

A Wheel Of Radius 15 Cm Has Sa Rotational Inertia Of 2.3 Kg M². The Wheel Is Spinning At A Rate Of 6.5

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Understanding the dynamics of rotating objects is fundamental in physics and engineering, especially when dealing with wheels, gears, or any rotating machinery. In this article, we will explore the implications of a wheel with specific properties: a radius of 15 cm, a rotational inertia of 2.3 kg·m², spinning at a rate of 6.5 radians per second. We will delve into the concepts of rotational inertia, angular velocity, kinetic energy, and the forces involved, providing a comprehensive understanding suitable for students, educators, and professionals alike.

Introduction to Rotational Motion and Inertia

What Is Rotational Inertia?

Rotational inertia, also known as the moment of inertia, is a measure of an object's resistance to changes in its rotational motion. It depends on both the mass of the object and how that mass is distributed relative to the axis of rotation. The greater the rotational inertia, the more torque is needed to change the angular velocity of the object.

Key points:

- Rotational inertia is expressed in kg·m².
- It varies with the shape and mass distribution of the object.
- For a given mass, objects with mass farther from the axis have higher rotational inertia.

Angular Velocity and Its Significance

Angular velocity indicates how fast an object is rotating, typically measured in radians per second (rad/s). In our scenario, the wheel's angular velocity is 6.5 rad/s.

- Angular velocity (ω): The rate of change of angular displacement.
- Relation to linear velocity: The tangential linear velocity (v) at a point on the wheel's rim is $v = r \omega$.

Analyzing the Given Data

The problem provides specific data points:

- Radius of the wheel (r): 15 cm = 0.15 meters
- Rotational inertia (I): 2.3 kg·m²
- Angular velocity (ω): 6.5 rad/s

Using this data, we can compute various physical quantities related to the wheel's motion.

Calculating the Wheel's Kinetic Energy

The rotational kinetic energy (KE) of a spinning object is given by:

$$KE = \frac{1}{2} I \omega^2$$

Plugging in the values:

$$KE = \frac{1}{2} \times 2.3 \, \text{kg} \cdot \text{m}^2 \times (6.5 \, \text{rad/s})^2$$

$$KE = 1.15 \times 42.25 = 48.588 \, \text{J}$$

Result: The wheel possesses approximately 48.59 joules of rotational kinetic energy.

Understanding the Dynamics: Torque, Angular Momentum, and Power

Angular Momentum of the Wheel

Angular momentum (L) is given by:

$$L = I \omega$$

\]

Calculating:

\[

$$L = 2.3 \, \text{kg}\cdot\text{m}^2 \times 6.5 \, \text{rad/s} = 14.95 \, \text{kg}\cdot\text{m}^2/\text{s}$$

\]

This value indicates the quantity of rotational motion that the wheel has, which is crucial in analyzing how it responds to external torques.

Torque and Angular Acceleration

If an external torque (τ) is applied, it causes angular acceleration (α):

\[

$$\alpha = \frac{\tau}{I}$$

\]

For example, suppose a torque of 1.15 Nm is applied; then:

\[

$$\alpha = \frac{1.15 \, \text{Nm}}{2.3 \, \text{kg}\cdot\text{m}^2} = 0.5 \, \text{rad/s}^2$$

This demonstrates how torque influences the rotational speed over time.

Power in Rotational Motion

Power (P) related to rotational motion is:

\[

$$P = \tau \omega$$

\]

If a torque of 1 Nm is applied at an angular velocity of 6.5 rad/s:

\[

$$P = 1 \, \text{Nm} \times 6.5 \, \text{rad/s} = 6.5 \, \text{W}$$

\]

This indicates the rate at which work is being done to the wheel.

Applications and Practical Considerations

Design Implications in Mechanical Engineering

Understanding the inertia and rotational dynamics of wheels is vital when designing machinery such as:

- Bicycles and motorcycles
- Automotive wheels
- Industrial turbines
- Flywheels for energy storage

Design considerations include balancing the mass distribution to control inertia, ensuring safety during acceleration and deceleration, and optimizing energy efficiency.

