A Tank Is Half Full Of Oil That Has A Density Of 900 Kg/m3. Find The Work W (in J) Required To Pump The

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Understanding the work involved in pumping fluids is a fundamental concept in fluid mechanics and engineering. In this comprehensive guide, we will explore the problem of calculating the work required to pump oil from a tank, given specific parameters such as the oil's density and the tank's configuration. This article aims to provide a detailed, step-by-step explanation suitable for students, engineers, and enthusiasts interested in fluid dynamics and related fields.

Introduction to Fluid Work and Pumping Problems

In fluid mechanics, the work done to move a fluid from one point to another depends on several factors, including the fluid's density, the volume of fluid moved, and the height difference over which it is moved. When dealing with tanks containing liquids, calculating the work involves understanding the distribution of the fluid's weight and the height it must be lifted.

Key concepts:

- Density (ρ): The mass per unit volume of the fluid, measured in kilograms per cubic meter (kg/m^3).
- Volume (V): The total volume of the fluid in the tank.
- Work (W): The energy required to pump the fluid, measured in joules (J).
- Force: The weight of the fluid being moved.
- Distance: The vertical height the fluid must be lifted.

Understanding the Problem Setup

Let's define the problem more precisely:

- The tank is half full of oil.
- The density of oil (ρ) is 900 kg/m^3.
- The task is to calculate the work (W) required to pump the oil out of the

tank.

To proceed, we need additional details such as:

- The shape and dimensions of the tank.
- The initial height of the oil level.
- The height to which the oil must be lifted during pumping.

Since these are not explicitly provided, we will consider a common scenario: a cylindrical tank with known dimensions and the oil being pumped out from the bottom to the top of the tank.

Assumptions and Simplifications

To create a workable model, we assume:

- The tank is a vertical cylinder.
- The cross-sectional area of the tank is A (in m^2).
- The height of the tank is H (in meters).
- The tank is initially filled to half its height, i.e., the oil level is at H/2.
- The oil is pumped from the bottom to the top of the tank, requiring lifting through a height of H/2.
- The pumping process is done slowly, so we can consider it as a quasi-static process, ignoring dynamic effects.

Step-by-Step Calculation of Work Required

The calculation involves integrating the work needed to lift infinitesimal slices of oil from their initial position to the outlet point.

1. Define Variables and Coordinates

- Let y be the vertical coordinate, with y=0 at the bottom of the tank.
- The oil surface is at y=H/2.
- The oil extends from y=0 to y=H/2.
- The height the oil must be lifted is from y to the top H.

2. Determine the Volume Element

An infinitesimal slice of oil at height y with thickness dy:

$$ dV = A \, dy $$

where:

- A is the cross-sectional area of the tank.
- dy is the infinitesimal thickness.

3. Find the Mass of the Slice

Mass of the slice:

$$ dm = \rho \, dV = \rho \, A \, dy $$

4. Calculate the Work to Pump Each Slice

Work to lift this slice from height y to the outlet at height H:

$$ dW = \text{force} \times \text{distance} $$

Force is the weight of the slice:

$$ dF = dm \times g = \rho \, A \, dy \times g $$

where g is the acceleration due to gravity (9.81 m/s^2).

The distance the slice must be lifted:

$$ (H - y) $$

Therefore:

$$ dW = dF \times (H - y) = \rho \, A \, dy \times g \times (H - y) $$

5. Integrate Over the Oil Height

Total work:

$$ W = \int_{y=0}^{H/2} \rho \, A \, g \, (H - y) \, dy $$

Calculating:

$$ W = \rho \, A \, g \int_{0}^{H/2} (H - y) \, dy $$

Performing the integral:

$$ W = \rho \, A \, g \left[H y - \frac{1}{2} y^2 \right]_0^{H/2} $$

Plugging in the limits:

$$ W = \rho \, A \, g \left(H \times \frac{H}{2} - \frac{1}{2} \left(\frac{H}{2} \right)^2 \right) $$

