A Tank Is Full Of Water. Find The Work W Required To Pump The Water Out Of The Spout. (Use 9.8 M/s2 For

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When dealing with fluid mechanics and physics problems involving lifting water from a tank, understanding the work required to pump water out is essential. Whether you're a student studying physics, an engineer designing water systems, or simply curious about how much effort it takes to move water, this article provides a comprehensive guide. Using the acceleration due to gravity as 9.8 m/s², we'll explore how to calculate the work done in pumping water out of a tank through its spout.

Understanding the Problem

Before diving into calculations, it's important to understand the key elements involved:

- The tank is assumed to be full of water.
- The water must be pumped out through a spout at a certain height.
- Gravity acts on the water, providing the force that must be overcome to lift the water.
- Work is defined as the force applied over a distance, and in this context, it refers to the energy needed to move water from its initial position to the spout's outlet.

Fundamental Concepts in Calculating Work

To determine the work W required to pump the water out, we need to understand some core physics principles:

Work Done in Lifting a Small Volume of Water

- Work (W) is calculated as:

$$
W = \text{Force} \times \text{Distance}
$$

- When lifting water, the force is equal to the weight of the water segment, which is:

$$
\text{Force} = \text{mass} \times g
$$

- The mass of a small volume of water (dV) is:

$$
dm = \rho \, dV
$$

where:

- ρ = density of water (~1000 kg/m³)
- dV = small volume element

- The work to lift this small volume from height h to the spout's outlet at height H is:

$$
dW = \text{Force} \times \text{height difference} = \rho g dV \times (H - h)
$$

Total Work as an Integral

Since the water is distributed throughout the tank, we integrate over the entire volume of water:

$$
W = \int \rho g (H - h) dV
$$

The key is to express dV in terms of the variable height h.

Modeling the Tank for Calculation

The specific shape of the tank influences how we set up the integral. Common shapes include:

- Vertical Cylinder
- Rectangular Prism
- Spherical Tank

For simplicity, we'll assume a common shape — a vertical cylindrical tank.

Parameters for the Cylindrical Tank

- Radius of the tank: r
- Height of the tank: H_{tank}
- The spout is located at the top, at height H_{spout}

Calculating Work for a Cylindrical Tank

Suppose:

- The tank is filled to height H_{tank}
- The spout is at height H_{spout}
- The water is uniformly distributed from the bottom ($h=0$) to the top ($h=H_{tank}$)

Step 1: Set Up the Differential Volume Element dV

- Consider a thin horizontal slice of water at height h with thickness dh
- The volume of this slice:

$$
dV = \text{area} \times dh = \pi r^2 dh
$$

Step 2: Express the Work Integral

- The work to lift this slice from height h to the spout at H_{spout}:

$$
dW = \rho g \pi r^2 (H_{spout} - h) dh
$$

- The total work:

$$
W = \int_{h=0}^{H_{tank}} \rho g \pi r^2 (H_{spout} - h) dh
$$

Step 3: Perform the Integration

$$
W = \rho g \pi r^2 \int_{0}^{H_{tank}} (H_{spout} - h) dh
$$

Calculating the integral:

$$
\int (H_{spout} - h) dh = H_{spout} h - \frac{h^2}{2}
$$

Evaluate from 0 to H_{tank}:

$$
W = \rho g \pi r^2 \left[H_{spout} H_{tank} - \frac{H_{tank}^2}{2} \right]
$$

Final Formula for Work W

The total work required to pump out all water from the tank through the spout is:

$$
\boxed{
W = \rho g \pi r^2 \left(H_{spout} H_{tank} - \frac{H_{tank}^2}{2} \right)
}
$$

Where:

- ρ = 1000 kg/m^3 (density of water)
- g = 9.8 m/s^2 (acceleration due to gravity)
- r = radius of the tank
- H_{tank} = height of water in the tank
- H_{spout} = height of the spout

Practical Example

Suppose:

- A cylindrical tank with radius $r = 2\, \text{m}$
- Water height $H_{tank} = 5\, \text{m}$
- Spout located at height $H_{spout} = 1\, \text{m}$

Calculate the work:

$$
W = 1000 \times 9.8 \times \pi \times (2)^2 \times \left(1 \times 5 - \frac{5^2}{2} \right)
$$

Calculate step-by-step:

1. $\pi r^2 = \pi \times 4 \approx 12.566$

2. $H_{spout} H_{tank} = 1 \times 5 = 5$

3. $\frac{H_{tank}^2}{2} = \frac{25}{2} = 12.5$

4. Inner bracket: $5 - 12.5 = -7.5$

5. Now compute:

$$
W = 1000 \times 9.8 \times 12.566 \times (-7.5) = -1000 \times 9.8 \times 12.566 \times 7.5
$$

Since work cannot be negative, the negative sign indicates the direction of work (lifting against gravity). The magnitude is:

$$
W \approx 1000 \times 9.8 \times 12.566 \times 7.5 \approx 1000 \times 9.8 \times 94.245 \approx 1000 \times 923.9 \approx 923,900\, \text{J}
$$

Thus, approximately 924 kJ of work is needed to pump all the water out.

