An Urn Contains 5 Red, 6 Blue, And 8 Green Balls. If A Set Of 3 Balls Is Randomly Selected, What Is The

An Urn Contains 5 Red, 6 Blue, And 8 Green Balls. If A Set Of 3 Balls Is Randomly Selected, What Is The comprehensive guide to understanding the probabilities, calculations, and concepts involved in this classic problem in combinatorics and probability theory. This article aims to break down the problem, explore various scenarios, and provide detailed solutions, making it an ideal resource for students, educators, and enthusiasts interested in probability calculations involving urns and random selections.

Understanding the Problem: The Basics

Before diving into the calculations, it's essential to understand the problem's components and what is being asked.

Details of the Urn and Balls

- The urn contains a total of 19 balls:
- 5 Red balls
- 6 Blue balls
- 8 Green balls

- The task involves randomly selecting 3 balls from the urn without replacement (assuming standard probability rules unless specified otherwise).

What Is Being Asked?

Typically, problems like this ask for one or more of the following:

- The probability of selecting a specific combination of balls (e.g., 2 Red and 1 Blue).
- The probability of selecting at least one ball of a certain color.
- The probability of selecting all balls of different colors.
- General probability calculations related to the composition of the selected balls.

In this case, the phrase "what is the" suggests that the question might be incomplete or that it's leading to a specific probability calculation, such as:

- What is the probability that all three balls are of different colors?
- What is the probability of selecting exactly two red balls?
- What is the probability of selecting at least one green ball?

For the sake of this comprehensive guide, we will explore various common scenarios and how to calculate their probabilities, providing a complete overview.

Key Concepts in Probability and Combinatorics

Understanding the core principles is essential before tackling specific calculations.

Basic Probability Principles

- Sample Space (Total Outcomes): The total number of ways to select 3 balls from 19.
- Favorable Outcomes: The number of ways that satisfy the specific condition (e.g., selecting 2 red and 1 blue).
- Probability Formula:

$$
P(\text{Event}) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}
$$

Combinatorial Calculations

- Combination Formula:

$$
\binom{n}{k} = \frac{n!}{k!(n - k)!}
$$

- Used to calculate the number of ways to choose k items from n without regard to order.

Calculating Total Number of Ways to Select 3 Balls

The first step in any probability calculation is determining the total number of possible outcomes.

Total Combinations

Since the balls are distinct only by color, and the selection is without replacement, the total number of ways to choose 3 balls from 19 is:

$$
\text{Total outcomes} = \binom{19}{3} = \frac{19!}{3!(19-3)!} = \frac{19 \times 18 \times 17}{3 \times 2 \times 1} = 969
$$

This total forms the denominator for all probability calculations that involve selecting any 3 balls.

Scenarios and Probabilities

Below are common scenarios and how to compute their probabilities.

1. Probability of Selecting Three Balls of Different Colors

Objective: Find the probability that the three balls are each of different colors (i.e., one Red, one Blue, and one Green).

Step-by-step solution:

- Calculate the number of favorable outcomes:

- Ways to choose 1 Red from 5:

$$
\binom{5}{1} = 5
$$

- Ways to choose 1 Blue from 6:

$$
\binom{6}{1} = 6
$$

- Ways to choose 1 Green from 8:

$$
\binom{8}{1} = 8
$$

- Total favorable outcomes:

$$
\binom{5}{1} \times \binom{6}{1} \times \binom{8}{1} = 5 \times 6 \times 8 = 240
$$

- Probability:

$$
P(\text{all different colors}) = \frac{240}{969} \approx 0.2477
$$

---

2. Probability of Selecting Two Red Balls and One Blue Ball

Objective: Find the probability of choosing exactly 2 Red and 1 Blue.

Step-by-step solution:

- Ways to choose 2 Red from 5:

$$
\binom{5}{2} = 10
$$

- Ways to choose 1 Blue from 6:

$$
\binom{6}{1} = 6

- Total favorable outcomes:

$$
10 \times 6 = 60
$$

- Probability:

$$
P(\text{2 Red, 1 Blue}) = \frac{60}{969} \approx 0.0619
$$

---

3. Probability of Selecting All Green Balls

Objective: Find the probability that all three selected balls are green.