Energy Storage and Conversion

Flywheels are used to store rotational energy efficiently. The amount of energy stored depends on the rotational inertia and angular velocity:

$$E_{\text{stored}} = \frac{1}{2} I \omega^2$$

In our case, approximately 48.59 joules can be stored, which can be used in various energy management systems.

Real-World Examples

- Wheels in vehicles: The inertia affects acceleration and braking.
- Gyroscopic devices: Maintain orientation due to angular momentum.
- Rotational machinery: Smoothing out power delivery and reducing vibrations.

Factors Affecting Rotational Inertia

While the given rotational inertia ($2.3 \text{ kg}\cdot\text{m}^2$) is specific, other factors influence this value:

- Shape of the object: Solid disc, hoop, rod, etc.
- Mass distribution: More mass farther from the axis increases inertia.
- Material density: Denser materials can increase overall inertia if distributed outward.

Different objects with the same mass can have vastly different inertias depending on their geometry.

Additional Calculations and Considerations

Linear Velocity at the Rim

The tangential or linear velocity at the wheel's rim is:

$$v = r \times \omega$$

Calculating:

$$v = 0.15 \text{ m} \times 6.5 \text{ rad/s} = 0.975 \text{ m/s}$$

This velocity indicates how fast a point on the edge of the wheel moves relative to the center.

Angular Acceleration and Deceleration

If the wheel slows down or speeds up, the rate of change of angular velocity (α) is important:

- Controlled deceleration involves applying torque opposite to the direction of rotation.
- The energy lost during braking converts into heat.

Role of Friction and External Forces

Friction at the axle and air resistance oppose rotation, gradually reducing angular velocity unless work is done to sustain it.

Summary and Key Takeaways

- The wheel's rotational kinetic energy is approximately 48.59 joules at an angular velocity of 6.5 rad/s.
- Its angular momentum is roughly 14.95 kg·m²/s.
- The linear velocity at the rim is about 0.975 m/s.
- Understanding rotational inertia helps optimize design and control of rotating systems.
- External torques influence angular acceleration, impacting how quickly a

wheel can start or stop spinning.

Conclusion

The physics of a spinning wheel involves a complex interplay of inertia, torque, angular velocity, and energy. Recognizing how these elements interact provides critical insights into mechanical design, energy management, and dynamic behavior of rotational systems. Whether in engineering applications or academic studies, mastering these principles enables the development of efficient, safe, and innovative rotating machinery.

Additional Resources for Learning

- Classical Mechanics by Herbert Goldstein
- Online tutorials on rotational dynamics
- Simulation tools for visualizing rotational motion
- Educational videos explaining moment of inertia and angular momentum

Understanding the fundamental concepts outlined in this article fosters a deeper appreciation of the physics behind everyday objects and advanced machinery alike.

Frequently Asked Questions

What is the angular velocity of the wheel in radians per second, given a rotation rate of 6.5 revolutions per second?

The angular velocity $\omega = 2\pi \times 6.5 \approx 40.84$ radians per second.

How much rotational kinetic energy does the wheel possess at this angular velocity?

Rotational kinetic energy, $KE = 0.5 \times I \times \omega^2 = 0.5 \times 2.3 \text{ kg}\cdot\text{m}^2 \times (40.84 \text{ rad/s})^2 \approx 1.15 \times 1670.53 \approx 1920.11$ Joules.

What is the linear (tangential) speed of a point on the rim of the wheel?

$v = r \times \omega = 0.15 \text{ m} \times 40.84 \text{ rad/s} \approx 6.13$ meters per second.

If the wheel's rotational inertia increases, how does that affect its rotational kinetic energy at the same angular velocity?

The rotational kinetic energy increases proportionally with the moment of inertia; $KE = 0.5 \times I \times \omega^2$, so a larger I results in higher KE .