Extensions and Additional Considerations

While the above calculation provides a solid foundation, in real-world applications, several factors can influence the actual work required:

- Tank shape: Spherical, conical, or irregular shapes require different integration approaches.
- Friction and pump efficiency: Mechanical losses mean actual energy expenditure exceeds the ideal calculation.
- Variable water density: In some cases, density may vary due to temperature.
- Height of the spout: If the spout is lower than the tank's top, less work is needed, and vice versa.

Conclusion

Calculating the work required to pump water out of a tank involves understanding the principles of physics and fluid mechanics. By modeling the tank shape and setting up appropriate integrals, you can accurately estimate the energy needed for such tasks. Remember to consider the specific dimensions and heights involved, and always account for real-world inefficiencies when planning practical operations.

This comprehensive approach not only enhances your understanding of fluid lifting problems but also equips you with the tools to tackle similar physics challenges in engineering and everyday life.

Keywords: work calculation, pumping water, fluid mechanics, gravitational force, integral calculus, cylindrical tank, physical principles, engineering problem, energy expenditure

Frequently Asked Questions

How do you determine the work required to pump water out of a tank through a spout?

The work is calculated by integrating the weight of each infinitesimal layer of water times the distance it must be lifted to reach the spout, using the formula W = ⍰ (weight per unit volume) height dV across the water's depth.

What information is needed to compute the work to pump water out of a tank?

You need the tank's dimensions (height and cross-sectional area), the density of water (typically 1000 kg/m³), gravitational acceleration (9.8 m/s²), and the height of the spout above the bottom of the tank.

Why is the value 9.8 m/s² used in calculating work for pumping water?

Because 9.8 m/s² is the acceleration due to gravity, which is essential for calculating the weight of water and the work needed to lift it during the pumping process.

How does the shape of the tank affect the work required to pump out water?

The tank's shape determines the volume distribution at different heights, affecting how much water needs to be lifted at each level, thus influencing the total work calculation.

Can the work required to pump water be minimized? If so, how?

Yes, by designing the tank with a shape that reduces the average height water must be lifted or by placing the spout at a lower height, the total work can be minimized.

How is the integral set up to find the work in this problem?

The integral is set up as W = ⍰ from bottom to top of (density volume per unit height) g height, where height varies from the bottom of the tank to the spout level, summing the work for each water layer.

What assumptions are made in calculating the work to pump water out of the tank?

Assumptions include uniform water density, neglecting friction and pump efficiency, and considering the tank as a static system with known dimensions and height of the spout.

Additional Resources

A Tank Is Full Of Water. Find The Work W Required To Pump The Water Out Of The Spout. (Use 9.8 m/s^2 for gravity)

Introduction

In the realm of physics and engineering, understanding the work required to move fluids within containers is fundamental to designing efficient systems and solving practical problems. One classic problem involves calculating the work needed to pump water out of a tank through a spout, especially when the tank is filled to capacity. This scenario exemplifies principles of fluid mechanics, energy conservation, and calculus. Here, we conduct a detailed investigation into the problem, elucidate the underlying physics, and derive a comprehensive formula for the work W required.

Problem Overview

Consider a tank filled completely with water. The goal is to determine the work W needed to pump all the water out through a spout located at the top of the tank. To approach this problem systematically, the following parameters are essential:

- Tank dimensions: The shape (e.g., cylindrical, rectangular) and size.
- Water properties: Density ρ, typically $1000\, \text{kg/m}^3$.
- Gravity: $g = 9.8\, \text{m/s}^2$.
- Height of water column: Varies depending on tank shape.
- Position of spout: Usually at the top of the tank.
- Flow considerations: Assumptions about ideal flow (no frictional losses), static conditions, and steady pumping.

In this analysis, we'll focus on a common scenario—a cylindrical tank of known height and radius—to derive a general formula applicable to similar cases.

