A Uniform Wave In Air Has E=10cos(2106tz)ay (a) Calculate And . (b) Sketch The Wave At Z=0,/4. (c) Find

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Introduction to Electromagnetic Waves in Air

Electromagnetic waves are fundamental to understanding various phenomena in physics and engineering, including communication systems, radar technology, and wireless transmission. A uniform wave propagating through air can be described mathematically by sinusoidal functions representing electric and magnetic fields. In this article, we analyze a specific wave characterized by the electric field expression:

E = 10cos(2106 t z) a_y

This wave propagates through air, and our goal is to evaluate certain parameters, sketch the wave at specified positions, and determine additional wave properties.

Understanding the Wave Equation

The given electric field vector:

- **Magnitude:** 10 V/m (volts per meter)
- **Angular frequency component:** 2106 tz
- **Direction of propagation:** Along the z-axis
- **Polarization:** Along the y-axis

The expression indicates a plane electromagnetic wave traveling in the z-direction with electric field oscillations polarized along the y-axis. The sinusoidal form implies the wave's amplitude, frequency, and phase are embedded within the cosine function.

Part A: Calculating the Wave's Parameters

1. Determine the Angular Frequency (ω)

The given wave's argument is:

$\cos(2106\ t\ z)$

In standard wave equation form, the phase is:

$\omega t - k z$

Comparing, the angular frequency (ω) and the wave number (k) are embedded in the coefficient 2106.

However, in the provided expression, it appears as:

$\cos(2106\ t\ z)$

which suggests the phase varies with both t and z. If the expression is:

$E(z, t) = 10 \cos(\omega t - k z)\ a_y$

then the coefficient in front of t and z should be split accordingly. But as per the notation, it appears as:

$E(z, t) = 10 \cos(2106\ t\ z)\ a_y$

which may be a typo or shorthand. Typically, the wave is expressed as:

$E(z, t) = E_0 \cos(\omega t - k z)$

Assuming that, the coefficients indicate:

- $\omega = 2106$ rad/sec (from the coefficient of t)
- $k = 2106$ rad/m (from the coefficient of z)

But the expression "2106 t z" suggests that the argument is a product of t and z, which is unusual. Usually, the phase is linear: $\omega t - k z$.

Assumption: The wave is described with phase:

$\Phi = \omega t - k z$

and the expression is simplified as:

$E(z, t) = 10 \cos(\omega t - k z)\, a_y$

with:

- $\omega = 2106$ rad/sec
- $k = 2106$ rad/m

Note: If the expression actually involves the product t z, then the form is different and would imply a different kind of wave. But standard wave analysis assumes linear phase dependence.

2. Calculate the Wave Number (k) and Wavelength (λ)

The wave number:

$k = 2\pi / \lambda$

The wave number is given as:

$k = 2106$ rad/m

Therefore, the wavelength:

$\lambda = 2\pi / k = 2\pi / 2106 \approx 0.00298$ meters ≈ 2.98 mm

3. Calculate the Wave's Speed (v)

The wave propagates through air, where the speed of light (c) is approximately:

$c \approx 3 \times 10^8$ m/s

The relation between wave speed, angular frequency, and wavelength:

$v = \omega / k$

Substituting values:

$v = 2106 / 2106 = 1$ m/sec

This suggests that the wave's phase velocity is 1 m/sec, which is much less than the speed of light, indicating perhaps a different context or a typo in the parameters. In typical electromagnetic waves in air, the speed should be close to 3 × 108 m/sec.

Alternatively, perhaps the coefficient is scaled differently. For accurate calculations, assuming the wave propagates at the speed of light:

- $\omega = c\,k$

Given the standard in free space:

$k = \omega / c$

And:

$\omega = 2\pi f$

In the absence of further clarification, we proceed with the given parameters.

Part B: Sketching the Wave at Z=0 and Z=π/4

1. Electric Field at Z = 0

At $z = 0$, the electric field simplifies to:

$E(0, t) = 10 \cos(\omega t)\, a_y$

This represents a sinusoidal oscillation with amplitude 10 V/m along the y-axis, varying over time.

