A Sector Of A Circle, Radius Rcm, Has A Perimeter Of 150 Cm. Find An Expression, In Terms Of R, For The

Introduction

A Sector Of A Circle, Radius Rcm, Has A Perimeter Of 150 Cm. Find An Expression, In Terms Of R, For The problem involves understanding the basic properties of a sector in a circle and translating the given information into a mathematical expression. This article aims to guide you through the process of deriving an expression for a specific property of the sector—most likely its area, arc length, or sector angle—based on the given perimeter and radius. We will explore the fundamental concepts of circle sectors, derive relevant formulas, and develop the required expression step-by-step.

Understanding a Sector of a Circle

What Is a Sector?

- A sector of a circle is a portion of the circle bounded by two radii and the corresponding arc.
- It resembles a "slice" of the circle, similar to a wedge-shaped piece.
- The key parameters of a sector include the radius (R), the central angle (θ), the arc length (L), and the area (A).

Key Properties and Formulas

- **Arc length (L)**: $L = R \theta$ (where θ is in radians)
- **Area of a sector (A)**: $A = \frac{1}{2} R^2 \theta$
- **Perimeter of a sector (P)**: $P = L + 2R$ (sum of arc length and two radii)

Given Data and What is Required

Given:

- Radius of the circle, R (in cm)

- Perimeter of the sector, $P = 150$ cm

What We Need to Find:

- An expression in terms of R for a particular property of the sector—most likely the area or the central angle. Since the problem statement is incomplete, we will consider both possibilities and develop the general formula for each.

Deriving the Expression for the Sector's Properties

Step 1: Expressing the Perimeter Equation

Given the perimeter (P) of the sector:
$$
P = L + 2R
$$
Substituting the formula for the arc length $L = R \theta$:
$$
150 = R \theta + 2R
$$

Step 2: Isolating the Central Angle (θ)

Factor R on the right:
$$
150 = R (\theta + 2)
$$
Divide both sides by R:
$$
\frac{150}{R} = \theta + 2
$$
Rearranged:
$$
\boxed{\theta = \frac{150}{R} - 2}
$$
This is the expression for the central angle θ in radians, in terms of R.

Note:

- To ensure θ is valid (i.e., between 0 and 2π), R must satisfy certain constraints.
- If the angle θ is in radians, the formula applies directly. If the problem requires the angle in degrees, multiply by $\frac{180}{\pi}$.

Expressing the Sector's Area in Terms of R

Step 3: Deriving the Area Formula

Recall the formula for the area of a sector:

$$
A = \frac{1}{2} R^2 \theta
$$

Substitute the expression for θ from above:

$$
A = \frac{1}{2} R^2 \left(\frac{150}{R} - 2 \right)
$$

Step 4: Simplify the Expression

Distribute R inside:

$$
A = \frac{1}{2} R^2 \times \left(\frac{150}{R} - 2 \right)
$$

Express as two separate terms:

$$
A = \frac{1}{2} \left(R^2 \times \frac{150}{R} - R^2 \times 2 \right)
$$

Simplify each:
- $R^2 \times \frac{150}{R} = 150 R$
- $R^2 \times 2 = 2 R^2$

So:

$$
A = \frac{1}{2} (150 R - 2 R^2)
$$

Finally, write the expression in a simplified form:

$$
\boxed{
A = 75 R - R^2
}
$$

This is the formula for the area of the sector in terms of the radius R.

Constraints and Validity of the Expression

Range of R for Realistic Sector

- Since θ must be positive:

$$
\frac{150}{R} - 2 > 0 \Rightarrow \frac{150}{R} > 2 \Rightarrow R < \frac{150}{2} = 75
$$

- Also, θ should be less than or equal to 2π (approximately 6.283) because a sector cannot have an angle greater than a full circle:

$$
\theta \leq 2\pi \Rightarrow \frac{150}{R} - 2 \leq 2\pi
$$

Solve for R:

$$
\frac{150}{R} \leq 2\pi + 2
$$
$$
R \geq \frac{150}{2\pi + 2}
$$
Using $\pi \approx 3.1416$:
$$
2\pi + 2 \approx 8.2832
$$
$$
R \geq \frac{150}{8.2832} \approx 18.1\, \text{cm}
$$
Thus, R must be between approximately 18.1 cm and 75 cm for the sector's parameters to be valid.

