

An Electron Moves With A Speed Of 3.0×10^4 M/s Perpendicular To A Uniform Magnetic Field Of 0.40 T. What

An Electron Moves With A Speed Of 3.0×10^4 M/s Perpendicular To A Uniform Magnetic Field Of 0.40 T. What implications does this scenario have in the realm of physics, particularly in understanding electromagnetic forces and particle motion? This question touches on fundamental principles of electromagnetism, notably the behavior of charged particles in magnetic fields, which is essential in various applications ranging from particle accelerators to magnetic resonance imaging (MRI). In this comprehensive article, we will explore the physics behind the motion of electrons in magnetic fields, deriving relevant formulas, analyzing the implications, and discussing practical applications.

Understanding the Basics: Electron Motion in Magnetic Fields

The Lorentz Force and Its Role

The foundation of understanding how electrons move in magnetic fields lies in the Lorentz force law. This law states that a charged particle experiences a force when it moves through electric and magnetic fields. When considering only magnetic fields, the force $\mathbf{F}$ on a particle with charge q and velocity $\mathbf{v}$ in a magnetic field $\mathbf{B}$ is given by:

$$\mathbf{F} = q \mathbf{v} \times \mathbf{B}$$

Since the velocity is perpendicular to the magnetic field in our scenario, the magnitude of the force simplifies to:

$$F = q v B$$

where:

- q = charge of the electron ($\sim -1.6 \times 10^{-19}$ C)
- v = velocity of the electron ($\sim 3.0 \times 10^4$ m/s)
- B = magnetic field strength (0.40 T)

This force acts perpendicular to the velocity, causing the electron to undergo circular or helical motion depending on the initial velocity vector.

Key Concepts in Electron Trajectory

When an electron moves perpendicular to a uniform magnetic field, it experiences a centripetal force that causes it to move in a circular path. The characteristics of this motion depend on the magnetic field strength and the electron's initial speed.

Circular Motion Characteristics:

- The magnetic force provides the centripetal force:

$$q v B = \frac{m v^2}{r}$$

- Rearranged to find the radius (r):

$$r = \frac{m v}{q B}$$

where:

- (m) = mass of the electron ($\sim (9.11 \times 10^{-31})$ kg)

Period of Motion:

- The time taken for one complete revolution:

$$T = \frac{2 \pi r}{v}$$

- The frequency:

$$f = \frac{1}{T}$$

Understanding these parameters allows us to analyze the behavior of the electron in the magnetic field and predict its trajectory.

Calculating the Radius of the Electron's Path

Given:

- Electron charge ($q = 1.6 \times 10^{-19}$ C)
- Electron mass ($m = 9.11 \times 10^{-31}$ kg)
- Velocity ($v = 3.0 \times 10^4$ m/s)
- Magnetic field ($B = 0.40$ T)

Applying the formula for the radius:

$$r = \frac{m v}{q B}$$

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Plugging in the values:

$$r = \frac{9.11 \times 10^{-31} \times 3.0 \times 10^4}{1.6 \times 10^{-19} \times 0.40}$$

Calculating numerator:

$$9.11 \times 10^{-31} \times 3.0 \times 10^4 = 2.733 \times 10^{-26}$$

Calculating denominator:

$$1.6 \times 10^{-19} \times 0.40 = 6.4 \times 10^{-20}$$

Now, dividing:

$$r = \frac{2.733 \times 10^{-26}}{6.4 \times 10^{-20}} \approx 4.27 \times 10^{-7} \text{ meters}$$

Interpretation:

The radius of approximately 0.427 micrometers indicates that the electron moves in a very tight circle under these conditions.

Determining the Electron's Period of Revolution

Using:

$$T = \frac{2 \pi r}{v}$$

Substitute $(r \approx 4.27 \times 10^{-7} \text{ m})$ and $(v = 3.0 \times 10^4 \text{ m/s})$:

$$T = \frac{2 \pi \times 4.27 \times 10^{-7}}{3.0 \times 10^4}$$

Calculate numerator:

$$2 \pi \times 4.27 \times 10^{-7} \approx 2 \times 3.1416 \times 4.27 \times 10^{-7} \approx 2.68$$

$\times 10^{-6}$

\]

Now divide:

\[

$T = \frac{2.68 \times 10^{-6}}{3.0 \times 10^4} \approx 8.93 \times 10^{-11} \text{ seconds}$

\]

Implication:

The electron completes a revolution in approximately 89.3 picoseconds, reflecting rapid motion constrained within a microscopic radius.

