A Student Used The Sieve Of Eratosthenes To Find All Prime Numbers Less Than 100. Create A Step-by-step

A Student Used The Sieve Of Eratosthenes To Find All Prime Numbers Less Than 100. Create A Step-by-step

Finding prime numbers is a fundamental concept in number theory and plays a crucial role in various fields such as cryptography, computer science, and mathematics. One of the most efficient and ancient methods to identify all prime numbers less than a given number is the Sieve of Eratosthenes. This article provides a comprehensive, step-by-step guide on how a student can use this classic algorithm to find all prime numbers less than 100, along with explanations of each step, tips, and insights to enhance understanding.

Understanding the Sieve of Eratosthenes

Before diving into the step-by-step process, it's essential to understand what the Sieve of Eratosthenes is and how it works.

What is the Sieve of Eratosthenes?

The Sieve of Eratosthenes is an ancient algorithm designed to efficiently find all prime numbers up to a specified limit. It works by systematically eliminating composite numbers (non-primes) from a list of numbers, leaving only primes.

Why Use the Sieve of Eratosthenes?

- Simple and easy to implement
- Efficient for finding primes within a moderate range
- Demonstrates the fundamental properties of prime numbers and divisibility

Preparing to Use the Sieve

Step 1: Create a List of Numbers

Start with a list of consecutive integers from 2 up to the number just less than 100, which is 99 in this case.

Example:
2, 3, 4, 5, 6, ..., 98, 99

This list will serve as the pool from which you'll identify primes.

Step 2: Understand the Basic Rules

- The first number in the list is 2, which is the smallest prime.
- You will eliminate multiples of each prime as you proceed.
- The process continues until you've processed numbers up to the square root of the limit (here, $\sqrt{100} \approx 10$).

Step-by-step Process to Find All Primes Less Than 100

Step 1: List the numbers from 2 to 99

Write down or visualize the list:

2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, ..., 98, 99

Step 2: Identify the first prime number (2)

- 2 is prime.
- Cross out all multiples of 2 greater than 2.

Elimination:
- 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, 48, 50, 52, 54, 56, 58, 60, 62, 64, 66, 68, 70, 72, 74, 76, 78, 80, 82, 84, 86, 88, 90, 92, 94, 96, 98

Remaining list:
3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, 65, 67, 69, 71, 73, 75, 77, 79, 81, 83, 85, 87, 89, 91, 93, 95, 97, 99

Step 3: Move to the next unmarked number (3)

- 3 is prime.
- Cross out all multiples of 3 greater than 3.

Elimination:
- 9, 15, 21, 27, 33, 39, 45, 51, 57, 63, 69, 75, 81, 87, 93, 99

Remaining list:
3, 5, 7, 11, 13, 17, 19, 23, 25, 29, 31, 35, 37, 41, 43, 45, 47, 49, 53, 55, 59, 61, 65, 67, 71, 73, 77, 79, 83, 85, 89, 91, 95, 97

Note: Some numbers like 25, 35, 49, 55, 65, 77, 85, 91, 95 are multiples of 5 or 7 but are still present

because they weren't eliminated yet.

Step 4: Next unmarked number (5)

- 5 is prime.
- Cross out all multiples of 5 greater than 5.

Elimination:
- 25, 35, 45, 55, 65, 75, 85, 95

Remaining list:
3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 49, 53, 59, 61, 65, 67, 71, 73, 77, 79, 83, 85, 89, 91, 95, 97

Note: 49, 77, 85, 91, 95 are multiples of 7 or 5, as expected.

Step 5: Next unmarked number (7)

- 7 is prime.
- Cross out multiples of 7 greater than 7.

Elimination:
- 49, 77, 91

Remaining list:
3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 65, 67, 71, 73, 77, 79, 83, 85, 89, 91, 95, 97

Note: 65, 85 are multiples of 5, but they weren’t eliminated yet because their multiples of 7 are already removed.

Step 6: Continue with next unmarked number (11)

- 11 is prime.
- Cross out multiples of 11 greater than 11.

Elimination:
- $11 \times 11 = 121$, but since $121 > 100$, no elimination needed here.