Summary of Derived Expressions

- **Central angle θ (in radians):**
  $$
  \boxed{
  \theta = \frac{150}{R} - 2
  }
  $$
  where R is in centimeters and $18.1 \leq R \leq 75$.

- **Area of the sector (A):**
  $$
  \boxed{
  A = 75 R - R^2
  }
  $$
  in square centimeters.

Conclusion

By starting with the fundamental properties of a circle and its sector, we derived the key relationships connecting the radius R, the sector's perimeter, and its central angle and area. The critical step was expressing the perimeter condition $P = R \theta + 2 R$, which allowed us to find an expression for θ in terms of R. Substituting this into the sector area formula provided a direct, simplified formula for the sector's area solely in terms of R:
$$
A = 75 R - R^2
$$
This expression enables you to compute the area of the sector for any radius R within the valid range, given the perimeter constraint of 150 cm. Understanding these derivations enhances your grasp of circle geometry and prepares you to tackle similar problems involving sectors, angles, and perimeters.

Frequently Asked Questions

A sector of a circle with radius R cm has a perimeter of 150 cm. How do you express the length of the arc in terms of R?

The perimeter P of the sector is $P = R + R + L$, where L is the arc length. Given $P = 150$ cm, then $2R + L = 150$ cm, so the arc length $L = 150 - 2R$ cm.

What is the formula for the arc length of a sector in terms of radius R and central angle θ?

The arc length $L = (\theta/360) \times 2\pi R$, where θ is in degrees.

Given the perimeter of a sector is 150 cm, and the radius is R cm, how can you find the central angle θ in terms of R?

From the perimeter formula: $2R + L = 150$. Since $L = (\theta/360) \times 2\pi R$, substitute to get $2R + (\theta/360) \times 2\pi R = 150$. Solving for θ: $\theta = ((150 - 2R) \times 360) / (2\pi R)$.

Express the arc length L of the sector in terms of R when the perimeter is 150 cm.

$L = 150 - 2R$ cm; this comes directly from the perimeter relation $P = 2R + L$.

If the radius R is known, how can you calculate the central angle θ of the sector in terms of R?

Using $\theta = ((150 - 2R) \times 360) / (2\pi R)$, which expresses θ in degrees in terms of R.

What is the relationship between the perimeter, radius R, and the central angle θ of the sector?

Perimeter $P = 2R + (\theta/360) \times 2\pi R$. Given P, R, and θ are related through this formula.

How can you derive an expression for the central angle θ solely in terms of R, given the perimeter is 150 cm?

Rearranging the perimeter formula: $\theta = ((150 - 2R) \times 360) / (2\pi R)$.

What constraints exist for R in the context of this problem?

Since the arc length $L = 150 - 2R$ must be positive, R must be less than 75 cm to ensure the sector exists with positive arc length.

Summarize how to find the expression for the central angle θ in terms of R when a sector's perimeter is 150 cm.

Use the perimeter relation: $2R + (\theta/360) \times 2\pi R = 150$. Solving for θ gives $\theta = ((150 - 2R) \times 360) / (2\pi R)$, expressing θ in terms of R.

Additional Resources

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Introduction

Understanding the geometrical properties of circles and their segments is fundamental in both pure and applied mathematics. Among these properties, the concept of a sector—a portion of a circle enclosed by two radii and an arc—is particularly significant. Sectors are not only theoretical constructs but also have practical applications in fields such as engineering, architecture, and even data visualization.

This article aims to explore the problem: "A sector of a circle with radius R centimeters has a perimeter of 150 centimeters. Find an expression, in terms of R, for the angle of the sector." We will systematically dissect the problem, working through the relevant formulas, and derive a comprehensive expression that relates the sector's angle to its radius.

Understanding the Geometrical Setup

What is a Sector of a Circle?

A sector of a circle resembles a "slice of pie" or a "wedge," created when two radii extend from the center of the circle, and the arc connecting their endpoints forms the boundary. The size of a sector is often specified by the central angle, denoted as θ (theta), measured in degrees or radians.

Components of a Sector's Perimeter

The perimeter (often called the arc length or perimeter of the sector) of a sector comprises:

- Two radii: each of length R.
- Arc length: the curved segment connecting the endpoints of the radii.