Plot Characteristics:

- Amplitude: 10 V/m
- Frequency: determined by ω
- Oscillation: sinusoidal

2. Electric Field at $Z = \pi/4$

At $z = \pi/4$, the phase shift becomes:

$\Phi = \omega t - k(\pi/4)$

The electric field:

$E(\pi/4, t) = 10 \cos(\omega t - k(\pi/4))\, a_y$

Since k = 2106 rad/m, then:

$k(\pi/4) = 2106 \times (\pi/4) \approx 2106 \times 0.7854 \approx 1654.7$ radians

This phase shift corresponds to a time delay or phase lag in the wave. The waveform at this position is similar in shape to that at z=0 but shifted in phase.

Sketching the Wave:

- The wave at z=0 starts at maximum amplitude when $\cos(\omega t) = 1$.
- At $z=\pi/4$, the wave is shifted by approximately 1654.7 radians, which is many cycles (since 2π radians = 1 cycle). To find the phase shift in terms of cycles:

Number of cycles:

Shift / $2\pi \approx 1654.7 / 6.283 \approx 263.2$ cycles

Thus, the wave at $z=\pi/4$ is shifted by approximately 263 full cycles, essentially returning to the same phase (since phase repeats every cycle). However, the key point is that at this position, the wave is shifted in phase relative to z=0, and the sketch would depict sinusoidal oscillations with this phase lag.

Part C: Additional Wave Properties

1. Determine the Magnetic Field (H)

In electromagnetic waves, electric and magnetic fields are perpendicular and related by:

$H = E / \eta$

where η is the intrinsic impedance of air (~377 ohms).

Given Emax = 10 V/m:

$H_{max} = 10 / 377 \approx 0.0265$ A/m

The magnetic field oscillates with the same frequency and phase as the electric field but perpendicular in direction (along the x-axis if electric is along y, magnetic is along x).

2. Direction of Propagation and Field Orientation

- Electric field: along y-axis
- Magnetic field: along x-axis
- Propagation: along z-axis

This configuration is typical for plane electromagnetic waves, satisfying the right-hand rule.

3. Calculating the Power Density

The average power per unit area carried by the wave (Poynting vector magnitude):

$S = E_{rms} \times H_{rms}$

Where:

- Erms = Emax/√2 ≈ 10/1.414 ≈ 7.07 V/m
- Hrms = Hmax/√2 ≈ 0.0265/1.414 ≈ 0.0188 A/m

Thus:

$S \approx 7.07 \times 0.0188 \approx 0.133\ W/m^2$

This represents the average power flux of the wave in air.

Conclusion

The analysis of the given uniform wave reveals several key aspects. The wave propagates along the z-axis with a wavelength of approximately 2.98 mm, and a phase velocity that, based on the parameters, appears to be around 1 m/sec—though in practice, electromagnetic waves in air travel at the speed of light, indicating potential discrepancies in the given

Frequently Asked Questions

What is the expression for the uniform wave in air given in the

problem?

The wave is given by E = 10 cos(2106 t z) ay.

How do you interpret the wave equation E = 10 cos(2106 t z) ay?

It describes a plane electromagnetic wave propagating in air with an amplitude of 10 units, oscillating with a phase factor depending on time t and position z, polarized along the y-axis.

What is the physical significance of the wave parameters in E = 10 cos(2106 t z) ay?

The amplitude is 10 units, the angular wave number is 2106 rad/m, and the wave propagates along the z-direction with the electric field polarized along the y-axis.

How do you calculate the phase velocity of the wave from the wave number?

The phase velocity $v_p = \omega / k$, where ω is the angular frequency and k is the wave number. However, as ω is not explicitly given, further information is needed to compute v_p.

How to find the electric field at z=0 and z=π/4?

Substitute z=0 and z=π/4 into the wave equation: at z=0, E=10 cos(2106 t 0) ay = 10 cos(0) ay; at z=π/4, E=10 cos(2106 t π/4) ay.

What is the wave's behavior at z=0?

At z=0, the electric field simplifies to E=10 cos(0) ay = 10 ay, indicating maximum amplitude at that position.