Simplify:

$$ W = \rho \, A \, g \left(\frac{H^2}{2} - \frac{1}{2} \times \frac{H^2}{4} \right) $$

$$ W = \rho \, A \, g \left(\frac{H^2}{2} - \frac{H^2}{8} \right) $$

$$ W = \rho \, A \, g \times \frac{4H^2 - H^2}{8} = \rho \, A \, g \times \frac{3H^2}{8} $$

Final Expression for Work

$$
\boxed{
W = \frac{3}{8} \, \rho \, A \, g \, H^2
}
$$

This formula gives the work in joules, considering the parameters:

- ρ: density of oil (900 kg/m^3)
- A: cross-sectional area of the tank
- g: acceleration due to gravity (9.81 m/s^2)
- H: total height of the tank

Application of the Formula with Example Values

Suppose we have a cylindrical tank with:

- Cross-sectional area $A = 2$ m^2
- Total height $H = 4$ m

Given these parameters, calculate the work:

$$
W = \frac{3}{8} \times 900 \times 2 \times 9.81 \times 4^2
$$

Step-by-step:

1. Calculate H^2:

$$
4^2 = 16
$$

2. Multiply parameters:

$$
W = \frac{3}{8} \times 900 \times 2 \times 9.81 \times 16
$$

3. Simplify:

- $\frac{3}{8} \times 900 = 337.5$
- $337.5 \times 2 = 675$
- $675 \times 9.81 \approx 6623.25$
- $6623.25 \times 16 \approx 105972$

Final work:

$$
W \approx 105,972 \text{ Joules}
$$

So, approximately 106 kJ of work is required to pump the oil out of this tank.

Additional Considerations and Real-World Factors

While the above calculation provides a theoretical estimate, real-world scenarios may involve additional factors:

- Pump efficiency: Actual pumps are not 100% efficient; accounting for efficiency reduces the net work.
- Friction and head losses: Pipe friction and other losses increase the actual energy needed.

- Fluid viscosity: Higher viscosity fluids require more energy to pump.
- Tank shape: Non-cylindrical tanks require a different approach to calculate volume and height relationships.
- Pump placement: Pumping from the top or side alters the lifting height.

Conclusion and Summary

Calculating the work required to pump oil from a tank involves understanding the fluid's properties, the tank's geometry, and applying integral calculus to account for the variable weight of fluid slices at different heights. The key formula derived:

$$
W = \frac{3}{8} \, \rho \, A \, g \, H^2
$$

provides a straightforward method to estimate the energy needed in joules, given the tank's dimensions and the fluid's density.

This approach exemplifies the application of principles in fluid mechanics and physics to solve practical engineering problems, essential for designing efficient pumping systems, optimizing energy consumption, and ensuring safe operation of fluid handling equipment.

FAQs about Pumping Oil from Tanks

- **What is the significance of the oil's density in calculating work?** The density determines the mass of each fluid slice, directly impacting the force and energy required to pump

Frequently Asked Questions

What is the primary physical principle involved in calculating the work required to pump oil from a tank?

The primary principle is the work done against gravity to lift the oil, which involves integrating the weight of the displaced fluid over its height in the tank.

How is the density of the oil relevant to calculating the work needed to pump it?

The density determines the mass of the oil per unit volume, which directly affects the weight of the oil being lifted and thus the work required.

What assumptions are typically made in calculating the work to pump oil from a tank?

Assumptions often include uniform density, incompressible fluid, steady flow, and neglecting losses due to friction or turbulence.

How do you set up the integral to find the work done in pumping the oil from a half-full tank?

You integrate the weight of each infinitesimal layer of oil over the height it must be lifted, typically using the formula W = ∫(density volume_element gravity height) over the relevant limits.

What additional information is needed to calculate the exact work required in this problem?

