{5}{4} + 3 \times \frac{5}{4} \right)
$$

Calculations Step-by-Step

1. Compute each product:

- $lw = \frac{17}{4} \times 3 = \frac{17 \times 3}{4} = \frac{51}{4}$

- $lh = \frac{17}{4} \times \frac{5}{4} = \frac{17 \times 5}{4 \times 4} = \frac{85}{16}$

- $wh = 3 \times \frac{5}{4} = \frac{3 \times 5}{4} = \frac{15}{4}$

2. Sum the three products:
$$
\frac{51}{4} + \frac{85}{16} + \frac{15}{4}
$$

3. Find a common denominator (16):

- Convert $\frac{51}{4}$ and $\frac{15}{4}$ to sixteenths:

$$
\frac{51}{4} = \frac{51 \times 4}{4 \times 4} = \frac{204}{16}
$$
$$
\frac{15}{4} = \frac{15 \times 4}{4 \times 4} = \frac{60}{16}
$$

Now, sum:
$$
\frac{204}{16} + \frac{85}{16} + \frac{60}{16} = \frac{204 + 85 + 60}{16} = \frac{349}{16}
$$

4. Multiply by 2:
$$
SA = 2 \times \frac{349}{16} = \frac{2 \times 349}{16} = \frac{698}{16}
$$

5. Simplify the fraction:
$$
\frac{698}{16} = \frac{349}{8}
$$

6. Convert to mixed number:
- $8 \times 43 = 344$
- Remainder: $349 - 344 = 5$
- Final surface area:
$$
43 \frac{5}{8} \text{ square inches}
$$

This detailed process yields the exact surface area.

Understanding the Role of Expressions in Geometry

Expressions such as those derived here serve pivotal roles in understanding and solving problems involving rectangular prisms:

- Facilitating calculations: Algebraic expressions allow for straightforward computation of volume and surface area, especially when dimensions are variable.

- Generalization: Using variables, such as \(l, w, h \), enables us to create formulas applicable to any rectangular prism, not just a specific instance.

- Converting measurements: Understanding how to convert mixed numbers to improper fractions ensures accuracy in calculations.

- Handling complex fractions: Recognizing

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