

A Sector Of A Circle Of Radius 3in Has An Area Of 25in^2 . Find The Central Angle Of The Sector. Do Not

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Understanding the Concept of a Sector in a Circle

A sector of a circle is a portion of the circle bounded by two radii and the corresponding arc. Think of it as a "slice" or "wedge" of the circle, much like a piece of a pie. The size of the sector is primarily determined by its central angle, which is the angle subtended at the center of the circle by the two radii forming the sector.

Components of a Sector

- Radius (r): The distance from the center of the circle to any point on its circumference.
- Central Angle (θ): The angle subtended at the circle's center by the two radii forming the sector.
- Arc Length (L): The length of the curved boundary of the sector.
- Area of Sector (A): The region enclosed within the two radii and the arc.

Given Data and Problem Statement

In this particular problem, you are provided with the following data:

- Radius (r): 3 inches
- Area of Sector (A): 25 square inches

The goal is to find the central angle (θ) of the sector.

Step-by-Step Solution Approach

To find the central angle, we need to understand the relationship between the area of a sector and its central angle.

Key Formula for the Area of a Sector

The area of a sector of a circle is given by:

$$A = \frac{\theta}{360^\circ} \times \pi r^2$$

\]

Where:

- (A) is the area of the sector.
- (θ) is the central angle in degrees.
- (r) is the radius of the circle.
- (π) is Pi, approximately 3.1416.

Rearranged, the formula to solve for (θ) is:

\[

$$\theta = \frac{A \times 360^\circ}{\pi r^2}$$

\]

Applying the Given Data

Substitute the known values into the formula:

\[

$$\theta = \frac{25 \times 360^\circ}{\pi \times (3)^2}$$

\]

Calculate the denominator:

\[

$$\pi \times (3)^2 = \pi \times 9 \approx 3.1416 \times 9 = 28.2744$$

\]

Now, compute (θ) :

\[

$$\theta = \frac{25 \times 360^\circ}{28.2744}$$

\]

Calculate numerator:

\[

$$25 \times 360^\circ = 9000^\circ$$

\]

Finally:

\[

$$\theta = \frac{9000^\circ}{28.2744} \approx 318.3^\circ$$

\]

Result

The central angle of the sector is approximately 318.3 degrees.

Additional Insights and Related Concepts

Understanding the Significance of the Central Angle

- The central angle determines how large the sector is relative to the entire circle.
- An angle of 360° corresponds to the full circle.
- Since 318.3° is close to 360° , the sector represents a significant portion of the circle, roughly 88% of the entire circle.

Comparing Sector Area with the Whole Circle

- Total area of the circle:

$$\text{Area} = \pi r^2 = 3.1416 \times 9 = 28.2744 \text{ in}^2$$

- The sector's area (25 in^2) is about 88.4% of the full circle, consistent with the calculated central angle.

Practical Applications of Sector Calculations

Understanding how to calculate the central angle of a sector has numerous real-world applications:

- Engineering and Design: Calculating segments of circular components.
- Architecture: Designing arcs and curved structures.
- Navigation and Geography: Determining portions of circular plots or regions.
- Mathematics and Education: Teaching concepts of fractions of a circle and angle measurement.

Common Mistakes to Avoid

When solving sector problems, students often make the following errors:

- Incorrect unit conversions: Ensure angles are in degrees when using the formula.
- Misapplication of the formula: Remember that the area formula involves the central angle in degrees, not radians unless specified.
- Forgetting to square the radius: The radius is squared in the formula; neglecting this leads to incorrect results.
- Ignoring the approximation of Pi: Use a consistent value for Pi for accuracy, or specify the level of precision.

Practice Problems to Reinforce Learning

To deepen understanding, here are some practice questions:

1. Given a sector with a radius of 5 inches and an area of 30 square inches, find the central angle.
2. If a sector of a circle with a radius of 4 inches has a central angle of 90° , what is its area?
3. Determine the length of the arc of a sector with a radius of 6 inches and a central angle of 60° .

