

A Triangle With An Area Of 23 Cm Is Dilated By A Factor Of 6. What Is The Area Of The Dilated Triangle?

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Understanding geometric transformations is fundamental in the study of geometry, especially when it involves shapes like triangles. When a triangle undergoes dilation—a type of transformation that enlarges or reduces a figure proportionally—its dimensions change in a predictable manner. Specifically, the area of the triangle scales by the square of the scale factor. In this article, we will explore how to determine the area of a triangle after dilation, using the example of a triangle with an initial area of 23 cm² that is dilated by a factor of 6. We will delve into the concepts of dilation, scale factors, and area calculation, providing a comprehensive understanding suitable for students, educators, and math enthusiasts.

Understanding Dilation and Scale Factors in Geometry

What Is Dilation?

Dilation is a transformation in geometry that produces an image that is the same shape as the original but is scaled larger or smaller. It involves a fixed point called the center of dilation and a scale factor that determines how much the figure is enlarged or reduced.

Key characteristics of dilation:

- It preserves the shape of the figure (similarity).
- It changes the size proportionally.
- The center of dilation is the point about which the figure is scaled.

What Is a Scale Factor?

The scale factor in dilation is a number that indicates how much the figure is expanded or contracted. It can be:

- Greater than 1: The figure enlarges.
- Equal to 1: The figure remains unchanged.
- Between 0 and 1: The figure reduces in size.

For example, a scale factor of 6 means every distance from the center of dilation is multiplied by 6.

Relationship Between Dilation and Area

Area Scaling in Dilation

While linear dimensions such as length or height are multiplied directly by the scale factor, the area of a figure scales differently. The area of a figure after dilation is proportional to the square of the scale factor.

Mathematically:

$$\text{New Area} = \text{Original Area} \times (\text{Scale Factor})^2$$

This is a fundamental rule in geometry concerning similar figures:

- If a figure is scaled by a factor of (k) , then its area scales by (k^2) .

Implication for Triangles

Since triangles are similar figures when scaled, their areas follow the same rule. If you know the original area and the scale factor, you can easily compute the new area after dilation.

Example:

- Original area: 23 cm²
- Scale factor: 6
- New area: $(23 \times 6^2 = 23 \times 36 = 828, \text{cm}^2)$

Next, we will walk through the step-by-step process to compute the area of the dilated triangle.

Step-by-Step Calculation of the Dilated Triangle's Area

Step 1: Identify the given data

- Original area of the triangle: 23 cm²
- Scale factor of dilation: 6

Step 2: Apply the area scaling rule

Use the formula:

$$\text{Dilated Area} = \text{Original Area} \times (\text{Scale Factor})^2$$

Substituting the known values:

$$\text{Dilated Area} = 23 \times 6^2$$

Step 3: Calculate the square of the scale factor

$$6^2 = 36$$

Step 4: Compute the new area

$$\text{Dilated Area} = 23 \times 36 = 828 \text{ cm}^2$$

Therefore, the area of the dilated triangle is 828 cm².

Additional Considerations and Real-World Applications

Why Does the Area Scale by the Square of the Scale Factor?

This occurs because area measures two-dimensional space, and when a figure is scaled linearly, both dimensions (length and width) are multiplied by the scale factor. As area depends on two dimensions, the scale factor impacts the area quadratically.

Illustrative analogy:

- Imagine a square with side length 1 cm.
- Its area is $1 \times 1 = 1 \text{ cm}^2$.
- If scaled by a factor of 3, the new side length is 3 cm.
- New area: $3 \times 3 = 9 \text{ cm}^2$.
- The area increased by a factor of $3^2 = 9$.

Practical Applications of Dilation and Area Calculations

Understanding dilation's effect on area is useful across various fields, such as:

- Architecture: scaling blueprints and calculating surface areas.
- Engineering: designing scaled models of structures.
- Art and Design: creating proportionate enlargements.
- Education: teaching concepts of similarity and scale.

Summary and Key Takeaways

- Dilation is a transformation that enlarges or reduces a figure proportionally, centered around a fixed point.
- The scale factor determines how much the figure is scaled.
- When a triangle (or any similar figure) is dilated, its area scales by the square of the scale factor.
- For a triangle with an initial area of 23 cm^2 dilated by a factor of 6, the new area is calculated as:

$$\boxed{828 \text{ cm}^2}$$

This understanding enables precise calculations in geometric transformations and helps visualize real-world scaling problems.

Conclusion

In conclusion, when dealing with geometric figures such as triangles, understanding the relationship between dilation and area is essential. The key takeaway is that the area scales by the square of the scale factor, making calculations straightforward once the scale factor and original area are known. For the given example of a triangle with an initial area of 23 cm^2 dilated by a factor of 6, the resulting area is 828 cm^2 . Mastery of these concepts aids in solving complex problems involving similar figures, scaling, and transformations in both academic and practical contexts.

Frequently Asked Questions

What is the area of the original triangle before dilation?

The area of the original triangle is 23 cm^2 .

By what factor is the triangle dilated?

The triangle is dilated by a factor of 6.

How does dilation affect the area of a triangle?

When a triangle is dilated by a factor of k , its area is multiplied by k^2 .

What is the formula for the area of a dilated

triangle?

The area of the dilated triangle = original area $\times$ (dilation factor)².