What is the torque required to bring the wheel to rest in 10 seconds from its current rotational speed?

Torque $\tau = I \times \alpha$, where $\alpha = (\text{final angular velocity} - \text{initial}) / \text{time} = (0 - 40.84 \text{ rad/s}) / 10 \text{ s} = -4.084 \text{ rad/s}^2$. Therefore, $\tau = 2.3 \text{ kg}\cdot\text{m}^2 \times (-4.084 \text{ rad/s}^2) \approx -9.39 \text{ Nm}$.

How does the radius of the wheel influence its linear speed for a given angular velocity?

Linear speed $v = r \times \omega$; increasing the radius r increases the linear speed proportionally, so larger wheels at the same ω move faster at the rim.

What is the significance of the rotational inertia value ($2.3 \text{ kg}\cdot\text{m}^2$) for the wheel's rotational dynamics?

The rotational inertia determines how much torque is needed to change the wheel's rotational speed; higher inertia means more torque is required for acceleration or deceleration.

Additional Resources

A Wheel Of Radius 15 Cm Has Sa Rotational Inertia Of 2.3 Kg M^2 . The Wheel Is Spinning At A Rate Of 6.5

In the realm of rotational dynamics, understanding how a wheel behaves under various forces is crucial for applications ranging from everyday machinery to advanced engineering systems. Recently, a particular wheel with specific parameters has garnered attention: it possesses a radius of 15 centimeters, a rotational inertia of $2.3 \text{ kg}\cdot\text{m}^2$, and spins at a rate of 6.5 radians per second. This combination of characteristics provides a compelling case study to explore fundamental principles of angular motion, energy, and torque.

This article delves into the physics behind such a system, offering a comprehensive explanation suitable for readers with a keen interest in mechanics, whether students, educators, or professionals seeking a deeper understanding of rotational phenomena.

Understanding the Key Parameters: Radius, Rotational Inertia, and Angular Velocity

Before analyzing the dynamics at play, it's vital to define and understand the key parameters involved:

Radius of the Wheel

The wheel's radius is 15 centimeters, which converts to 0.15 meters. The radius influences the wheel's moment of inertia and the torque required to alter its rotational state. Smaller radii generally mean less rotational inertia for a given mass distribution, but in this case, the inertia value already accounts for the wheel's mass distribution.

Rotational Inertia (Moment of Inertia)

The rotational inertia, given as $2.3 \text{ kg}\cdot\text{m}^2$, quantifies the wheel's resistance to changes in its rotational motion. It encapsulates the distribution of mass relative to the axis of rotation. For a wheel, this depends on the shape, mass distribution, and size.

Mathematically, the moment of inertia (I) is expressed as:

$$I = \sum m_i r_i^2$$

for discrete mass elements, or for continuous objects, via integral calculus based on the shape and mass distribution.

Angular Velocity

The wheel's angular velocity is 6.5 radians per second, indicating how rapidly it completes rotations. Since one full rotation corresponds to (2π) radians, this angular velocity translates to:

$$\text{Rotational speed in revolutions per second} = \frac{6.5}{2\pi} \approx 1.035 \text{ rev/sec}$$

or approximately 62 revolutions per minute (RPM).

Calculating the Kinetic Energy of the Spinning

Wheel

One fundamental aspect of rotational motion is the kinetic energy stored in a spinning object. The rotational kinetic energy (KE_{rot}) is given by:

$$KE_{\text{rot}} = \frac{1}{2} I \omega^2$$

where:

- (I) is the moment of inertia,
- (ω) is the angular velocity.

Plugging in the known values:

$$KE_{\text{rot}} = \frac{1}{2} \times 2.3 \, \text{kg}\cdot\text{m}^2 \times (6.5 \, \text{rad/sec})^2$$

Calculating step-by-step:

1. Square the angular velocity:

$$6.5^2 = 42.25$$

2. Multiply by the inertia:

$$2.3 \times 42.25 = 97.175$$

3. Multiply by 0.5:

$$\frac{1}{2} \times 97.175 = 48.5875$$

Therefore,

$$KE_{\text{rot}} \approx 48.59 \, \text{Joules}$$

This amount of energy represents the rotational energy stored in the wheel due to its current motion, highlighting how much work would be needed to bring it to a stop or to accelerate it further.

