# An Urn Contains 4 Red Balls And 3 Green Balls. Two Balls Are Drawn Randomly. A. Let Z Denote The Number

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## Introduction to the Problem

Understanding probability and combinatorial analysis is essential when dealing with problems involving random draws from a finite set. In this scenario, we have an urn containing 4 red balls and 3 green balls, making a total of 7 balls. Two balls are drawn simultaneously or sequentially without replacement, and the goal is to analyze the distribution of a random variable Z, which denotes a particular outcome related to these draws.

This article aims to explore the probability distribution associated with Z, elucidate the various possible outcomes, and demonstrate how to compute relevant probabilities. Such analysis is fundamental in fields like statistics, operations research, and decision science, where understanding random processes and their outcomes is crucial.

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## Understanding the Composition of the Urn

### Initial Setup

The urn contains:

- Red Balls: 4
- Green Balls: 3

Total number of balls: 7.

Drawing two balls from the urn involves combinatorial considerations, especially since the balls are drawn without replacement, affecting the probabilities of subsequent outcomes.

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# Defining the Random Variable Z

## Possible Interpretations of Z

The variable Z can be defined in multiple ways depending on the problem's context. Common interpretations include:

- The number of red balls drawn in the two draws.
- The position of a specific color in the sequence of draws.
- An indicator variable for a particular event, such as drawing at least one red ball.

In this discussion, we will assume:

- Z denotes the number of red balls drawn in the two draws.

This interpretation allows us to analyze the probability distribution of Z, which can take values of 0, 1, or 2.

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# Possible Values of Z and Their Probabilities

## Range of Z

Since only two balls are drawn, the possible values of Z are:

- $Z = 0$: No red balls drawn (both green)

- Z = 1: One red and one green ball drawn
- Z = 2: Both red balls drawn

Understanding the probability of each scenario involves calculating the number of favorable combinations over the total possible combinations.

## Calculating Total Number of Outcomes

The total number of ways to draw two balls from seven is:

$$
\binom{7}{2} = \frac{7 \times 6}{2} = 21
$$

Each of these combinations is equally likely when drawing randomly without replacement.

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# Calculating Probabilities for Each Value of Z

## Probability that Z = 0 (No Red Balls)

This occurs when both balls are green. The number of ways to choose 2 green balls out of 3:

$$
\binom{3}{2} = 3
$$

Therefore:

$$
P(Z=0) = \frac{\text{Number of ways to pick 2 green}}{\text{Total ways to pick 2 balls}} = \frac{3}{21} = \frac{1}{7}
$$

## Probability that Z = 2 (Both Red Balls)

This occurs when both balls are red. The number of ways:

$$
\binom{4}{2} = 6
$$

Thus:

$$
P(Z=2) = \frac{6}{21} = \frac{2}{7}
$$

### Probability that Z = 1 (One Red and One Green)

This scenario involves selecting one red and one green ball:

$$
\binom{4}{1} \times \binom{3}{1} = 4 \times 3 = 12
$$

Hence:

$$
P(Z=1) = \frac{12}{21} = \frac{4}{7}
$$

### Summary of Probabilities

| Z Value | Description | Number of Favorable Outcomes | Probability |
|---|---|---|---|
| 0 | No red balls | 3 | 1/7 |
| 1 | One red ball | 12 | 4/7 |
| 2 | Both red balls | 6 | 2/7 |

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## Expected Value and Variance of Z

### Calculating the Expected Value

The expected value $E[Z]$ measures the average number of red balls drawn:

$$
E[Z] = 0 \times P(Z=0) + 1 \times P(Z=1) + 2 \times P(Z=2)
$$

$$
E[Z] = 0 \times \frac{1}{7} + 1 \times \frac{4}{7} + 2 \times \frac{2}{7} = 0 + \frac{4}{7} + \frac{4}{7} = \frac{8}{7} \approx 1.14
$$

This indicates that, on average, slightly more than one red ball is expected in each draw of two balls.

## Calculating the Variance

Variance quantifies the spread of Z around its mean:

$$
Var(Z) = E[Z^2] - (E[Z])^2
$$

Where:

$$
E[Z^2] = 0^2 \times P(Z=0) + 1^2 \times P(Z=1) + 2^2 \times P(Z=2)
$$

$$
E[Z^2] = 0 + 1 \times \frac{4}{7} + 4 \times \frac{2}{7} = \frac{4}{7} + \frac{8}{7} = \frac{12}{7}
$$

Thus:

$$
Var(Z) = \frac{12}{7} - \left(\frac{8}{7}\right)^2 = \frac{12}{7} - \frac{64}{49} = \frac{84}{49} - \frac{64}{49} = \frac{20}{49} \approx 0.408
$$

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# Applications and Significance of the Analysis

## Real-World Applications

Understanding such probability distributions has numerous practical applications:

- Quality Control: Assessing the likelihood of defective items (red balls) in batches.
- Card Games and Gambling: Calculating probabilities of specific hands or outcomes.
- Sampling and Surveys: Estimating proportions of certain traits in populations.
- Inventory Management: Predicting the chance of drawing specific items from a stock.