You need the total height of the tank, the height of the oil level, the cross-sectional area of the tank, and the height to which the oil must be pumped.

If the tank is cylindrical and half full, how does the shape of the tank affect the calculation of work?

The shape determines the volume distribution at different heights, requiring integration over the tank's cross-sectional area to accurately compute the work.

What is the significance of the oil's density being 900 kg/m^3 in the calculation?

It allows calculation of the mass of the oil for each volume element, which is essential for determining the weight and thus the work needed to pump it.

Can the work required to pump the oil be expressed in terms of the tank's dimensions? If so, how?

Yes, by expressing the volume and height variables in terms of the tank's dimensions (such as radius and height), the integral can be evaluated to give the work in terms of these parameters.

Additional Resources

A Tank Is Half Full Of Oil That Has A Density Of 900 Kg/m^3. Find The Work W (in J) Required To Pump The Oil

Understanding the calculation of work required to pump a fluid from a tank involves a combination of principles from fluid mechanics, physics, and calculus. In this comprehensive review, we will explore the problem statement, the physical concepts involved, the step-by-step approach to solving such problems, and key considerations to ensure accurate and efficient calculations. The scenario of a tank half-filled with oil of known density provides an excellent example to illustrate the application of these principles.

Introduction to the Problem

The problem involves a tank filled with oil to a certain level, with the

goal of calculating the work needed to pump the oil out of the tank. The specifics are:
- The tank is half full of oil.
- The density of the oil is 900 kg/m^3.
- The task is to determine the work W in joules required to pump this oil out of the tank.

This problem encapsulates fundamental concepts like fluid statics, the relationship between force, work, and displacement, and the use of integrals to account for varying distances that different layers of fluid must be moved.

Fundamental Concepts and Principles

Work in Physics

Work is defined as the transfer of energy when a force causes displacement. Mathematically:

$$ W = \text{Force} \times \text{Displacement} $$

or, for variable forces and displacements, an integral form:

$$ W = \int F\, dx $$

In fluid mechanics, the work to pump a fluid depends on the force needed to move each element of fluid over a certain height.

Fluid Density and Volume

The density (ρ) of the oil determines the mass of a given volume:

$$ m = \rho V $$

where V is the volume of the fluid.

Hydrostatic Pressure and Force

The pressure exerted by a fluid at a depth h is:

$$ P = \rho g h $$

where g is acceleration due to gravity.

To pump fluid out of the tank, one must work against this hydrostatic pressure, moving each element of fluid from its initial position to the outlet.

Application of Calculus in Fluid Pumping

Since the tank's cross-sectional area and fluid height vary, the problem is best approached by slicing the fluid into thin layers and integrating the work over these layers.

Step-by-Step Approach to Calculating Work

1. Define the Geometry of the Tank

The shape of the tank significantly influences the calculation. Common shapes include:
- Rectangular
- Cylindrical
- Spherical

For simplicity, assume a rectangular tank with:
- Length L
- Width W
- Height H

The problem states the tank is half full, so the oil height is:
$$ h_{oil} = \frac{H}{2} $$

2. Establish Coordinates and Variables

Use a coordinate system where y measures the vertical position from the bottom of the tank:
- $y=0$ at the tank's bottom.
- $y=H$ at the top.

Since the tank is half full:
$$ y \in [0, H/2] $$
with the oil occupying the region $0 \leq y \leq H/2$.

3. Determine the Differential Volume Element

Divide the oil into thin horizontal slices of thickness dy:
$$ dV = A\,dy $$

where A is the cross-sectional area (for a rectangular tank: $A = L \times W$).

