A Woman Who Weighs 500 N Stands On A 6.0-m-long Board That Weighs 100 N. The Board Is Supported At Each

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Understanding forces, equilibrium, and the principles of mechanics is essential for analyzing real-world scenarios involving statics and dynamics. One such intriguing problem involves a woman standing on a long board, which is supported at both ends. This scenario exemplifies fundamental physics concepts like force balance, torque, and the distribution of loads. In this article, we will explore the problem in detail, analyze the forces involved, and discuss the principles that govern such systems. Whether you're a student studying physics or an enthusiast interested in mechanics, this comprehensive guide aims to clarify the concepts with clarity and depth.

Scenario Overview

Let's first understand the scenario:

- A woman weighing 500 N stands on a board.
- The board itself weighs 100 N and measures 6.0 meters in length.
- The board is supported at each end, implying two support points located at the extremities.
- The goal is to analyze the forces acting on the system, determine the reactions at the supports, and understand how the load is distributed.

This setup is a classic example of a simply supported beam subjected to a load, which is a common problem in structural engineering and physics.

Fundamental Concepts Involved

Before diving into calculations, it's important to revisit some fundamental principles.

1. Equilibrium of Forces

A system in equilibrium experiences no net force or net torque. Mathematically:

- Sum of vertical forces = 0
- Sum of torques about any point = 0

2. Support Reactions

Support reactions are the forces exerted by supports on the beam. In this case, each support provides a vertical reaction force:

- R_A at the left end
- R_B at the right end

These reactions counteract the downward forces acting on the beam.

3. Torque (Moment) Calculations

Torque is the rotational equivalent of force. To find support reactions, taking moments about one support allows solving for the other reaction.

Step-by-Step Analysis of the System

Let's proceed systematically to analyze the problem.

Step 1: Identify All Forces

- Woman's weight: $W_{woman} = 500\,N$
- Board's weight: $W_{board} = 100\,N$
- Support reactions: R_A, R_B

Assuming the woman stands somewhere along the length of the board, but since her position isn't specified, we'll analyze the general case.

Step 2: Assume Position of the Woman

To proceed, let's consider a typical scenario where the woman stands at the midpoint of the board (at 3.0 meters from either end). This simplifies calculations and provides insight into load distribution.

Step 3: Draw Free-Body Diagram

A visual diagram helps to understand the forces and moments. The diagram would include:

- The beam (length 6.0 m)
- Woman's weight acting at 3.0 m from the left end
- Board's weight acting at its center (also at 3.0 m)
- Reactions R_A at the left support and R_B at the right support

Step 4: Apply Equilibrium Equations

Vertical Force Balance:

$$
R_A + R_B = W_{woman} + W_{board} = 500\,N + 100\,N = 600\,N
$$

Moment Balance:

Taking moments about the left support (A) to solve for R_B:

$$
\text{Sum of moments about A} = 0
$$

$$
R_B \times 6.0\,m = W_{woman} \times 3.0\,m + W_{board} \times 3.0\,m
$$

$$
R_B \times 6.0 = 500 \times 3.0 + 100 \times 3.0
$$

$$
R_B \times 6.0 = (1500 + 300) = 1800
$$

$$
R_B = \frac{1800}{6.0} = 300\,N
$$

Now, using the vertical force equilibrium:

$$
R_A = 600\,N - R_B = 600 - 300 = 300\,N
$$

Result:

- Left support reaction, $R_A = 300\,N$
- Right support reaction, $R_B = 300\,N$

This symmetric load distribution confirms the assumption that the woman is standing at the midpoint.

Effect of Different Positions of the Woman

If the woman stands at a different point along the board, the support reactions will change accordingly. Let’s analyze this general case.

Position of the Woman: x meters from the left end

The moments about support A:

$$
R_B \times 6.0 = W_{woman} \times x + W_{board} \times 3.0
$$

Vertical forces:

$$
R_A + R_B = 600\,N
$$

Rearranged:

$$
R_B = 600 - R_A
$$

Substitute into the moment equation:

$$
(600 - R_A) \times 6.0 = 500 \times x + 100 \times 3.0
$$

Solve for R_A:

$$
(600 \times 6.0) - R_A \times 6.0 = 500x + 300
$$

$$
3600 - 6 R_A = 500 x + 300
$$

$$
6 R_A = 3600 - (500 x + 300) = 3300 - 500 x
$$

$$
R_A = \frac{3300 - 500 x}{6}
$$

Similarly,

$$
R_B = 600 - R_A = 600 - \frac{3300 - 500 x}{6} = \frac{3600 - (3300 - 500 x)}{6} = \frac{300 + 500 x}{6}
$$

This general formula shows how support reactions vary depending on the woman's position.

Structural and Engineering Implications

Understanding how load distribution works on simple beams has practical applications in structural engineering, safety assessments, and design.

1. Load Distribution

The reactions at supports must be sufficient to withstand the applied loads. If the reactions exceed the support's capacity, structural failure may occur.