Applications and Significance in Physics and Technology

Particle Accelerators and Beam Steering

Understanding the motion of electrons in magnetic fields is fundamental in designing particle accelerators. Magnetic fields are used to steer and focus electron beams with high precision. Calculations of radius and period help determine the magnetic field strengths required for desired trajectories.

Magnetic Resonance Imaging (MRI)

Electrons' behavior in magnetic fields underpins MRI technology, where strong magnetic fields align nuclear spins. Although electrons themselves are not directly imaged, the principles of charged particle motion inform the design of magnetic systems.

Fundamental Physics Research

Studying electron trajectories in magnetic fields helps in testing quantum mechanics, particle physics theories, and understanding phenomena like synchrotron radiation emitted when charged particles accelerate in magnetic fields.

Additional Considerations and Related Concepts

Effect of Electric Fields

If an electric field is present, the electron's motion becomes more complex, resulting in helical trajectories. The combined effects of electric and magnetic fields lead to phenomena like the $\mathbf{E} \times \mathbf{B}$ drift, which is exploited in plasma confinement.

Relativistic Effects

At high velocities approaching the speed of light, relativistic effects alter the mass and behavior of electrons. In our case, the velocity is non-relativistic, but in accelerators, these effects become significant and require adjustments in calculations.

Magnetic Moment and Spin

Beyond classical motion, electrons possess intrinsic spin magnetic moments, contributing to their magnetic interactions. Understanding classical trajectories complements quantum mechanical descriptions.

Summary and Final Thoughts

Analyzing the movement of an electron moving perpendicular to a magnetic field reveals critical insights into electromagnetic forces, particle trajectories, and their practical applications. The key takeaways include:

- The radius of the electron's circular motion is approximately 0.427 micrometers.
- The period of revolution is about 89.3 picoseconds.
- The Lorentz force governs the dynamics, providing a foundation for numerous technological innovations.
- Precise calculations enable the design of devices like particle accelerators and magnetic confinement systems.

Understanding these principles not only deepens our grasp of fundamental physics but also drives technological advancement across various fields. The behavior of electrons in magnetic fields remains a cornerstone topic in electromagnetism, emphasizing the elegant interplay between electric charges and magnetic forces in shaping the microscopic and macroscopic worlds.

References:

- Griffiths, D. J. Introduction to Electrodynamics. 4th Edition.
- Serway, R. A., & Jewett, J. W. Physics for Scientists and Engineers.
- Tipler, P. A., & Mosca, G. Physics for Scientists and Engineers.

Frequently Asked Questions

What is the radius of the electron's circular path in the magnetic field?

Using the formula $r = mv / (qB)$, where $m = 9.11 \times 10^{-31}$ kg, $v = 3.0 \times 10^4$ m/s, $q = 1.6 \times 10^{-19}$ C, and $B = 0.40$ T, the radius $r \approx 0.43$ mm.

What is the magnetic force acting on the electron?

The magnetic force $F = qvB = (1.6 \times 10^{-19} \text{ C})(3.0 \times 10^4 \text{ m/s})(0.40 \text{ T}) = 1.92 \times 10^{-15} \text{ N}$.

Does the magnetic field do work on the electron?

No, because the magnetic force acts perpendicular to the electron's velocity, performing no work and only changing its direction.

What is the frequency of the electron's circular motion in the magnetic field?

The frequency $f = v / (2\pi r)$. Using $r \approx 0.43 \text{ mm}$, $f \approx 3.0 \times 10^4 / (2\pi \times 0.00043) \approx 11.1 \text{ kHz}$.

How long does it take for the electron to complete one full circle?

Time period $T = 1 / f \approx 1 / 11,100 \text{ Hz} \approx 90 \text{ microseconds}$.

Would increasing the magnetic field strength affect the electron's path? How?

Yes, increasing B would decrease the radius of the electron's circular path, causing it to curve more sharply.