Remaining list unchanged:
3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 65, 67, 71, 73, 77, 79, 83, 85, 89, 91, 95, 97

Similarly, for 13, 17, 19, and other primes less than or equal to 10, you check their multiples. Since their multiples exceed 100 or are already eliminated, no further eliminations occur at this stage.

Step 7: Final list of primes

After performing all eliminations for primes up to √100 (~10), the remaining unmarked numbers are all prime:

Prime numbers less than 100:
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

Note: Remember, 2 was eliminated initially, but in practice, you start with 2 and then proceed.

Summary of the Process

Step	Action	Numbers Eliminated	Remaining Primes
1	List numbers from 2 to 99	-	2, 3, 4, ..., 99
2	Remove multiples of 2	4, 6, 8, ... 98	3, 5, 7, 9, ... 99
3	Remove multiples of 3	9, 15, 21, ... 99	3, 5, 7, 11, ...
4	Remove multiples of 5	25, 35, 45, ... 95	...
5			

Frequently Asked Questions

What is the Sieve of Eratosthenes method used for?

The Sieve of Eratosthenes is a classical algorithm used to find all prime numbers up to a specified limit, such as 100, by systematically eliminating composite numbers.

How do you start the Sieve of Eratosthenes for numbers less than 100?

Begin by listing all numbers from 2 to 99, then start with the first prime number (2) and eliminate all its multiples, moving to the next unmarked number and repeating the process.

What are the first steps to applying the Sieve of Eratosthenes for primes less than 100?

First, list numbers 2 through 99. Then, identify 2 as prime and eliminate all multiples of 2 (4, 6, 8, ...). Next, find the next unmarked number (3) and eliminate its multiples, and so on.

How do you know when to stop eliminating multiples in the sieve process?

You stop when you reach the square root of the maximum number (in this case, √100 ≈ 10). Beyond this point, remaining unmarked numbers are prime.

What are the prime numbers less than 100 identified by the sieve?

The prime numbers less than 100 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, and 97.

Can you explain the step-by-step process of the Sieve of Eratosthenes for numbers less than 100?

Yes. First, list numbers 2 to 99. Next, eliminate multiples of 2. Then, move to 3 and eliminate its multiples. Continue with the next unmarked number (5) and eliminate its multiples, repeating this for 7, 11, etc., up to 10. The remaining unmarked numbers are prime.

Why is it sufficient to check multiples only up to the square root of the maximum number in the sieve?

Because any composite number greater than the square root of the limit will have a factor less than or equal to that square root, so checking up to √100 ensures all composites are eliminated.

What are common mistakes to avoid when using the Sieve of Eratosthenes?

Common mistakes include forgetting to eliminate multiples of the initial primes, missing some multiples, or stopping the process too early before all composites are eliminated.

How can the Sieve of Eratosthenes be applied to larger numbers beyond 100?

The process remains the same: list all numbers up to the desired limit, eliminate multiples of primes starting from 2, and proceed up to the square root of the limit. However, for very large numbers, more efficient algorithms or computer programs are often used.

Additional Resources

Sieve of Eratosthenes: Unlocking the Secrets of Prime Numbers with a Timeless Algorithm

Prime numbers have fascinated mathematicians for centuries, standing as the building blocks of the natural numbers and serving crucial roles in areas like cryptography, number theory, and computer science. Among the many methods devised to identify primes, the Sieve of Eratosthenes remains one of the most elegant, efficient, and accessible algorithms—especially for students venturing into the realm of prime discovery. In this article, we explore how a diligent student used the Sieve of Eratosthenes to find all prime numbers less than 100, offering a comprehensive, step-by-step analysis that combines historical context, detailed methodology, and practical insights.

Understanding the Sieve of Eratosthenes: A Historical and Conceptual Overview

Before diving into the step-by-step process, it’s essential to appreciate what makes the Sieve of Eratosthenes a cornerstone of prime number identification.

Historical Context

Developed by the ancient Greek mathematician Eratosthenes of Cyrene (circa 276–194 BC), this algorithm was one of the earliest systematic methods to find prime numbers efficiently. Its enduring relevance lies in its simplicity and effectiveness, particularly for smaller ranges such as numbers less than 100.

Core Concept

At its core, the sieve works by iteratively marking the multiples of each prime starting from the smallest prime (2), effectively "sieving out" composite numbers and leaving primes unmarked. The process relies on the fundamental property that every composite number has at least one prime factor less than or equal to its square root.