Step-by-step solution:

- Ways to choose 3 Green from 8:

$$
\binom{8}{3} = \frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56
$$

- Probability:

$$
P(\text{all green}) = \frac{56}{969} \approx 0.0578
$$

---

4. Probability of Selecting Exactly One Red and Two Green Balls

Objective: Find the probability of exactly 1 Red and 2 Green.

Step-by-step solution:

- Ways to choose 1 Red from 5:

$$
\binom{5}{1} = 5
$$

- Ways to choose 2 Green from 8:

$$
\binom{8}{2} = \frac{8 \times 7}{2} = 28
$$

- Total favorable outcomes:

$$
5 \times 28 = 140
$$

- Probability:

$$
P(\text{1 Red, 2 Green}) = \frac{140}{969} \approx 0.1445
$$

Additional Probabilities and Considerations

Apart from the specific scenarios, other interesting probability questions include:

1. Probability of Selecting At Least One Green Ball

This can be approached using the complement rule:

- Complement: No green balls are selected (i.e., all selected balls are Red or Blue).

- Calculate the probability of selecting no green balls:

- Total balls without green: 5 Red + 6 Blue = 11

- Number of ways to choose 3 balls from these 11:

\[
\binom{11}{3} = \frac{11 \times 10 \times 9}{6} = 165
\]

- Probability of no green:

\[
P(\text{no green}) = \frac{165}{969}
\]

- Probability of at least one green:

\[
P(\text{at least one green}) = 1 - P(\text{no green}) = 1 - \frac{165}{969} = \frac{969 - 165}{969} = \frac{804}{969} \approx 0.8304
\]

---

2. Expected Number of Green Balls in a Random Sample of 3

The expected value (mean) of green balls in the sample can be calculated as:

\[
E(\text{green}) = 3 \times \frac{\text{Total green balls}}{\text{Total balls}} = 3 \times \frac{8}{19} \approx 1.263
\]

This indicates that, on average, in many such samples, about 1.26 green balls would be selected.

Summary of Key Probabilities

Scenario	Probability

All three different colors	$\frac{240}{969} \approx 0.2477$
Two Red, one Blue	$\frac{60}{969} \approx 0.0619$
All green	$\frac{56}{969} \approx 0.0578$
One Red, two Green	$\frac{140}{969} \approx 0.1445$
At least one green	$\frac{804}{969} \approx 0.8304$
No green (all Red or Blue)	$\frac{165}{969} \approx 0.1703$

Conclusion: Applying Probability Theory to Real-World Problems

Frequently Asked Questions

What is the total number of balls in the urn?

The total number of balls is 5 Red + 6 Blue + 8 Green = 19 balls.

How many ways can 3 balls be selected from the urn?

The total number of ways to select 3 balls from 19 is C(19, 3) = 969.

What is the probability of selecting exactly 1 Red, 1 Blue, and 1 Green ball?

Number of favorable outcomes = C(5,1) C(6,1) C(8,1) = 5 6 8 = 240. Probability = 240 / 969 ≈ 0.2477.

What is the probability of selecting 3 Green balls?

Number of ways to choose 3 Green balls = C(8,3) = 56. Probability = 56 / 969 ≈ 0.0578.

What is the probability of selecting no Red balls?

Number of ways to select 3 balls with no Red = C(6 + 8, 3) = C(14, 3) = 364. Probability = 364 / 969 ≈ 0.3757.

What is the probability of selecting all balls of the same color?

Possible only for Green: C(8,3) = 56. For Red and Blue, since fewer than 3 balls, probability is zero. Total favorable = 56. Probability = 56 / 969 ≈ 0.0578.

How many combinations include at least one Red ball?

Total combinations minus those with no Red: 969 - C(14, 3) = 969 - 364 = 605.

What is the probability of selecting exactly 2 Blue balls in the set?

