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Introduction

Analyzing discrete-time systems is fundamental in signal processing, control systems, and digital communications. When given a difference equation such as

\[Y(n) = 0.5 Y(n-1) - 0.3 Y(n-2) + 2 X(n-1) + X(n-3), \]

one of the primary objectives is to understand the system's behavior, including calculating its response to specific inputs, finding its system function, and analyzing stability. This article provides a comprehensive guide to analyzing the given discrete-time system, focusing on calculation methods, system properties, and solution techniques.

Understanding the System Equation

Discrete Time System Equation Breakdown

The given system is described by a linear difference equation:

\[Y(n) = 0.5 Y(n-1) - 0.3 Y(n-2) + 2 X(n-1) + X(n-3). \]

Here:

- \(Y(n) \) is the output at discrete time \(n \).
- \(X(n) \) is the input at time \(n \).
- The coefficients \(0.5 \) and \(-0.3 \) are associated with feedback terms involving previous outputs.
- The terms \(2 X(n-1) \) and \(X(n-3) \) represent the influence of past inputs on current output.

Key Points to Note

- The system is linear and time-invariant (LTI), given the constant coefficients.
- The order of the system is determined by the highest delay in the output terms, which is 2 (since \(Y(n-2) \) appears).
- The input delays are 1 and 3, indicating the system's dependence on past input signals.

Step 1: Deriving the System's Homogeneous Solution

Homogeneous Equation

To analyze the system, begin by considering the homogeneous equation where input signals are zero:

$$ Y_h(n) = 0.5 Y_h(n-1) - 0.3 Y_h(n-2). $$

The characteristic equation associated with this recurrence relation is:

$$ r^2 - 0.5 r + 0.3 = 0. $$

Solving the Characteristic Equation

Applying quadratic formula:

$$ r = \frac{0.5 \pm \sqrt{(0.5)^2 - 4 \times 1 \times 0.3}}{2} = \frac{0.5 \pm \sqrt{0.25 - 1.2}}{2}. $$

Calculating discriminant:

$$ 0.25 - 1.2 = -0.95, $$

which is negative, indicating complex conjugate roots:

$$ r = \frac{0.5 \pm j \sqrt{0.95}}{2}. $$

Expressed explicitly:

$$
r = \frac{0.5}{2} \pm j \frac{\sqrt{0.95}}{2} = 0.25 \pm j \times 0.487.
$$

Homogeneous Solution

The homogeneous solution is:

$$
Y_h(n) = A \left(r_1 \right)^n + B \left(r_2 \right)^n,
$$

where:

$$
r_{1,2} = 0.25 \pm j \times 0.487,
$$

and A, B are constants determined by initial conditions.

Expressing roots in polar form:

$$
|r| = \sqrt{0.25^2 + 0.487^2} \approx 0.546,

\]

$$
\theta = \arctan \left(\frac{0.487}{0.25} \right) \approx 62.8^\circ.
$$

Thus,

$$
r_{1,2} = 0.546 \angle \pm 62.8^\circ.
$$

The homogeneous solution becomes:

$$
Y_h(n) = M \times (0.546)^n \cos(62.8^\circ n + \phi),
$$

where M, ϕ depend on initial conditions.

Step 2: Calculating Particular Solution with Given Input

Input Signal Specification

To proceed with calculations, specify the input $X(n)$. Common choices include:

- Unit step input: $X(n) = u(n)$,
- Impulse input: $X(n) = \delta(n)$,
- Sinusoidal input: $X(n) = \sin(\omega n)$.

Suppose the input is a unit step: $X(n) = u(n)$, where $u(n) = 1$ for $n \geq 0$, zero otherwise.

Form of the Particular Solution

Given the linearity of the system, the particular solution $Y_p(n)$ can be found using methods such as:

- Method of Undetermined Coefficients: suitable for inputs like sinusoidal or polynomial signals.
- Z-Transform Method: more comprehensive, especially for arbitrary inputs.

For simplicity, consider the input as a constant (unit step), then the particular solution may tend to a steady-state value if the system is stable.

Step 3: Applying the Z-Transform

The Z-transform simplifies solving difference equations, especially with inputs.