Assumptions and Simplifications

To make the problem tractable, we adopt the following assumptions:

1. Ideal fluid: No viscosity or frictional losses.
2. Static water: The water is initially at rest.
3. Steady pumping: The pump operates slowly enough that kinetic energy transfer can be neglected, and the work calculation is based solely on potential energy considerations.
4. Vertical spout at the top: The water is lifted from various depths to the spout at the top.
5. Uniform density: The water density remains constant throughout.

These assumptions allow us to employ calculus-based methods to compute the work.

Geometry of the Tank

For clarity, let's define the tank as a vertical cylinder:

- Height of tank: H
- Radius of tank: R

The coordinate system is set with the bottom of the tank at $y = 0$ and the top at $y = H$.

The Conceptual Approach: Differential Work Calculation

The total work required to pump all water out of the tank can be computed by integrating the work needed to lift infinitesimal elements of water from their initial position to the outlet.

Key idea:
- Consider a thin horizontal slice of water at height y, with thickness dy.
- The volume of this slice: $dV = A \, dy$, where A is the cross-sectional area.
- The mass of this slice: $dm = \rho \, dV = \rho \, A \, dy$.
- The weight of this slice: $dW_{\text{weight}} = dm \, g = \rho \, A \, g \, dy$.

Since the water must be lifted from height y to the spout at the top H, the lifting distance is $(H - y)$.

Work to lift this slice:
$$dW = \text{(Force)} \times \text{(Distance)} = dW_{\text{weight}} \times (H - y) = \rho \, A \, g \, (H - y) \, dy$$

The total work is obtained by integrating y from 0 to H:

$$
W = \int_{0}^{H} \rho \, A \, g \, (H - y) \, dy
$$

Derivation of the Work Formula

Step 1: Set up the integral

$$
W = \rho \, A \, g \int_{0}^{H} (H - y) \, dy
$$

Step 2: Evaluate the integral

\[

$$
\int_{0}^{H} (H - y) \, dy = H y - \frac{1}{2} y^2 \bigg|_{0}^{H} = H \times H - \frac{1}{2} H^2 = H^2 - \frac{1}{2} H^2 = \frac{1}{2} H^2
$$

Step 3: Write the total work

$$
W = \rho \, A \, g \times \frac{1}{2} H^2
$$

Final Formula

$$
\boxed{
W = \frac{1}{2} \rho \, A \, g \, H^2
}
$$

This formula indicates that the work required is proportional to the cross-sectional area A, the square of the height H, the density ρ, and gravity g.

Applying the Formula to a Cylindrical Tank

Given:

- $\rho = 1000\, \text{kg/m}^3$
- $g = 9.8\, \text{m/s}^2$
- Cross-sectional area: $A = \pi R^2$
- Height of water: H

The work becomes:

$$
W = \frac{1}{2} \times 1000 \times \pi R^2 \times 9.8 \times H^2
$$

or

$$
W = 4900 \pi R^2 H^2
$$

This expression provides a direct way to compute the work once the tank dimensions are known.

Practical Considerations and Extensions

While the idealized model provides a solid foundation, real-world scenarios often involve additional complexities:

- Frictional losses in pipes and hoses.
- Variable cross-sections if the tank shape differs.
- Flow rate constraints affecting the pumping speed.
- Energy efficiency considerations.

- Pump power ratings needed for specific applications.

Moreover, if the spout is located at a different height or the tank shape is irregular, the integral limits and cross-sectional area $A(y)$ will vary accordingly, requiring numerical methods for precise calculations.

Additional Insights: Comparing Theoretical and Practical Values

In practical engineering, the calculation of work often involves considering energy losses and inefficiencies. For example:

- Frictional losses can significantly increase the energy required.
- Pump efficiencies typically range from 70% to 90%, meaning actual energy input exceeds the theoretical work.

Therefore, engineers incorporate safety factors and efficiency coefficients into their design calculations.

Conclusion

Understanding the work involved in pumping water out of a full tank is a classic problem that elegantly illustrates the application of calculus and physics principles. The derived formula highlights the relationship between the physical dimensions of the tank, the properties of water, and the energy expenditure required. This analysis serves as a foundational tool for engineers designing pumping systems, ensuring energy efficiency and operational effectiveness.

By modeling the tank as a series of thin slices and integrating the work needed to lift each slice to the spout height, we obtain a clear, quantitative measure of the energy costs associated with such operations. Extending this approach to more complex geometries or real-world conditions involves additional factors but remains rooted in the same fundamental principles demonstrated here.

References

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Note: The specific numerical results depend on actual tank dimensions. For tailored calculations, substitute your parameters into the formulas provided.

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