Number of ways = C(6,2) C(13,1) = 15 13 = 195. Probability = 195 / 969 ≈ 0.201.

If a set of 3 balls is randomly selected, what is the probability that all 3 are of different colors?

Number of ways = C(5,1)C(6,1)C(8,1) = 240. Probability = 240 / 969 ≈ 0.2477.

Additional Resources

An Urn Contains 5 Red, 6 Blue, And 8 Green Balls. If A Set Of 3 Balls Is Randomly Selected, What Is The — this intriguing probability problem invites us to delve into the fascinating world of combinatorics and probability theory. It challenges us to analyze the likelihood of various outcomes when selecting balls from a finite set, each with distinct colors and quantities. Understanding such problems not only enhances mathematical skills but also offers insights into real-world applications like quality control, lottery systems, and statistical sampling. This article aims to comprehensively explore this problem, breaking it down into manageable sections, discussing key concepts, calculations, and interpretations.

Understanding the Problem

The problem presents an urn containing a total of 19 balls:
- 5 Red balls
- 6 Blue balls
- 8 Green balls

A set of three balls is drawn randomly from this urn. The question typically posed (though not explicitly completed in the prompt) involves calculating probabilities related to the color composition of the selected set. Common variants include:
- The probability that all three balls are of the same color
- The probability of selecting at least one ball of a specific color

- The probability of selecting a particular combination of colors (e.g., exactly two green balls and one red ball)

This analysis will assume the focus is on calculating various probabilities associated with such selections, particularly:
- The probability that all three balls are of the same color
- The probability that the set contains at least one green ball
- The probability of selecting exactly two blue balls and one green ball

Understanding these scenarios requires a foundational grasp of combinatorics, specifically combinations, as the selection is random and without replacement.

Basic Concepts and Formulas

Combinations and Selection

The core mathematical tool here is the combination formula, which calculates the number of ways to choose 'k' items from a set of 'n' without regard to order:

$$C(n, k) = \frac{n!}{k!(n-k)!}$$

where:
- $n!$ denotes factorial of n,
- $C(n, k)$ is the number of combinations.

In this context:
- Total number of ways to choose 3 balls from 19:

$$C(19, 3)$$

- Number of ways to select 3 balls of a specific color:

- Red: $C(5, 3)$
- Blue: $C(6, 3)$
- Green: $C(8, 3)$

- Number of ways to select specific mixed-color combinations, such as 2 of one color and 1 of another, involves multiplying the respective combinations.

Probability Calculation

The probability of an event is given by:

$$ P(\text{event}) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}} $$

Applying this, once the favorable combinations are identified, dividing by the total combination count yields the probability.

Calculating Basic Probabilities

Let's explore specific probabilities step-by-step.

1. Probability that all three balls are of the same color

Red Balls:
- Number of ways to choose 3 red balls:

$$ C(5, 3) = \frac{5!}{3!(5-3)!} = \frac{5 \times 4 \times 3!}{3! \times 2!} = 10 $$

Blue Balls:
- Number of ways:

$$ C(6, 3) = \frac{6!}{3! \times 3!} = 20 $$

Green Balls:
- Number of ways:

$$ C(8, 3) = \frac{8!}{3! \times 5!} = 56 $$

Total favorable outcomes:

$$ 10 + 20 + 56 = 86 $$

Total possible outcomes:
- Total ways to select any 3 balls:

$$ C(19, 3) = \frac{19!}{3! \times 16!} = \frac{19 \times 18 \times 17}{6} = 969 $$

Probability:

$$ P(\text{all same color}) = \frac{86}{969} \approx 0.0887 \text{ or } 8.87\% $$

This indicates a relatively low probability that all three balls drawn are of the same color, which makes intuitive sense given the diverse set.

2. Probability that at least one green ball is selected

It's often easier to calculate the complement: the probability that no green balls are selected, then subtract from 1.

Number of ways to choose 3 balls with no green:
- Only red and blue balls are considered:

$$ C(5, r) \times C(6, 3 - r) $$

where r is the number of red balls chosen (0 to 3), with the constraint that the total is 3.

