A Uniform Rod Is Hung At Onen End And Is Partially Submerged In Water. If The Density Of The Rod Is 5/9

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Understanding the principles of buoyancy, density, and equilibrium is crucial when analyzing the behavior of objects immersed in fluids. In particular, studying a uniform rod that is hung at one end and partially submerged in water provides valuable insights into how forces act on submerged bodies, how density influences buoyant forces, and how to determine the conditions for equilibrium. This article delves into these concepts, explaining the physics behind the scenario, deriving essential formulas, and exploring related applications.

Introduction to Buoyancy and Density

Before examining the specific case of a partially submerged rod, it is important to grasp the fundamental principles involved:

What Is Buoyancy?

Buoyancy is the upward force exerted by a fluid on an object placed within it. According to Archimedes' principle:

- The buoyant force acting on an object submerged in a fluid is equal to the weight of the fluid displaced by the object.

Mathematically:

$$F_b = \rho_{water} \times V_{displaced} \times g$$

where:

- F_b is the buoyant force,
- ρ_{water} is the density of water,
- $V_{displaced}$ is the volume of water displaced,
- g is acceleration due to gravity.

Density and Its Role

Density (ρ) is defined as mass per unit volume:

$$\rho = \frac{m}{V}$$

The ratio of the density of the object to the density of the fluid determines whether the object floats, sinks, or remains in equilibrium.

- If $\rho_{object} < \rho_{water}$, the object tends to float.
- If $\rho_{object} > \rho_{water}$, the object tends to sink.
- If $\rho_{object} = \rho_{water}$, the object remains neutrally buoyant.

In our case, the density of the rod is given as a fraction of water's density: $\frac{5}{9} \rho_{water}$.

Scenario Description: A Uniform Rod Hung at One End

Imagine a uniform rod of length L, made of a material with density $\rho_{rod} = \frac{5}{9} \rho_{water}$. The rod is suspended at one end, say the top end, and is partially submerged in water, such that a certain length l_{sub} of the rod is underwater.

This setup raises several questions:

- What is the position of the center of gravity of the rod?
- How much of the rod is submerged at equilibrium?
- What are the conditions for the rod to remain in equilibrium?
- How do the forces balance, considering buoyancy and weight?

Analyzing the Submersion of the Rod

Determining the Volume and Mass of the Rod

Let’s define:

- A: cross-sectional area of the rod (assumed uniform),
- L: total length of the rod,
- $V_{rod} = A \times L$: volume of the rod,
- $m_{rod} = \rho_{rod} \times V_{rod}$: mass of the rod.

Given $\rho_{rod} = \frac{5}{9} \rho_{water}$, the mass becomes:

$$m_{rod} = \frac{5}{9} \rho_{water} \times A \times L$$

The weight of the rod:

$$W_{rod} = m_{rod} \times g = \frac{5}{9} \rho_{water} \times A \times L \times g$$

Conditions for Equilibrium

For the rod to remain in equilibrium while hanging at one end with partial submersion:

- The sum of forces must be zero.
- The sum of moments about the pivot point must be zero.

The forces involved:

- The weight of the entire rod acts at its center of gravity, which for a uniform rod is at its midpoint — at $L/2$ from the hanging end.
- The buoyant force acts upward at the centroid of the submerged volume, which is at the midpoint of the submerged part ($l_{sub}/2$ from the water surface), but since the rod is hung at one end, the buoyant force acts at the centroid of the submerged part relative to the pivot.

By analyzing moments about the pivot point, we can derive the conditions for equilibrium.

Calculating the Submerged Length and Buoyant Force

Assuming the rod is just in equilibrium, the buoyant force F_b must balance the weight component tending to rotate the rod:

- The buoyant force acts upward at the centroid of the submerged part.
- The weight acts downward at the center of mass of the rod.