Torque and Angular Acceleration: How Forces Influence Rotation

Understanding how the wheel responds to external forces involves examining torque and angular acceleration.

Torque and Its Relation to Rotational Motion

Torque (τ) is the rotational equivalent of force, representing how much a force causes an object to rotate about an axis. It is calculated as:

$$\tau = r \times F \times \sin \theta$$

where:

- r is the lever arm (radius),
- F is the applied force,
- θ is the angle between the force and the lever arm.

For example, if a force is applied tangentially at the wheel's rim, the torque simplifies to:

$$\tau = r \times F$$

Since the wheel's radius is 0.15 meters, even modest forces can generate significant torque.

Angular Acceleration and Its Calculation

When a net torque acts on a wheel, it produces angular acceleration (α), according to Newton's second law for rotation:

$$\tau = I \times \alpha$$

Rearranged to find (α):

$$\alpha = \frac{\tau}{I}$$

Suppose an external torque of 1.15 Nm (for illustration):

$$\alpha = \frac{1.15}{2.3} = 0.5 \text{ rad/sec}^2$$

This means the wheel's angular velocity would increase by 0.5 radians per second every second under this torque.

Implications:

- The higher the torque, the faster the wheel will accelerate.
- The moment of inertia acts as a "rotational mass," resisting changes in angular velocity.

Energy Transfer and Efficiency in Rotational

Systems

The energy dynamics of spinning wheels are integral to numerous mechanical systems, such as flywheels, turbines, and vehicle wheels.

Energy Storage and Retrieval

A spinning wheel can store kinetic energy, which can be converted into useful work or dissipated as heat through friction or braking. The efficiency of this energy transfer depends on factors like frictional losses, bearing resistance, and air drag.

Braking and Deceleration

To slow down or stop a spinning wheel, applying a braking torque reduces its angular momentum:

$$\tau_{\text{brake}} = I \times \alpha_{\text{decel}}$$

The work done by brakes (or any resistive force) is equal to the reduction in rotational kinetic energy. For instance, halting the wheel from 6.5 rad/sec requires dissipating approximately 48.59 Joules of energy.

Practical Considerations

In real-world applications, engineers must account for:

- Mechanical losses due to friction,
- Material fatigue over repeated cycles,
- Control systems for precise acceleration/deceleration.

Real-World Applications and Significance

Understanding the physics of this wheel extends beyond theoretical interest, influencing practical machinery design and safety considerations.

Flywheels and Energy Storage

Flywheels, which utilize spinning disks or wheels like this one, serve as energy reservoirs in power grids and regenerative braking systems. Their ability to store and release energy efficiently depends on the moment of inertia and rotational speed.

Automotive and Mechanical Engineering

Vehicle wheels and engine flywheels must be precisely engineered to balance mass distribution, rotational inertia, and durability to ensure smooth operation and energy efficiency.

Industrial Machinery

Rotating components in manufacturing equipment rely on similar physics principles to optimize performance, minimize wear, and maximize safety.

Conclusion: The Interplay of Parameters in Rotational Dynamics

The case of this 15 cm radius wheel spinning at 6.5 radians per second exemplifies fundamental rotational principles. With a rotational inertia of $2.3 \text{ kg}\cdot\text{m}^2$, it stores approximately 48.59 Joules of kinetic energy at that speed. Understanding how torque influences its angular acceleration, how energy is stored and dissipated, and how physical parameters interplay provides valuable insights for engineers and physicists alike.

From theoretical calculations to practical applications, the physics of rotational motion remains a cornerstone of mechanical engineering. As systems become more sophisticated, comprehending these foundational concepts ensures innovation, safety, and efficiency in designing the machinery that powers our world.

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