Calculate the area of the dilated triangle.

The area = $23 \text{ cm}^2 \times 6^2 = 23 \times 36 = 828 \text{ cm}^2$.

Why do we square the dilation factor when calculating the new area?

Because area depends on two dimensions, and each dimension is scaled by the dilation factor, so the area scales by its square.

Is the area of the dilated triangle larger or smaller than the original?

It is larger; specifically, 828 cm^2 compared to the original 23 cm^2 .

If the original area was 23 cm^2 , what would be the area after a dilation of 3?

It would be $23 \text{ cm}^2 \times 3^2 = 23 \times 9 = 207 \text{ cm}^2$.

Can the area of the dilated triangle be calculated using linear scale factor alone?

No, the linear scale factor affects only length, but area scales with the square of the scale factor.

What is the importance of understanding scale factors in geometry?

Understanding scale factors helps in accurately calculating how dimensions and areas change during dilation or similar transformations.

Additional Resources

A Triangle With An Area Of 23 cm^2 Is Dilated By A Factor Of 6. What Is The Area Of The Dilated Triangle?

Understanding the effects of dilation on geometric figures is a fundamental concept in geometry, especially when it comes to triangles. In this article, we will explore the process of calculating the area of a triangle after dilation, focusing on the specific problem where a triangle with an initial area of 23 cm^2 is dilated by a factor of 6. We will delve into the principles

of dilation, the relationship between scale factors and area changes, and work through the problem step-by-step to enhance comprehension.

Introduction to Dilation and Its Impact on Area

Dilation is a transformation that produces an image that is the same shape as the original but is a different size. It is characterized by a scale factor, which determines how much the figure is enlarged or reduced. When a figure undergoes dilation:

- The shape remains similar to the original.
- The size of the figure changes proportionally based on the scale factor.
- The distances from the center of dilation are multiplied by the scale factor.

Understanding how dilation affects area is crucial because the area does not change linearly with the scale factor; instead, it changes according to the square of the scale factor.

Key Principles of Dilation and Area Transformation

- Scale Factor (k): The ratio by which lengths are multiplied during dilation.
- Area Scale Factor: The ratio of the areas of the dilated figure to the original figure. It equals the square of the scale factor.
- Effect on Area: If the original figure has an area (A) , then after dilation by a scale factor (k) , the new area (A') is given by:

$$A' = A \times k^2$$

This principle is particularly important in problems involving dilations because it allows us to calculate the new area once the scale factor is known.

Applying the Concept to the Given Problem

Let's analyze the specific problem:

> A triangle with an area of 23 cm² is dilated by a factor of 6. What is the area of the dilated triangle?

To solve this, we will follow a systematic approach.

Step 1: Identify Known Values

- Original area, $(A = 23\text{ cm}^2)$
- Scale factor, $(k = 6)$

Step 2: Understand What is Needed

- Find the area of the dilated triangle, (A') .

Step 3: Apply the Area Scale Factor Formula

Using the principle that the area scales by the square of the scale factor:

$$A' = A \times k^2$$

Substituting the known values:

$$A' = 23 \times 6^2$$

Calculating:

$$A' = 23 \times 36$$

$$A' = 828\text{ cm}^2$$

Therefore, the area of the dilated triangle is 828 cm^2 .

Understanding the Result and Its Implications

The calculation shows that when a figure is dilated by a scale factor of 6, its area increases by a factor of $(6^2 = 36)$. This quadratic relationship emphasizes how even moderate changes in linear dimensions lead to significant changes in area.

Features and Significance of the Calculation

- Proportional Increase: The area increases proportionally with the square of the scale factor.
- Predictability: The formula allows easy prediction of the new area after dilation.
- Applicability: This principle applies universally to similar figures, not just triangles.

Pros and Cons of Using the Dilation Area Formula

Pros:

- Simple to use once the scale factor and original area are known.
- Provides quick and accurate results.
- Reinforces understanding of similarity and proportional reasoning.

Cons:

- Assumes the dilation is centered and uniform, which might not always be the case in real-world applications.
- Less intuitive when dealing with irregular figures or non-uniform transformations.
- Requires understanding of the relationship between linear scale factors and area scale factors.

Extended Applications and Related Concepts

Understanding how dilation affects area is foundational in various fields:

- Geometry and Design: Ensuring proportions are maintained when resizing shapes.
- Physics and Engineering: Scaling models and understanding how size changes affect area-related properties.
- Art and Architecture: Maintaining proportions during scaling of plans or models.

Additionally, this concept extends beyond 2D figures to 3D objects, where volume scales with the cube of the scale factor.

Summary and Final Thoughts

In conclusion, the problem of determining the area of a triangle after dilation illustrates a fundamental geometric principle: the area of a figure

scales with the square of the linear scale factor. For a triangle with an initial area of 23 cm^2 dilated by a factor of 6, the new area becomes 828 cm^2 , highlighting the significant impact of dilation on size and area. This understanding is essential for students and professionals working with geometric transformations, design, and modeling.

By mastering these concepts, one can confidently solve similar problems, analyze transformations, and appreciate the beauty and consistency of geometric principles across various contexts. Remember, the key takeaway is to always consider the square of the scale factor when dealing with area changes in dilation, ensuring accurate and efficient calculations.

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