The volume of displaced water:

$$V_{displaced} = A \times l_{sub}$$

Corresponding buoyant force:

$$F_b = \rho_{water} \times A \times l_{sub} \times g$$

The moments about the pivot point (the top end):

- Moment due to weight:

$$ M_{weight} = W_{rod} \times \frac{L}{2} $$

- Moment due to buoyant force:

$$ M_{buoyancy} = F_b \times \frac{l_{sub}}{2} $$

For equilibrium (no rotation):

$$ M_{buoyancy} = M_{weight} $$

Substituting the expressions:

$$ \rho_{water} \times A \times l_{sub} \times g \times \frac{l_{sub}}{2} = \frac{5}{9} \rho_{water} \times A \times L \times g \times \frac{L}{2} $$

Simplify:

$$ l_{sub}^2 = \frac{5}{9} L^2 $$

Thus:

$$ l_{sub} = L \times \sqrt{\frac{5}{9}} = L \times \frac{\sqrt{5}}{3} $$

This result indicates the submerged length in terms of the total length of the rod.

Implications and Practical Applications

Understanding the behavior of such a system has numerous real-world applications, including:

Design of Floating and Partially Submerged Structures

Engineers use similar principles when designing ships, submarines, and floating platforms, ensuring they achieve the correct submersion levels for stability and buoyancy.

Material Selection Based on Density

Choosing materials with specific densities allows for control over how much of an object will be submerged, which is essential in manufacturing and construction.

Analyzing Stability and Equilibrium

The analysis of moments and forces helps determine whether an object will tip over or remain stable when partially submerged, crucial for safety assessments.

Additional Considerations

While the above analysis provides a theoretical framework, real-world scenarios often involve additional factors:

- Fluid dynamics and turbulence, affecting stability.
- Surface tension and adhesion, especially for small objects.
- Non-uniform density if the material isn't perfectly uniform.
- External forces, such as currents, wind, or added weights.

Summary of Key Formulas and Results

Parameter	Formula / Explanation
Density of rod	$\rho_{rod} = \frac{5}{9} \rho_{water}$
Volume of rod	$V_{rod} = A \times L$
Mass of rod	$m_{rod} = \rho_{rod} \times V_{rod}$
Weight of rod	$W_{rod} = m_{rod} \times g$
Volume of displaced water	$V_{displaced} = A \times l_{sub}$
Buoyant force	$F_b = \rho_{water} \times V_{displaced} \times g$
Submerged length at equilibrium	$l_{sub} = L \times \frac{\sqrt{5}}{3}$

In conclusion, analyzing a uniformly dense rod partially submerged in water involves understanding the interplay between buoyant forces, weight, and moments for equilibrium. The specific density ratio of $\frac{5}{9}$ leads to

Frequently Asked Questions

What is the significance of the density of the rod being 5/9 in the context of buoyancy?

A density of 5/9 indicates that the rod is less dense than water (density of water is 1), so it will be partially submerged rather than sinking completely, affecting the buoyant force experienced by the rod.

How does the partial submersion of the rod relate to its density?

The extent to which the rod is submerged depends on its density relative to water; since its density is less than water, only a part of it is submerged to balance the weight with the buoyant force.

What role does the position of the rod's fulcrum or pivot play in this scenario?

If the rod is hung at one end and partially submerged, its equilibrium depends on the distribution of weight and buoyant force, which are influenced by the rod’s density and the submerged length, affecting its balance point.

How can we determine the submerged volume of the rod given its density?

By comparing the weight of the rod with the buoyant force, which equals the weight of the displaced water, we can calculate the submerged volume using the relation: submerged volume = (mass of rod) / (density of water).

Why does the density of the rod being less than water matter in designing floating structures?

A lower density allows the structure to float with less submerged volume, making it more stable and efficient for designing floating bridges, boats, or pontoons.

What is the condition for the rod to be in equilibrium when partially submerged?

The sum of torques about the pivot must be zero, meaning the weight of the rod acts downward at its center of mass, and the buoyant force acts upward at the center of buoyancy, balancing each other.

How does the length of the rod affect its stability when partially submerged?