Summary and Key Takeaways

- The formula for the area of a sector:

$$A = \frac{\theta}{360^\circ} \times \pi r^2$$

- To find the central angle:

$$\theta = \frac{A \times 360^\circ}{\pi r^2}$$

- In the given problem, with $(r=3, \text{in})$ and $(A=25, \text{in}^2)$, the central angle is approximately 318.3° .

- Understanding the relationship between sector area and central angle is crucial in various fields, from mathematics to engineering.

Final Thoughts

Calculating the central angle of a sector is an essential skill in geometry. It combines knowledge of circle properties, algebra, and formula manipulation. Mastery of these concepts enables problem-solving in practical scenarios involving circular segments. Always double-check your calculations, pay attention to units, and practice with different values to build confidence and proficiency in sector-related problems.

Frequently Asked Questions

How do you find the central angle of a sector when given its area and radius?

Use the formula for the area of a sector, $A = (\theta/360) \times \pi \times r^2$, then solve for θ : $\theta = (A \times 360) / (\pi \times r^2)$.

What is the central angle of a sector with radius 3 inches and

area 25 square inches?

The central angle $\theta = (25 \times 360) / (\pi \times 3^2) = (25 \times 360) / (\pi \times 9) \approx (9000) / (28.274) \approx 318.31$ degrees.

Can the central angle of a sector exceed 360°? Why or why not?

No, because a circle's total angle is 360°, so a sector's central angle cannot exceed 360°.

What is the significance of the radius in calculating the sector's central angle?

The radius determines the size of the sector; it is used in the area formula to relate the sector's area to its central angle.

Is the calculated central angle of approximately 318.31 degrees plausible for a sector with the given dimensions?

Yes, because the sector's area (25 in²) is close to the total area of the circle ($\pi \times 3^2 \approx 28.27$ in²), so a central angle slightly less than 360° makes sense.

How does changing the radius affect the central angle for a fixed sector area?

Increasing the radius decreases the required central angle for the same sector area, while decreasing the radius increases the central angle.

Additional Resources

A Sector Of A Circle Of Radius 3in Has An Area Of 25in². Find The Central Angle Of The Sector. Do Not

Understanding the geometry of circles and their sectors is fundamental in various fields such as engineering, architecture, and mathematics. The problem presented—finding the central angle of a sector given its radius and area—serves as a classic example of how geometric formulas can be applied to solve real-world problems. In this article, we will explore the concepts involved in this problem comprehensively, analyze the steps to find the central angle, and discuss related concepts, tips, and common pitfalls to enhance your understanding of circle sectors.

Understanding the Basics of a Sector of a Circle

What is a Sector?

A sector of a circle is a portion of a circle bounded by two radii and the arc between them. Imagine slicing a pizza: each slice is essentially a sector. The size of a sector depends on the angle between the two radii (called the central angle) and the radius of the circle itself.

Key Features of a Sector

- Radius (r): The distance from the center of the circle to any point on its circumference.
- Central angle (θ): The angle subtended at the center of the circle by the two radii forming the sector.
- Area of the sector (A): The portion of the circle's total area corresponding to the sector.

Understanding these features helps in applying the correct formulas and solving related problems accurately.

Mathematical Formulas for a Sector

Area of a Sector

The area (A) of a sector with radius (r) and central angle (θ) (in degrees) is given by:

$$A = \frac{\theta}{360^\circ} \times \pi r^2$$

Alternatively, when (θ) is in radians:

$$A = \frac{1}{2} r^2 \theta$$

It's crucial to be consistent with units—either degrees or radians—and to convert appropriately if necessary.

Arc Length of a Sector

The length of the arc (L) subtended by the central angle (θ) is:

- In degrees:

$$L = \frac{\theta}{360^\circ} \times 2\pi r$$

- In radians:

$$L = r \theta$$

While arc length isn't directly needed for this problem, understanding it enriches your grasp of sectors.

Problem Breakdown: Given Data and Objective

Given:

- Radius $(r = 3, \text{in})$
- Area $(A = 25, \text{in}^2)$

Find:

- The central angle (θ) in degrees corresponding to this sector.