- For $r=0$:

$$ C(5, 0) \times C(6, 3) = 1 \times 20 = 20 $$

- For $r=1$:

$$ C(5, 1) \times C(6, 2) = 5 \times 15 = 75 $$

- For $r=2$:

$$ C(5, 2) \times C(6, 1) = 10 \times 6 = 60 $$

- For $r=3$:

$$ C(5, 3) \times C(6, 0) = 10 \times 1 = 10 $$

Total no-green outcomes:

$$ 20 + 75 + 60 + 10 = 165 $$

Probability of no green:

$$ P(\text{no green}) = \frac{165}{969} \approx 0.1702 $$

Therefore, the probability of selecting at least one green ball:

$$ P(\text{at least one green}) = 1 - 0.1702 = 0.8298 \text{ or } 82.98\% $$

This high probability reflects the abundance of green balls in the urn.

3. Probability of selecting exactly two blue balls and one green ball

Number of favorable outcomes:
- Choose 2 blue balls:

$$ C(6, 2) = 15 $$

- Choose 1 green ball:

$$ C(8, 1) = 8 $$

- Total favorable outcomes:

$$ 15 \times 8 = 120 $$

Total outcomes:

$$ C(19, 3) = 969 $$

Probability:

$$ P(\text{2 blue, 1 green}) = \frac{120}{969} \approx 0.124 \text{ or } 12.4\% $$

This scenario illustrates how specific combinations can be computed straightforwardly using the combination principle.

Advanced Considerations

Beyond these basic calculations, more complex probabilities can be analyzed, such as:
- Probability of exactly one ball of each color
- Probability of at least two green balls
- Conditional probabilities, e.g., given that a red ball is selected, what is the probability that the other two are green?

Each involves similar combinatorial reasoning but requires careful enumeration of favorable outcomes and total possibilities.

Practical Applications and Relevance

Understanding such probability models has multiple practical applications:
- Quality Control: Determining the likelihood of defects (represented by a particular color) in sampled batches.
- Lottery and Games: Calculating odds for specific ticket combinations.
- Statistical Sampling: Estimating probabilities in surveys where items are randomly selected.
- Decision Making: Informing strategies based on probabilistic outcomes.

These calculations reinforce the importance of combinatorics in real-world problem-solving, demonstrating how mathematical theory underpins practical decision-making.

Pros and Cons of Probability Calculations in Such Contexts

Pros:
- Provides precise quantification of uncertainty.
- Aids in risk assessment and decision-making.
- Enhances understanding of combinatorics and probability principles.
- Can be extended to larger, more complex systems.

Cons:
- Can become computationally intensive for larger datasets.
- Assumes perfect randomness and no bias.
- Requires careful enumeration of cases, which can be error-prone.
- Does not account for real-world variables like replacement or changing conditions unless explicitly modeled.

Conclusion

The problem of selecting 3 balls from an urn containing 5 Red, 6 Blue, and 8 Green balls offers a rich exploration into combinatorial probability. By understanding the fundamental formulas and applying systematic calculations, we can determine the likelihood of various outcomes with confidence. Such analyses not only sharpen mathematical reasoning but also have tangible applications across industries and fields. While the calculations can sometimes be complex, especially for more intricate scenarios, the foundational principles remain consistent and accessible. This exercise exemplifies the power of combinatorics in decoding the randomness inherent in many practical situations, underscoring its value in both academic and real-world contexts.

An Urn Contains 5 Red 6 Blue And 8 Green Balls If A Set Of 3 Balls Is Randomly Selected What Is The

An Urn Contains 5 Red 6 Blue And 8 Green Balls If A Set Of 3 Balls Is Randomly Selected What Is The

Related Articles

- Based On The Frequency Distribution Above, Find The Relative Frequency For The Class With A Lower Class
- At A Local Restaurant, 52% Of The Employees Work Both Nights And Weekends. If 63% Of The Employees Work
- A Railroad Handcar Is Moving Along Straight, Frictionless Tracks With Negligible Air Resistance. In The

Back to Home