A longer rod with the same density and submersion level may have a higher center of gravity, affecting stability; the distribution of mass and buoyant force determine whether it remains balanced.

Can you calculate the fraction of the rod submerged if its density is given as 5/9?

Yes, using Archimedes' principle, the fraction submerged equals the ratio of the rod's density to water's density: (5/9) / 1 = 5/9, so approximately 55.56% of the rod is submerged.

Additional Resources

A Uniform Rod Is Hung At One End And Is Partially Submerged In Water. If The Density Of The Rod Is 5/9

Introduction

In the realm of classical mechanics and fluid statics, the study of buoyancy and stability of submerged objects remains foundational. A common yet instructive problem involves analyzing the behavior of a uniform rod suspended at one end with part of it submerged in water, especially when the density of the rod is known as a fraction of the water’s density. This problem not only underscores principles of Archimedes’ buoyant force but also explores concepts related to equilibrium, stability, and center of mass.

Specifically, the scenario where a uniform rod is hung at one end and is partially submerged in water with the density of the rod being 5/9 that of water provides an excellent context for understanding these principles in depth. This investigation aims to dissect this problem comprehensively, examining the conditions for equilibrium, the implications for the rod’s stability, and the physical intuition behind these phenomena.

Background and Fundamental Principles

Density and Specific Gravity

Density (ρ) is a measure of mass per unit volume, expressed in units such as kg/m^3. When comparing the densities of two materials, it is often helpful to use the concept of specific gravity, which is a ratio of the density of an object to the density of water (assuming water’s density as $\rho_{water} \approx 1000\, \text{kg/m}^3$).

Given:
$$
\rho_{rod} = \frac{5}{9} \rho_{water}
$$

\]

This means the rod is less dense than water, which influences whether it floats, sinks, or reaches an equilibrium position when partially submerged.

Archimedes’ Principle

A fundamental principle governing buoyancy states that a body submerged in a fluid experiences an upward buoyant force equal to the weight of the displaced fluid:
$$
F_b = \rho_{fluid} \times g \times V_{displaced}
$$

where:
- F_b is the buoyant force,
- ρ_{fluid} is the water’s density,
- g is acceleration due to gravity,
- $V_{displaced}$ is the volume of water displaced.

This force acts vertically upward through the centroid of displaced fluid, which is critical when analyzing equilibrium and stability.

Physical Setup and Assumptions

Consider a uniform, slender rod of length L, mass m, and density $\rho_{rod} = \frac{5}{9} \rho_{water}$. The rod is rigidly hung at one end (say, the left end) and is partially submerged in water, with the water level reaching some point along its length.

Key assumptions are:

- The rod is uniform, so its mass distribution is constant along its length.
- The rod is rigid and does not bend.
- The water is at rest, and effects such as currents are negligible.
- The contact point at the hanging end does not slip, ensuring static equilibrium.
- The rod is sufficiently slender to ignore effects like twisting or bending.

Analyzing Equilibrium Conditions

Vertical Force Balance

For the rod to be in equilibrium, the sum of vertical forces must be zero:

\[
\text{Weight of the rod} = \text{Buoyant force}

\]

Expressed mathematically:

\[
mg = \rho_{water} \times g \times V_{submerged}
\]

where:
- \(m = \rho_{rod} \times V_{rod}\),
- \(V_{rod} = A \times L\), with \(A\) being the cross-sectional area (assumed constant).

Since the density of the rod is \(\frac{5}{9}\) that of water:

\[
m = \frac{5}{9} \rho_{water} \times V_{rod}
\]

Substituting into force balance:

\[
\frac{5}{9} \rho_{water} \times V_{rod} \times g = \rho_{water} \times g \times V_{submerged}
\]

Dividing both sides by \(\rho_{water} \times g\):

\[
\frac{5}{9} V_{rod} = V_{submerged}
\]

Given \(V_{rod} = A \times L\), the volume submerged is:

\[
V_{submerged} = \frac{5}{9} A L
\]

This indicates that approximately 55.56% of the rod’s volume must be submerged for the weights and buoyant forces to balance.