### Educational Importance

Analyzing simple models like the urn problem helps build foundational understanding of:

- Combinatorics: Counting arrangements and combinations.
- Probability Theory: Calculating likelihoods of events.
- Statistics: Understanding expected values and variance.

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## Extensions and Variations of the Problem

### Drawing with Replacement

If balls are replaced after each draw, probabilities change because the composition remains constant. For example, the probability of drawing a red ball in each draw becomes \(\frac{4}{7}\), and the distribution of Z follows a binomial model.

### Drawing More Than Two Balls

Expanding to three or more draws introduces more complex probability distributions, including hypergeometric distributions for exact counts.

### Different Composition of the Urn

Changing the number of red or green balls affects the probabilities significantly, modeling various real-world scenarios.

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## Conclusion

The problem of drawing two balls from an urn containing 4 red and 3 green balls exemplifies fundamental principles of probability and combinatorics. By defining the random variable Z as the number of red balls drawn, we can precisely calculate its probability distribution, expected value, and variance. These calculations provide valuable insights into the likelihood of different outcomes, supporting decision-making processes in various fields.

Understanding such models enhances our ability to analyze stochastic processes, optimize sampling strategies, and interpret random phenomena. Whether applied in quality control, game theory, or statistical sampling, the principles illustrated here form a cornerstone of probabilistic reasoning.

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References


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# Frequently Asked Questions

## What is the probability of drawing exactly one red ball and one green ball when two balls are drawn from the urn?

The probability is calculated as the number of favorable outcomes divided by total outcomes. There are 4 red and 3 green balls, so total balls are 7. The number of ways to draw one red and one green is (4 choose 1) (3 choose 1) = 4 3 = 12. The total number of ways to choose any 2 balls from 7 is (7 choose 2) = 21. Therefore, probability = 12 / 21 = 4 / 7.

## What is the probability of drawing two red balls in succession without replacement?

The probability of drawing two red balls without replacement is (4/7) (3/6) = (4/7) (1/2) = 4/14 = 2/7.

## If Z denotes the number of red balls drawn in two draws, what are the possible values of Z?

The possible values of Z are 0 (no red balls), 1 (one red ball), or 2 (both balls are red).

## What is the probability that both drawn balls are green?

The probability is (3/7) (2/6) = (3/7) (1/3) = 1/7.

## How do you interpret the random variable Z in this context?

Z represents the number of red balls drawn in the two randomly selected balls from the urn, taking values 0, 1, or 2.

# Additional Resources

An urn contains 4 red balls and 3 green balls. Two balls are drawn randomly. A. Let Z denote the number of red balls drawn in the sample. This classic problem in probability theory offers a rich ground for exploring concepts such as probability distributions, combinatorics, and statistical expectations. Understanding the distribution of Z, the number of red balls drawn, allows us to analyze various scenarios, calculate probabilities, and understand the underlying stochastic processes involved. In this comprehensive review, we will delve into the problem's setup, mathematical formulations, probability calculations, and broader applications, providing clarity for students, educators, and enthusiasts alike.

# Problem Setup and Basic Concepts

## Understanding the Composition of the Urn

The problem begins with an urn containing a total of 7 balls: 4 red and 3 green. The initial setup is straightforward:

- Total balls: 7
- Red balls: 4
- Green balls: 3

Two balls are drawn randomly without replacement, meaning once a ball is drawn, it is not returned to the urn before the second draw. The key random variable here is Z, which represents the number of red balls in the two draws.

## Defining the Random Variable Z

The variable Z can take on the following possible values:

- $Z = 0$: No red balls drawn (both balls are green)
- $Z = 1$: Exactly one red ball and one green ball
- $Z = 2$: Both balls are red

Since only two balls are drawn, these are the only possible outcomes for Z.

# Calculating Probabilities for Z

## Using Combinatorics

To find the probability distribution of Z, combinatorics provides a systematic approach:

- Total number of ways to draw 2 balls from 7: $\binom{7}{2} = 21$
- Number of favorable ways for each value of Z:

- For Z=0 (both green): select 2 green balls from 3: $\binom{3}{2} = 3$

- For Z=1 (one red, one green): select 1 red from 4 and 1 green from 3: $\binom{4}{1} \times \binom{3}{1} = 4 \times 3 = 12$

- For Z=2 (both red): select 2 red from 4: $\binom{4}{2} = 6$

Calculating Probabilities:

$$
\begin{aligned}
P(Z=0) &= \frac{\text{Number of ways to get 0 red}}{\text{Total ways}} = \frac{3}{21} = \frac{1}{7} \\
P(Z=1) &= \frac{12}{21} = \frac{4}{7} \\
P(Z=2) &= \frac{6}{21} = \frac{2}{7}
\end{aligned}
$$

This probability distribution captures the likelihood of each outcome, enabling further analysis.