What is the direction of the electron's motion relative to the magnetic field?

Since the electron moves perpendicular to the magnetic field, its motion is circular in a plane perpendicular to the field, with the direction determined by the right-hand rule (considering the negative charge of the electron).

If the electron's speed was doubled, how would that affect its radius of curvature?

The radius would double, since $r \propto v$; increasing speed increases the radius of the circular path.

Additional Resources

Electron Motion in Magnetic Fields: Unraveling the Dynamics of Charged Particles

Understanding the behavior of electrons in magnetic fields is fundamental to many areas of physics and engineering, from the operation of cathode-ray tubes to the principles underlying particle accelerators and magnetic confinement in fusion reactors. At the core of these phenomena lies the Lorentz force, which governs how charged particles such as electrons respond to electromagnetic influences. In this article, we delve into a specific scenario: an electron moving with a speed of $(3.0 \times 10^4) \text{ m/s}$ perpendicular to a uniform magnetic field of 0.40 Tesla. We will explore the physics governing this motion, examine the resulting dynamics, and analyze the implications of such behavior in scientific and technological contexts.

Fundamentals of Electron Motion in Magnetic Fields

The Lorentz Force and Its Role

The motion of a charged particle in electromagnetic fields is dictated by the Lorentz force, expressed mathematically as:

$$\vec{F} = q (\vec{E} + \vec{v} \times \vec{B})$$

where:

- q is the charge of the particle (for an electron, $q = -1.6 \times 10^{-19}$ C),
- $\vec{E}$ is the electric field vector,
- $\vec{v}$ is the velocity of the particle,
- $\vec{B}$ is the magnetic field vector.

In the scenario under consideration, the electric field is absent or negligible, simplifying the force to:

$$\vec{F} = q \vec{v} \times \vec{B}$$

This force acts perpendicular to both the velocity of the electron and the magnetic field, resulting in a characteristic circular or helical motion depending on the initial conditions.

Motion Characteristics: Circular and Helical Paths

When a charged particle moves perpendicular to a magnetic field, the Lorentz force acts as a centripetal force, causing the particle to undergo uniform circular motion. The key features are:

- Radius of the circular path (Larmor radius): The radius depends on the particle's velocity, charge, mass, and magnetic field strength.
- Period of revolution: The time it takes for the electron to complete one circle.
- Frequency (Cyclotron frequency): How often the electron completes a revolution.

If the initial velocity has a component parallel to the magnetic field, the resultant path is a helix, combining circular motion in the plane perpendicular to the field with linear motion along the field direction.

Analyzing the Given Scenario

Given Data and Assumptions

- Electron velocity perpendicular to magnetic field: $(v = 3.0 \times 10^4) \text{ m/s}$
- Magnetic field strength: $(B = 0.40) \text{ T}$
- Electric field: assumed negligible $(E \approx 0)$
- Charge of electron: $(q = -1.6 \times 10^{-19}) \text{ C}$
- Mass of electron: $(m = 9.11 \times 10^{-31}) \text{ kg}$

The negative charge of the electron indicates that the force direction will be opposite to the conventional right-hand rule, but magnitude calculations remain the same regardless of sign.

Calculating the Radius of Electron's Path

The radius (r) of the circular motion is derived from balancing the magnetic Lorentz force with the required centripetal force:

$$q v B = \frac{m v^2}{r}$$

Rearranged to solve for (r) :

$$r = \frac{m v}{|q| B}$$

Plugging in the known values:

$$r = \frac{(9.11 \times 10^{-31} \text{ kg}) \times (3.0 \times 10^4 \text{ m/s})}{(1.6 \times 10^{-19} \text{ C}) \times 0.40 \text{ T}}$$

Calculating numerator:

$$9.11 \times 10^{-31} \times 3.0 \times 10^4 = 2.733 \times 10^{-26}$$

Calculating denominator:

$$1.6 \times 10^{-19} \times 0.40 = 6.4 \times 10^{-20}$$

Therefore:

$$r = \frac{2.733 \times 10^{-26}}{6.4 \times 10^{-20}} \approx 4.27 \times 10^{-7} \text{ m}$$

This radius is approximately 427 nanometers, indicating that the electron's circular path is confined within sub-micrometer scales, typical of high-energy physics and nanotechnology applications.