Implications for Partial Submersion and Stability

Submersion Depth and Position of the Waterline

Since the rod is uniform, the volume submerged corresponds directly to the submerged length:

\[

$$
L_{submerged} = \frac{V_{submerged}}{A} = \frac{5}{9} L
$$

Thus, the waterline cuts across the rod at a point:

$$
\text{Water level position from the hanging end} = \frac{5}{9} L
$$

Center of Buoyancy and Center of Gravity

- The center of gravity of the uniform rod is at its midpoint, at $L/2$ from the hanging end.
- The center of buoyancy (the centroid of the displaced volume) is at the centroid of the submerged portion, which for uniform cross-section is at:

$$
\text{Center of buoyancy} = \frac{L_{submerged}}{2} = \frac{5}{9} \times \frac{L}{2} = \frac{5L}{18}
$$

from the hanging end.

Equilibrium and Stability Analysis

For the rod to be in stable equilibrium, the metacenter must be above the center of gravity. The key conditions are:

- The vertical line of action of the buoyant force passes through the center of buoyancy.
- The weight acts downward through the center of gravity.
- For stability, the center of gravity must lie below the metacenter, which depends on the geometry and the distribution of buoyant force.

In a simplified analysis, the stability criterion reduces to whether a small angular displacement results in a restoring torque. This is evaluated through the relative positions of the center of gravity and center of buoyancy.

The Role of Density in Stability

The density ratio ($5/9$) critically influences whether the rod floats or sinks and its equilibrium configuration:

- Since $\rho_{rod} < \rho_{water}$, the rod is less dense than water.
- The equilibrium condition derived indicates that the rod must be partially submerged to balance weight and buoyant force.
- If the rod were more dense, it would sink fully; if less dense, it would float with varying degrees of submersion depending on its shape and density.

Practical Scenarios and Experimental Considerations

Scenario 1: Rod Fully Submerged

If the rod is fully submerged (length L), the total volume displaced equals the entire volume:

$$
V_{displaced} = V_{rod}
$$

and the buoyant force exceeds the weight:

$$
\rho_{water} \times g \times V_{rod} > \frac{5}{9} \rho_{water} \times V_{rod} \times g
$$

$$
\Rightarrow \text{Rod tends to float} \quad \text{(since buoyant force > weight)}
$$

However, the exact equilibrium position depends on the initial configuration and whether the rod can pivot or rotate to find a stable position.

Scenario 2: Partially Submerged and in Equilibrium

Given the earlier calculations, the rod reaches equilibrium when approximately 55.56% of its volume is submerged, corresponding to a submerged length of $5L/9$. This state balances the forces and moments, but whether this equilibrium is stable depends on the center of gravity and the metacenter.

Stability Analysis: Torque and Metacenter

The stability of the rod in this equilibrium position hinges on the relative positions of the center of gravity (G) and the metacenter (M):

- Center of Gravity (G): Located at $L/2$.
- Center of Buoyancy (B): Located at $5L/18$.
- Metacenter (M): The point about which the buoyant force acts when the rod is tilted slightly; its position depends on the geometry of the submerged volume.

In classical fluid mechanics, the metacentric height (GM) determines stability:

$$
GM = BM - BG
$$

\]

where:
- \(BM\) depends on the geometry of the submerged volume,
- \(BG\) is the distance between the center of buoyancy and the center of gravity.

If \(GM > 0\), the rod is stable; if \(GM < 0\), it is unstable.

Given the uniformity and the partial submersion, the detailed calculation of \(GM\) involves integrating the displaced volume’s centroid and moments, but the key insight is that since the rod’s density is less than water, it naturally tends to float with a certain portion submerged, and the stability depends critically on the distribution of mass and buoyant volume.

Conclusion

The analysis of a uniform rod hung at one end and partially submerged in water, with the density of the rod being 5/9

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