Step-by-Step Solution Approach

Step 1: Recall the area formula for a sector

$$A = \frac{\theta}{360^\circ} \times \pi r^2$$

Given the values, substitute:

$$25 = \frac{\theta}{360^\circ} \times \pi \times 3^2$$

which simplifies to:

$$25 = \frac{\theta}{360^\circ} \times \pi \times 9$$

$$25 = \frac{\theta}{360^\circ} \times \pi \times 9$$

\]

Step 2: Isolate θ

Rearranging the equation:

\[

$$\theta = \frac{25 \times 360^\circ}{\pi \times 9}$$

\]

Calculating numerator and denominator separately:

\[

$$\theta = \frac{9000^\circ}{9\pi}$$

\]

Simplify numerator:

\[

$$\theta = \frac{9000^\circ}{9\pi} = \frac{1000^\circ}{\pi}$$

\]

Step 3: Final calculation

Using $\pi \approx 3.1416$:

\[

$$\theta \approx \frac{1000^\circ}{3.1416} \approx 318.31^\circ$$

\]

Therefore, the central angle $\theta \approx 318.31^\circ$.

This large angle indicates that the sector almost encompasses the entire circle, consistent with an area of 25 in^2 given the circle's radius.

Understanding the Result and Its Significance

The calculated central angle of approximately 318.31° signifies a sector that is a substantial portion

of the circle—specifically, over 88% of the full circle (which is 360°). This insight can be valuable in practical applications, such as designing sectors for circular segments or understanding portions of circular structures.

Alternative Methods: Using Radians

While the above solution uses degrees, it's instructive to approach the problem in radians for a more elegant formula.

Recall:

$$A = \frac{1}{2} r^2 \theta$$

where θ is in radians.

Rearranged for θ :

$$\theta = \frac{2A}{r^2}$$

Substituting the known values:

$$\theta = \frac{2 \times 25}{3^2} = \frac{50}{9} \approx 5.5556, \text{ radians}$$

Converting radians to degrees:

$$\text{Degrees} = \theta \times \frac{180^\circ}{\pi} \approx 5.5556 \times \frac{180^\circ}{3.1416} \approx 318.31^\circ$$

This confirms the previous result and demonstrates the consistency of both methods.

Common Pitfalls and Tips

Pitfalls:

- Mixing units: Confusing degrees and radians can lead to incorrect results. Always convert units appropriately.
- Incorrect formula application: Using the entire circle's area (πr^2) without considering the proportionality based on the angle can lead to errors.
- Arithmetic errors: Be cautious with fractions and multiplications, especially when simplifying.

Tips:

- Always double-check units before plugging into formulas.
- Use approximate values of π carefully; keep enough decimal places for accuracy.
- Practice similar problems to strengthen understanding.

Additional Related Concepts

Finding the Length of the Arc

Once the central angle θ is known, you can find the arc length:

$$L = \frac{\theta}{360^\circ} \times 2\pi r$$

Using the approximate $\theta \approx 318.31^\circ$:

$$L \approx \frac{318.31}{360} \times 2 \times 3.1416 \times 3 \approx 16.66, \text{ in}$$

This indicates the arc length corresponding to the sector.

Other Applications

- Designing circular segments in engineering.
- Calculating materials needed for curved structures.
- Analyzing sectors in pie charts for data visualization.

Summary and Conclusion

In solving the problem of identifying the central angle of a sector given the radius and area, the key

steps involve:

- Understanding the sector's fundamental properties.
- Applying the correct formula for the area of a sector.
- Solving algebraically for the central angle.
- Converting units when necessary, especially between degrees and radians.

The calculated central angle of approximately 318.31° reveals that the sector nearly spans the entire circle, emphasizing the importance of understanding the relationship between area, radius, and angle. Mastery of these concepts enables precise calculations for various practical and theoretical applications in geometry and beyond.

In conclusion, this problem exemplifies how fundamental geometric formulas serve as powerful tools for solving real-world problems with circles and sectors. Whether in academic exercises or engineering design, a solid grasp of these principles ensures accuracy and confidence in tackling similar challenges.

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