2. Bending Moments

The bending moment at any point along the beam can be calculated to assess the maximum stress experienced by the material.

Bending Moment at a Point x:

$$
M(x) = R_A \times x - \text{sum of moments due to loads between 0 and } x
$$

For the case with the woman at position x:

$$
M(x) = R_A \times x - W_{woman} \times (x - x_{woman}) - W_{board} \times (x - 3.0)
$$

where x_{woman} is the position of the woman.

3. Designing for Safety

Engineers must ensure that the materials used for the board and supports can handle the maximum bending moments and shear forces during use.

Real-World Applications and Related Concepts

The principles discussed extend to various real-world applications:

- Construction and Structural Engineering: Designing beams, bridges, and frameworks
- Furniture Design: Ensuring stability and load capacity
- Mechanical Systems: Analyzing forces in cranes, lifts, and other machinery
- Educational Demonstrations: Teaching static equilibrium and mechanics fundamentals

Summary and Key Takeaways

- The support reactions depend on the load distribution and the position of the load.
- Symmetric loading (woman at midpoint) results in equal reactions at supports.
- As load position shifts, reactions change accordingly, which is critical in structural design.
- Calculations involve applying equilibrium conditions—force balance and moment balance.
- Understanding these principles aids in designing safe, efficient structures.

Conclusion

Analyzing the forces acting on a woman standing on a supported board exemplifies fundamental principles of statics. By understanding how to compute support reactions and bending moments, engineers and students can design safer structures and deepen their grasp of mechanics. Whether in academic settings or practical engineering, mastering these concepts is essential for ensuring stability, safety, and efficiency in various applications.

If you want to explore more about forces, moments, and structural analysis, consider studying topics like shear force diagrams, bending moment diagrams, and material strength properties to expand your understanding of structural mechanics.

Frequently Asked Questions

How do you determine the force exerted on each support when a woman and a board are in equilibrium?

Use the principles of static equilibrium, summing forces and torques to solve for the support reactions. The sum of vertical forces equals the total weight, and the sum of torques about any point is zero.

What is the weight of the woman and the board combined?

The woman's weight is 500 N and the board's weight is 100 N, so combined they weigh 600 N.

If the woman stands at the center of the 6.0 m board, what are the reaction forces at each support?

Assuming the woman stands at the center, each support bears half of the total weight: 300 N each. The board's weight is evenly distributed, so the reactions balance the weights accordingly.

How does the position of the woman on the board affect the support reactions?

The closer the woman is to one support, the greater the reaction force at that support, due to the torque balance. Moving her position shifts the distribution of support forces accordingly.

What is the importance of considering both the weight of the board and the woman in calculating support reactions?

Both weights contribute to the total downward force that must be supported; neglecting either would lead to incorrect calculations of support reactions and could compromise structural stability.

Additional Resources

A Woman Who Weighs 500 N Stands On A 6.0-m-long Board That Weighs 100 N. The Board Is Supported At Each End. — a classic physics problem that combines concepts of static equilibrium, torque, and force analysis. Understanding how forces distribute in such a scenario is fundamental to grasping the principles of mechanics, which are applicable in everyday life, engineering, and safety assessments.

In this comprehensive guide, we'll explore the problem step-by-step, break down the physics principles involved, and provide a detailed analysis of the forces at play. Whether you're a student preparing for an exam or a curious mind interested in understanding real-world mechanics, this article aims to clarify the intricacies of balancing forces on a beam and how to approach similar problems systematically.

The Scenario in Detail

Imagine a long, rigid board that is 6.0 meters in length. It is supported at each end, meaning there are supports or fulcrums located at both extremities of the board. A woman, whose weight is 500 N, stands somewhere along the length of the board. The board itself has a weight of 100 N, which acts at its center of gravity (assuming uniform mass distribution).

The key points to understand here are:
- The location of the woman along the board.

- The positions of the supports.
- The forces exerted by the woman, the weight of the board, and the reactions at the supports.

Fundamental Concepts

Before diving into calculations, it’s essential to review the core physics principles relevant to this problem:

1. Static Equilibrium

For the system to be in equilibrium (not accelerating or rotating), two conditions must be satisfied:
- Sum of all vertical forces must be zero:
$\sum F_{y} = 0$
- Sum of all torques (moments) about any point must be zero:
$\sum \tau = 0$

2. Force and Torque

- Force: A push or pull that causes or tends to cause an object to move or change shape.
- Torque: The rotational equivalent of force; it depends on the force applied and the distance from the pivot point, calculated as:

$$
\tau = F \times d
$$

where:
- F is the applied force,
- d is the perpendicular distance from the pivot point to the line of action of the force.

Step-by-Step Analysis

Step 1: Defining Variables and Assumptions

- Length of board, $L = 6.0\, \text{m}$
- Weight of the board, $W_b = 100\, \text{N}$
- Weight of the woman, $W_w = 500\, \text{N}$
- Supports at both ends: Let's denote the left support as Support A and the right support as Support B.
- Position of the woman, x_w: Distance from Support A.
- Reaction forces at supports:
- R_A at Support A,
- R_B at Support B.