How does the wave sketch look at z=0 and z=π/4?

At z=0, the wave oscillates between +10 and -10 along the y-axis with time. At z=π/4, the wave oscillates with a phase shift of (2106 π/4), resulting in a shifted sine wave.

What is the significance of the phase shift at z=π/4?

The phase shift indicates the wave's progression in space, causing the wave at z=π/4 to be shifted in time compared to z=0, affecting the wave's instantaneous electric field.

How do you determine the value of E at any given time and position?

Use the wave equation E=10 cos(2106 t z) ay, substitute specific values of t and z to find the electric field at that instant and location.

What are the missing steps to fully analyze the wave's properties?

To fully analyze, you need to identify the wave's angular frequency ω, compute phase velocity, amplitude, and phase shifts, and plot the wave at specified positions and times based on the given equation.

Additional Resources

In-Depth Analysis of a Uniform Air Wave with E=10cos(2106tz)ay: Calculations, Visualization, and Physical Insights

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Introduction

Electromagnetic wave propagation in air and other media is a foundational topic in physics and engineering, combining principles of wave mechanics, electromagnetism, and mathematical analysis. The given wave expression:

$$
\mathbf{E} = 10 \cos(2106 t z) \, \mathbf{\hat{a}_y}
$$

represents a uniform wave traveling in air, oriented along the y-axis, with a spatial and temporal dependence. This review delves into the detailed analysis of this wave, including its physical interpretation, calculations of key parameters, graphical visualization at specific points, and derivation of related quantities such as magnetic field components and energy densities.

Physical Description of the Wave

The given wave expression indicates:

- Amplitude (E_0): 10 units (likely volts per meter, V/m)
- Wave dependence: The cosine function varies with both time t and position z
- Propagation direction: Along the z-axis (as indicated by the z in the argument)
- Polarization: Along the y-axis ($\mathbf{\hat{a}_y}$)

- Wave number (k): 2106 rad/m
- Angular frequency (ω): 2106 rad/sec (since the argument is $\omega t - k z$), but here it's written as $2106 t z$); this warrants clarification)

Understanding these parameters helps to analyze the wave's physical behavior, including its phase velocity, wavelength, and energy transport.

Clarification of the Wave Equation

The form:

$$
\mathbf{E} = E_0 \cos(k z - \omega t) \, \mathbf{\hat{a}_y}
$$

is typical for a plane wave propagating in the positive z-direction. However, in the provided expression, the argument is $2106 t z$), which suggests a different form. Assuming the standard convention, the wave should be:

$$
\mathbf{E} = 10 \cos(k z - \omega t) \, \mathbf{\hat{a}_y}
$$

with:

- $k = 2106 \, \text{rad/m}$
- $\omega = 2106 \, \text{rad/sec}$

This implies the wave travels along the z-axis, with spatial variation characterized by k, and temporal variation characterized by ω.

Part (a): Calculations of Key Wave Parameters

1. Wave Number (k)

Given directly as:

$$
k = 2106 \, \text{rad/m}
$$

\]

This parameter relates to the wavelength \(\lambda \):

\[
\lambda = \frac{2\pi}{k} = \frac{2\pi}{2106} \approx \frac{6.2832}{2106} \approx 0.00298 \, \text{m}
\]

which is approximately 2.98 mm, indicating a high-frequency electromagnetic wave in the microwave region.

2. Angular Frequency (\(\omega\))

Given as:

\[
\omega = 2106 \, \text{rad/sec}
\]

The frequency \(f \) is:

\[
f = \frac{\omega}{2\pi} = \frac{2106}{6.2832} \approx 335.5\, \text{Hz}
\]

This indicates the wave oscillates roughly 335.5 times per second, which is in the low RF or radio frequency range.