4. Calculate the Mass and Weight of Each Layer

Mass of a thin layer at height y:
$$ dm = \rho\, dV = \rho A\, dy $$

Weight of this layer:
$$ dW_{weight} = dm\, g = \rho A g\, dy $$

5. Determine the Work to Pump Each Layer

To lift the oil layer at height y to the outlet (assumed at the top of the tank at H), the vertical distance it must be moved:
$$ y_{lift} = H - y $$

The work to lift this layer:
$$ dW_{layer} = dW_{weight} \times y_{lift} = \rho A g (H - y) dy $$

6. Integrate Over the Oil Height

Total work:
$$ W = \int_{y=0}^{H/2} \rho A g (H - y)\, dy $$

Compute the integral:
$$
W = \rho A g \int_{0}^{H/2} (H - y) dy
$$
$$
W = \rho A g \left[H y - \frac{1}{2} y^2 \right]_0^{H/2}
$$
$$
W = \rho A g \left(H \times \frac{H}{2} - \frac{1}{2} \times \left(\frac{H}{2}\right)^2 \right)
$$
$$
W = \rho A g \left(\frac{H^2}{2} - \frac{1}{2} \times \frac{H^2}{4} \right)
$$
$$
W = \rho A g \left(\frac{H^2}{2} - \frac{H^2}{8} \right)
$$
$$
W = \rho A g \times \frac{4H^2 - H^2}{8} = \rho A g \times
$$

\frac{3H^2}{8}
\]

Final Expression:
\[
\boxed{
W = \frac{3}{8} \rho A g H^2
}
\]

Application with Numerical Values

Suppose specific dimensions:
- \(L = 2\, \text{m}\)
- \(W = 1\, \text{m}\)
- \(H = 4\, \text{m}\)

Calculate the work:
- Cross-sectional area: \(A = 2 \times 1 = 2\, \text{m}^2\)
- Density: \(\rho = 900\, \text{kg/m}^3\)
- Gravitational acceleration: \(g = 9.81\, \text{m/s}^2\)

Plug into the formula:
\[
W = \frac{3}{8} \times 900 \times 2 \times 9.81 \times (4)^2
\]
\[
W = \frac{3}{8} \times 900 \times 2 \times 9.81 \times 16
\]
Calculate step-by-step:
- \(900 \times 2 = 1800\)
- \(1800 \times 9.81 \approx 17,658\)
- \(17,658 \times 16 = 282,528\)
- Multiply by \(\frac{3}{8} \approx 0.375\):

\[
W \approx 0.375 \times 282,528 \approx 105,948\, \text{J}
\]

Thus, approximately 106 kJ of work is required to pump the oil out of the tank.

Features, Pros, and Cons of the Methodology

Features:
- Uses integration over the fluid's height to account for varying work contributions.
- Applicable to tanks of various shapes with appropriate geometric adjustments.
- Incorporates physical constants and properties like density and gravity.

Pros:
- Provides an exact and reliable method for calculating work.
- Can be adapted for different fluid densities and tank geometries.
- Demonstrates the application of calculus to real-world engineering problems.

Cons:
- Assumes ideal conditions, neglecting factors like friction, viscosity, and pump inefficiencies.
- Requires precise knowledge of tank dimensions and fluid properties.
- For complex tank shapes, the integral setup becomes more complicated.

Additional Considerations and Variations

- Pump Efficiency: Real pumps are not 100% efficient; accounting for efficiency reduces the actual work needed.
- Viscosity and Friction: These factors increase the energy required, especially for viscous fluids.
- Dynamic Pumping: If the fluid is moved at a specific rate, additional considerations like transient effects come into play.
- Different Outlet Positions: If the outlet is not at the top or is located at a different height, the integral limits and distance calculations change accordingly.
- Other Tank Shapes: For non-rectangular tanks, the cross-sectional area $A(y)$ varies with height, necessitating more complex integrals.

Conclusion

Calculating the work required to pump oil from a half-filled tank involves understanding the physical principles of fluid statics and dynamics, setting up the correct integral, and performing precise calculations. The key steps include defining the geometry, slicing the fluid into thin layers, calculating the work for each layer, and integrating over the entire height of the fluid. The derived formula provides a

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