Step 2: Establish Coordinates and Known Data

- Assume Support A is at $x=0\, \text{m}$,
- Support B at $x=6\, \text{m}$,
- The woman stands at position x_w (unknown for now).

Step 3: Write the Equilibrium Equations

Vertical Force Balance:

$$
R_A + R_B = W_b + W_w = 100\, \text{N} + 500\, \text{N} = 600\, \text{N}
$$

Moment Balance:

Choose Support A at $x=0$ as the pivot point to simplify calculations:

$$
\text{Sum of torques about Support A} = 0
$$

Clockwise torques are positive; counterclockwise are negative (or vice versa—consistency is key).

The torques include:
- Woman's weight: W_w acts at x_w,
- Board's weight: acts at its center at $x=3.0\, \text{m}$,
- Reactions R_A and R_B produce moments that balance these.

Because R_A acts at the pivot point, its torque is zero.

The torque balance:

$$
W_w \times x_w + W_b \times 3.0\, \text{m} = R_B \times 6.0\, \text{m}
$$

Rearranged:

$$
R_B = \frac{W_w \times x_w + W_b \times 3.0}{6.0}
$$

Once R_B is found, R_A can be calculated from the force balance:

$$
R_A = 600\, \text{N} - R_B
$$

Exploring Specific Scenarios

Scenario 1: Woman Standing at the Center of the Board

Suppose the woman stands exactly at the midpoint:

$$
x_w = 3.0\, \text{m}
$$

Calculate R_B:

$$
R_B = \frac{500 \times 3.0 + 100 \times 3.0}{6.0} = \frac{(1500 + 300)}{6} = \frac{1800}{6} = 300\, \text{N}
$$

Calculate R_A:

$$
R_A = 600 - 300 = 300\, \text{N}
$$

Result:

- Both supports bear equal reactions, each supporting 300 N.
- This symmetry makes sense because the woman is at the center, distributing the weight evenly.

Scenario 2: Woman Closer to Support A (e.g., at 1.0 m)

Suppose:

$$
x_w=1.0\, \text{m}
$$

Calculate R_B:

$$
R_B = \frac{500 \times 1.0 + 100 \times 3.0}{6.0} = \frac{(500 + 300)}{6} = \frac{800}{6} \approx 133.33\, \text{N}
$$

Calculate R_A:

$$
R_A = 600 - 133.33 \approx 466.67\, \text{N}
$$

Interpretation:

- The support closer to the woman (Support A) bears a larger reaction force.
- The support farther (Support B) bears less.

Scenario 3: Woman Near Support B (e.g., at 5.0 m)

Suppose:

$$
x_w=5.0\, \text{m}
$$

Calculate R_B:

$$
R_B = \frac{500 \times 5.0 + 100 \times 3.0}{6.0} = \frac{(2500 + 300)}{6} = \frac{2800}{6} \approx 466.67\, \text{N}
$$

Calculate R_A:

$$
R_A = 600 - 466.67 \approx 133.33\, \text{N}
$$

Interpretation:

- The support near the woman (Support B) bears most of the load.
- The other support (Support A) bears less.

Visualizing the Results

Creating a force diagram helps in understanding the distribution of forces:

- Reaction forces at supports: R_A and R_B
- Applied weights: Woman at x_w, board's weight at 3.0 m

Plotting these forces along the beam provides intuition about how the load shifts depending on the woman's position.

Additional Considerations

1. Effects of Non-Uniform Load Distribution

If the board is not uniform or has additional loads, the analysis becomes more complex, requiring the calculation of the centroid of the load distribution.

2. Safety and Structural Integrity

In real-world applications, engineers must ensure the reactions do not exceed material strength limits. Calculations like these inform design choices for safe load handling.

3. Dynamic Situations

This analysis assumes static equilibrium. If the woman moves or the board is in motion, additional factors like inertia and acceleration come into play.

Summary and Key Takeaways

- When analyzing a beam supported at both ends with a load somewhere along its length, the key is to set up the force and torque balance equations.
- The reactions at supports depend on the load's position, with closer proximity to a support increasing that support's share of the load.
- The problem demonstrates fundamental physics principles, such as equilibrium, torque, and the distribution of forces.

Final Thoughts

Understanding the mechanics of a woman standing on a long board supported at both ends is more than an academic exercise; it reflects real-world engineering challenges. Whether designing bridges, scaffolding, or simple structures, the principles discussed here are vital for ensuring safety and functionality.

By systematically applying static equilibrium equations, carefully choosing pivot points, and analyzing load positions, you can solve similar problems confidently. Practice with different configurations, weights, and support placements to strengthen your grasp of these essential physics concepts.

Remember: Always verify your calculations, consider the physical context, and appreciate the elegance of how forces balance in static systems.

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