Implications of the Radius Calculation

- The small radius highlights the electron's tight curvature in a magnetic field of this strength.
- Such confinement is exploited in devices like cyclotrons and magnetic resonance imaging (MRI), where precise control over charged particle paths is essential.
- The radius also reflects the delicate balance between magnetic forces and particle inertia, influencing design considerations in experimental setups.

Determining the Cyclotron Frequency and Period

Cyclotron Frequency (f)

The frequency at which the electron completes each revolution, known as the cyclotron frequency, is given by:

$$f = \frac{|q| B}{2 \pi m}$$

Substituting the known values:

$$f = \frac{1.6 \times 10^{-19} \times 0.40}{2 \pi \times 9.11 \times 10^{-31}}$$

Calculations:

Numerator:

$$1.6 \times 10^{-19} \times 0.40 = 6.4 \times 10^{-20}$$

Denominator:

$$2\pi \times 9.11 \times 10^{-31} \approx 5.73 \times 10^{-30}$$

Thus:

$$f \approx \frac{6.4 \times 10^{-20}}{5.73 \times 10^{-30}} \approx 1.12 \times 10^{10} \text{ Hz}$$

This frequency, approximately 11.2 GHz, corresponds to the microwave range and underpins phenomena such as electron cyclotron resonance.

Period of Revolution (T)

The period T is the reciprocal of the frequency:

$$T = \frac{1}{f} \approx \frac{1}{1.12 \times 10^{10}} \approx 8.93 \times 10^{-11} \text{ s}$$

or roughly 89.3 picoseconds. This rapid rotation emphasizes the need for high-precision instrumentation to observe and manipulate electron trajectories at such scales.

Broader Context and Applications

Magnetic Confinement and Particle Accelerators

Understanding the motion of electrons under magnetic influence is central to the development of particle accelerators, where magnetic fields steer and focus particle beams. The calculated radius and frequency inform the design of cyclotrons and synchrotrons, enabling high-energy physics experiments that probe the fundamental constituents of matter.

Medical Imaging and Diagnostics

In MRI technology, strong magnetic fields cause proton spins to precess at characteristic frequencies. While electrons are not directly used in MRI, the principles governing their trajectories underpin the broader field of magnetic resonance and the precise control of charged particles in medical devices.

Plasma Physics and Fusion Research

Magnetic confinement fusion devices, such as tokamaks, rely on the motion of charged particles in magnetic fields to sustain plasma at extremely high temperatures. The detailed understanding of electron dynamics informs efforts to achieve stable confinement necessary for sustainable fusion energy.

Fundamental Physics and Quantum Mechanics

At quantum scales, the classical calculations give way to quantum behaviors, including Landau quantization and electron cyclotron resonance effects. The classical radius and frequency serve as baseline parameters, bridging the gap between macroscopic physics and quantum phenomena.

Conclusion: Significance and Future Directions

The analysis of an electron moving perpendicularly to a uniform magnetic field reveals a fascinating interplay of forces and motion, characterized by rapid circular paths confined to nanometer scales. The calculated radius of approximately 427 nanometers and the cyclotron frequency of about 11.2 GHz exemplify how electromagnetic principles govern particle behavior at microscopic scales. These insights not only deepen our understanding of fundamental physics but also underpin a wide array of technological innovations—from medical imaging to particle physics research.

As technology advances, the ability to manipulate electrons and other charged particles with ever-increasing precision will continue to unlock new realms of scientific discovery. Whether in developing more efficient fusion reactors, designing next-generation accelerators, or exploring quantum phenomena, the principles illuminated by this scenario remain at the heart of modern physics. The ongoing exploration of charged particle dynamics promises to yield breakthroughs that could redefine our understanding of the universe and lead to transformative technological applications.

References

1. Griffiths, D. J. Introduction to Electrodynamics. 4th Edition, Pearson Education, 2013.
2. Tipler, P. A., & Mos

[An Electron Moves With A Speed Of \$3.0 \times 10^4\$ m/s Perpendicular To A Uniform Magnetic Field Of 0.40 T What](#)

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