3. Wave Velocity (\(v\))

The phase velocity \(v\) of the wave is:

\[
v = \frac{\omega}{k} = \frac{2106}{2106} = 1 \, \text{m/sec}
\]

This is physically consistent with electromagnetic waves in air, given that the speed of light \(c \) is approximately \(3 \times 10^8 \, \text{m/sec} \). The discrepancy suggests either a typo in the original data or that the wave is not propagating at the speed of light. Typically, in free space:

\[
v = c \approx 3 \times 10^8 \, \text{m/sec}
\]

Given the context, the actual wave should have:

$$
k = \frac{\omega}{c} \Rightarrow \omega = c k
$$

which would be:

$$
\omega = (3 \times 10^8) \times 2106 \approx 6.318 \times 10^{11} \, \text{rad/sec}
$$

This discrepancy suggests an inconsistency, but for the purpose of the analysis, we accept the provided values as an abstract wave with $v = 1\, \text{m/sec}$.

Part (b): Sketching the Wave at $z = 0$ and $z = \frac{\pi}{4}$

1. Wave at $z=0$

At $z=0$, the wave reduces to:

$$
\mathbf{E}(t, 0) = 10 \cos(0 - \omega t) \, \mathbf{\hat{a}_y} = 10 \cos(-\omega t) \, \mathbf{\hat{a}_y}
$$

Since cosine is even:

$$
\mathbf{E}(t, 0) = 10 \cos(\omega t) \, \mathbf{\hat{a}_y}
$$

This describes a simple harmonic oscillation in the y-direction, varying with time.

Graphical interpretation:

- The wave oscillates sinusoidally with amplitude 10 V/m.
- The waveform is centered at zero, with positive and negative peaks at ± 10 V/m.
- The period T:

$$
T = \frac{2 \pi}{\omega} = \frac{6.2832}{2106} \approx 0.00298\, \text{sec}
$$

\]

- The wave completes approximately 335 oscillations per second.

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2. Wave at $z= \frac{\pi}{4}$

At $z = \pi/4$, the wave becomes:

$$
\mathbf{E}(t, \pi/4) = 10 \cos(k \times \pi/4 - \omega t) \, \mathbf{\hat{a}_y}
$$

Calculating the phase:

$$
k \times \frac{\pi}{4} = 2106 \times \frac{\pi}{4} \approx 2106 \times 0.7854 \approx 1654.5\, \text{rad}
$$

Thus:

$$
\mathbf{E}(t, \pi/4) = 10 \cos(1654.5 - \omega t) \, \mathbf{\hat{a}_y}
$$

This phase shift indicates the wave at $z = \pi/4$ is ahead or behind the wave at $z=0$ by a phase of approximately 1654.5 radians, corresponding to many oscillation cycles.

Implication for visualization:

- The wave at $z = \pi/4$ is a sinusoid similar in shape but shifted in phase compared to the wave at $z=0$.
- In a plot over time, the wave at $z = \pi/4$ would be a sinusoid oscillating with the same amplitude, but shifted horizontally by:

$$
\Delta t = \frac{\text{phase shift}}{\omega} = \frac{1654.5}{2106} \approx 0.785\, \text{sec}
$$

which is a quarter of the wave period.

Part (c): Additional Calculations and Physical Quantities

1. Magnetic Field ($\mathbf{H}$)

In free space or air, electromagnetic waves satisfy Maxwell's equations, linking electric ($\mathbf{E}$) and magnetic ($\mathbf{H}$) fields:

$$
\mathbf{H} = \frac{1}{\eta_0} \, \mathbf{\hat{k}} \times \mathbf{E}
$$

where:

- $\eta_0 \approx 377\, \Omega$ is the intrinsic impedance of free space,
- $\mathbf{\hat{k}}$ is the unit vector in the direction of wave propagation (along z).

Given:

$$
\mathbf{E} = E_y \, \mathbf{\hat{a}_y}
$$

and wave propagates along $+z$, then:

$$
\mathbf{H} = \frac{1}{\eta_0} \, \mathbf{\hat{z}} \times \mathbf{E} = \frac{1}{377} \, \mathbf{\hat{z}} \times (E_y \, \mathbf{\hat{a}_y}) = \frac{E_y}{377} \, \mathbf{\hat{z}} \times \mathbf{\hat{a}_y}
$$

Since:

\[
\mathbf{\hat{z}} \times \mathbf{\hat{a}_y

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