# Features and Insights

- The probabilities are derived solely from combinatorial reasoning, assuming each pair is equally likely.
- The distribution is discrete and sums to 1, confirming its validity.
- The most probable outcome is drawing exactly one red ball, with a probability of $\frac{4}{7}$.

# Expected Value and Variance of Z

## Calculating the Expected Value

The expected value $E[Z]$ indicates the average number of red balls in numerous trials and is computed as:

$$
E[Z] = 0 \times P(Z=0) + 1 \times P(Z=1) + 2 \times P(Z=2)
$$

Substituting the probabilities:

$$
E[Z] = 0 \times \frac{1}{7} + 1 \times \frac{4}{7} + 2 \times \frac{2}{7} = 0 + \frac{4}{7} + \frac{4}{7} = \frac{8}{7} \approx 1.14
$$

This tells us that, on average, just over one red ball is expected in the two draws.

## Calculating Variance

Variance measures the spread of the distribution:

$$
Var(Z) = E[Z^2] - (E[Z])^2
$$

Where:

$$
E[Z^2] = 0^2 \times \frac{1}{7} + 1^2 \times \frac{4}{7} + 2^2 \times \frac{2}{7} = 0 + \frac{4}{7} + \frac{8}{7} = \frac{12}{7}
$$

Therefore:

$$
Var(Z) = \frac{12}{7} - \left(\frac{8}{7}\right)^2 = \frac{12}{7} - \frac{64}{49} = \frac{84}{49} -
$$

$$\frac{64}{49} = \frac{20}{49} \approx 0.408$$

The relatively low variance indicates that the number of red balls in two draws does not fluctuate widely.

# Conditional Probabilities and Further Analysis

## Conditional on the First Draw

Suppose we are interested in the probability of Z, given the result of the first draw:

- If the first ball is red, then the second draw has:

- 3 red and 3 green remaining

- Probability that the second is red: $\frac{3}{6} = \frac{1}{2}$

- Probability that the second is green: $\frac{3}{6} = \frac{1}{2}$

- If the first ball is green, then:

- 4 red and 2 green remaining

- Probability that the second is red: $\frac{4}{6} = \frac{2}{3}$

- Probability that the second is green: $\frac{2}{6} = \frac{1}{3}$

This conditional analysis helps in understanding the stepwise probabilities and can be extended to Bayesian updating or decision-making scenarios.

## Features of Conditional Probabilities

- Reflects how prior outcomes influence subsequent probabilities.
- Useful for sequential sampling or real-time decision processes.

# Broader Applications and Contexts

## Real-World Scenarios

This problem models several practical situations:

- Quality control: selecting defective vs. non-defective items.
- Card games: drawing specific suits or colors from a deck.
- Medical testing: probability of detecting a certain condition from a subset of tests.

## Teaching and Pedagogical Value

- Illustrates fundamental probability concepts clearly.
- Demonstrates combinatorics in practice.
- Encourages understanding of discrete probability distributions.
- Facilitates discussions on sampling without replacement.

## Extensions and Variations

- Drawing more than two balls.
- Changing the composition of the urn.
- Sampling with replacement.
- Introducing weights or biases in the drawing process.
- Analyzing hypergeometric versus binomial distributions.

# Pros and Cons of the Approach

Pros:

- Straightforward combinatorial calculations provide exact probabilities.
- Clear understanding of the distribution of Z.
- Applicable to various scenarios involving finite populations.
- Foundation for more advanced probabilistic models like hypergeometric distribution.

Cons:

- Assumes uniform probability of each pair; may not hold in real-world biased sampling.
- Does not consider additional complexities such as multiple draws or dependencies.
- Limited to small populations; computational challenges arise with larger datasets.

## Conclusion

The problem of drawing two balls from an urn containing 4 red and 3 green balls, with the variable Z denoting the number of red balls drawn, exemplifies core principles of probability, combinatorics, and statistical analysis. By carefully enumerating outcomes and calculating probabilities, one gains a deeper understanding of discrete distributions and their properties. The concepts extend beyond this simple setup to a wide range of practical and theoretical applications, making it an essential topic in the study of probability. Whether for teaching, research, or real-world decision-making, analyzing such sampling problems enhances analytical skills and fosters appreciation for the elegant structure of probabilistic systems.

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