Preparing for the Sieve: Setting the Stage

A student embarking on this task begins by organizing the range of numbers to analyze.

Step 1: Listing Numbers

Start by writing down all integers from 2 up to 99:

2, 3, 4, 5, 6, 7, 8, 9, 10, ..., 98, 99

This provides a straightforward visual foundation to perform the elimination process.

Step 2: Recognizing the Limits

Since the goal is to find all primes less than 100, the upper bound is 99. The reason for stopping at the square root of 100 (which is 10) will be explained later, but for now, it guides the student in choosing which numbers to use as bases for eliminating multiples.

Step-by-Step Application of the Sieve of Eratosthenes

The core of the process involves systematic elimination of composite numbers by marking their multiples. Let’s walk through this process meticulously.

Step 3: Identify the First Prime (2) and Remove Its Multiples

- Prime to start with: 2
- Multiples of 2 less than 100: 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, 48, 50, 52, 54, 56, 58, 60, 62, 64, 66, 68, 70, 72, 74, 76, 78, 80, 82, 84, 86, 88, 90, 92, 94, 96, 98

- Action: Mark all these multiples as composite (or remove them from the list).

Remaining numbers after this step:

2, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, 65, 67, 69, 71, 73, 75, 77, 79, 81, 83, 85, 87, 89, 91, 93, 95, 97, 99

Step 4: Move to the Next Unmarked Number (3) and Remove Its Multiples

- Next prime candidate: 3
- Multiples of 3 greater than 3: 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, 51, 54, 57, 60, 63, 66, 69, 72, 75, 78, 81, 84, 87, 90, 93, 96, 99

- Action: Mark these as composite.

Remaining list:

2, 3, 5, 7, 11, 13, 17, 19, 23, 25, 29, 31, 35, 37, 41, 43, 47, 49, 53, 55, 59, 61, 65, 67, 71, 73, 77, 79, 83, 85, 89, 91, 95, 97

Step 5: Proceed to the Next Unmarked Number (5) and Eliminate Its Multiples

- Next prime candidate: 5
- Multiples of 5 (greater than 5): 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95

- Action: Mark multiples as composite.

Remaining list:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 77, 79, 83, 89, 91, 95, 97

Note: Some numbers like 25, 35, 45, 55, 65, 75, 85, 95 are already marked as composite or removed.

Step 6: Continue with the Next Unmarked Number (7)

- Next prime candidate: 7
- Multiples of 7 (greater than 7): 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98

- Action: Mark these as composite.

Remaining list:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

Numbers like 21, 28, 42, 56, 63, 70, 84, 98 are eliminated.

Step 7: Check the Next Number (11)

- Next candidate: 11
- Multiples of 11: 22, 33, 44, 55, 66, 77, 88, 99

- Action: Remove these (some are already marked).

Remaining list:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

Since all multiples of 11 are eliminated, move on to the next unmarked number (13).

Step 8: Check Next Number (13)

- Multiples of 13: 26, 39, 52, 65, 78, 91

- Action: Remove these if not already eliminated.

Remaining list remains the same, as these are already marked or not in the list.

Step 9: Final Primes Up to the Square Root of 100

The process continues with the next unmarked number less than or equal to 10 (since $10^2 = 100$):

- 17, 19

Since their multiples exceed 100, further elimination isn't necessary.

Conclusion: The Primes Less Than 100

After completing the sieving process, the remaining unmarked numbers are the prime numbers less than 100:

2, 3, 5,

A Student Used The Sieve Of Eratosthenes To Find All Prime Numbers Less Than 100 Create A Step By Step

A Student Used The Sieve Of Eratosthenes To Find All Prime Numbers Less Than 100 Create A Step By Step

Related Articles

- Calculate The Number Of Moles Of Each Substance In Sample With The A) 2.4g Of HeB)40g Of SulphurC)1.5kg
- A Proton Moves In The Negative X-direction Through A Uniform Magnetic Field In The Negative Y-direction
- A Package Boodins Weighs 6.2 Ounces. A Package Of Rews Weighs 9.97 Ounces. How Much More Does A Package

Back to Home