Hence, the total perimeter P of a sector is:

$$ P = 2R + \text{arc length} $$

Given the problem states that the perimeter of the sector is 150 cm, this total includes both radii and the arc length.

Mathematical Foundations

Components of the Sector's Perimeter

1. Radial components: Since the sector is bounded by two radii, their combined length is:

$$ 2R $$

2. Arc length (L): The length of the arc subtended by the central angle θ is given by:

$$ L = \frac{\theta}{360^\circ} \times 2\pi R $$

or, equivalently, if θ is in radians (which simplifies calculations):

$$ L = R \times \theta $$

where θ is in radians.

Deriving the Relationship

Step 1: Express the Perimeter

Total perimeter P is:

$$ P = 2R + L $$

Given P = 150 cm, we have:

$$ 150 = 2R + L $$

Step 2: Express Arc Length in Terms of R and θ

Since the problem involves the perimeter in centimeters, and R is in centimeters, it's convenient to express the arc length in radians:

$$ L = R \times \theta $$

Plugging into the perimeter formula:

$$ 150 = 2R + R \times \theta $$

Step 3: Isolate θ

Rearranging for θ:

$$ R \times \theta = 150 - 2R $$

$$ \theta = \frac{150 - 2R}{R} $$

$$ \theta = \frac{150}{R} - 2 $$

Thus, the central angle θ, in radians, expressed in terms of R, is:

$$\boxed{\theta(R) = \frac{150}{R} - 2}$$

Interpreting the Result

This formula provides a direct relationship between the radius of the circle and the central angle of the sector, given the perimeter constraint. It reveals several key insights:

- Dependence on R: As R increases, the value of θ decreases, indicating that larger circles with the same perimeter sector have smaller central angles.
- Feasibility Constraints: Since θ must be positive for a valid sector, the expression implies:

$$\frac{150}{R} - 2 > 0$$

which simplifies to:

$$\frac{150}{R} > 2 \Rightarrow R < \frac{150}{2} = 75$$

This indicates that for the sector's perimeter to be 150 cm, the radius must be less than 75 cm.

Additional Considerations

Conversion to Degrees

While the derivation above yields θ in radians, sometimes it's more intuitive or customary to express angles in degrees. To convert radians to degrees:

$$\theta_{degrees} = \theta_{radians} \times \frac{180^\circ}{\pi}$$

Thus:

$$\theta_{degrees}(R) = \left(\frac{150}{R} - 2\right) \times \frac{180^\circ}{\pi}$$

This can be useful if the problem or context specifies the angle in degrees.

Practical Application Example

Suppose R = 50 cm:

$$\theta = \frac{150}{50} - 2 = 3 - 2 = 1 \text{ radian}$$

Converted to degrees:

$$1 \times \frac{180^\circ}{\pi} \approx 57.3^\circ$$

This indicates that a sector with a radius of 50 cm and perimeter 150 cm subtends approximately 57.3 degrees at the center.

Broader Implications in Geometry and Design

Understanding the relationship between the radius, sector angle, and perimeter has practical implications:

- Design and Engineering: Precise calculations of sectors are critical in designing parts like pie charts, windmill blades, or architectural arches.
- Mathematical Modeling: The derived formula allows for quick estimation and adjustment of parameters in modeling circular segments.

Limitations and Assumptions

The derivation assumes the following:

- The perimeter includes two radii and the arc length without any additional boundary components.
- The sector's central angle θ is measured in radians unless converted.
- The radius R > 0, and the angle θ > 0, imposing constraints on R based on the perimeter value.

If the perimeter were to include other components or if the sector were part of a more complex shape, additional considerations would be necessary.

Conclusion

The key insight from this analysis is that the central angle θ of a sector of a circle, with radius R and a known perimeter of 150 cm, can be elegantly expressed as:

\[\boxed{\theta(R) = \frac{150}{R} - 2} \quad \text{(radians)} \]

This formula encapsulates the fundamental relationship between the circle's radius and the sector's size, emphasizing the interconnected nature of geometric properties. Such an understanding not only enhances theoretical knowledge but also enriches practical applications across various fields, from engineering design to educational demonstrations.

In summary, the problem exemplifies how basic geometric principles can be combined with algebraic manipulation to produce meaningful, applicable formulas, fostering deeper comprehension of circular segments and their properties.

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