Z-Transform of the System Equation

Applying the Z-transform:

$$
Y(z) = 0.5 z^{-1} Y(z) - 0.3 z^{-2} Y(z) + 2 z^{-1} X(z) + z^{-3} X(z).
$$

Rearranged:

$$
Y(z) - 0.5 z^{-1} Y(z) + 0.3 z^{-2} Y(z) = 2 z^{-1} X(z) + z^{-3} X(z).
$$

Factor out $Y(z)$:

$$
Y(z) \left(1 - 0.5 z^{-1} + 0.3 z^{-2} \right) = X(z) \left(2 z^{-1} + z^{-3} \right).
$$

Define the system function:

$$
H(z) = \frac{Y(z)}{X(z)} = \frac{2 z^{-1} + z^{-3}}{1 - 0.5 z^{-1} + 0.3 z^{-2}}.
$$

Multiply numerator and denominator by z^{3} to remove negative powers:

$$
H(z) = \frac{2 z^{2} + 1}{z^{3} - 0.5 z^{2} + 0.3 z}.
$$

This transfer function describes the system's behavior in the frequency domain.

System Stability

Stability depends on the roots of the denominator:

$$
z^{3} - 0.5 z^{2} + 0.3 z = 0,
$$

or

$$
z (z^{2} - 0.5 z + 0.3) = 0.
$$

Roots are:

- $z = 0$,

- Roots of $z^{2} - 0.5 z + 0.3 = 0$.

Solve quadratic:

$$
z = \frac{0.5 \pm \sqrt{0.25 - 1.2}}{2} = \frac{0.5 \pm j \sqrt{0.95}}{2}.
$$

Same roots as earlier, with magnitude approximately 0.546, less than 1, indicating the system is stable.

Step 4: Computing the System Response

Response to the Step Input

For the unit step input $X(n) = u(n)$, the Z-transform is:

$$
X(z) = \frac{z}{z - 1}.
$$

The output in Z-domain:

$$
Y(z) = H(z) \times X(z) = \frac{2 z^{2} + 1}{z^{3} - 0.5 z^{2} + 0.3 z} \times \frac{z}{z - 1}.
$$

Inverse Z-transform yields $Y(n)$, i.e., the system's response.

Numerical Calculation

To find explicit values, partial fraction decomposition or numerical inverse Z-transform methods can be employed.

Step 5: Summary of Calculation Steps

1. Determine the homogeneous solution by solving the characteristic equation.
2. Find particular solutions based on the input signal.
3. Use Z-transform to convert the difference equation into an algebraic equation.
4. Solve for $Y(z)$ and then perform inverse Z-transform to obtain $Y(n)$.
5. Analyze stability by examining pole locations.
6. Compute system response for specific inputs.

Additional Considerations

Stability and Causality

- The roots of the characteristic equation (system poles) determine the stability.
- Since all roots are within the unit circle, the system is stable.
- Causality is inherent in the difference equation, as current output depends only on past inputs and

Frequently Asked Questions

What is the difference equation for the given discrete-time system?

The difference equation is Y(n) = 0.5 Y(n-1) - 0.3 Y(n-2) + 2 X(n-1) + X(n-3).

How do you find the homogeneous solution for this system?

Set the input X(n) to zero and solve the characteristic equation: r^2 - 0.5 r + 0.3 = 0, to find the homogeneous solution based on roots r.

What is the method to compute the system's transfer function H(z)?

Apply the Z-transform to the difference equation, assuming zero initial conditions, to express Y(z) and X(z), then compute H(z) = Y(z)/X(z).

How can stability be determined for this system?

Check the roots of the characteristic equation; the system is stable if all roots lie inside the unit circle in the z-plane.

What is the role of initial conditions in solving this difference equation?

Initial conditions Y(n-1) and Y(n-2) are necessary to find the particular solution for the difference equation when analyzing the system's response.

How do you compute the particular solution for this system when the input is known?

Use methods like Z-transform or convolution, substituting the known input sequence X(n) to solve for Y(n).

What is the significance of the coefficients 0.5, -0.3, 2, and 1 in the difference equation?

They represent the system's parameters that influence its behavior, such as damping (0.5, -0.3) and input gain (2, 1), affecting stability and response characteristics.

Additional Resources

Investigative Analysis of the Discrete-Time System: $Y(n)=0.5 Y(n-1)-0.3 Y(n-2)+2 X(n-1)+X(n-3)$

Introduction

Discrete-time systems are fundamental in digital signal processing, control systems, and various engineering applications. Understanding their behavior, stability, and response characteristics is essential for designing reliable and efficient systems. The system under investigation:

$$
Y(n) = 0.5\, Y(n-1) - 0.3\, Y(n-2) + 2\, X(n-1) + X(n-3)
$$

is a linear, time-invariant (LTI) difference equation that relates the current output $Y(n)$ to past outputs and inputs. This article aims to thoroughly analyze this system, focusing on calculating key properties such as its homogeneous solution, particular solution, transfer function, and stability criteria.

System Overview and Mathematical Foundations

Understanding the Difference Equation

The provided system is a second-order linear difference equation with input terms:

- Past outputs: $Y(n-1)$, $Y(n-2)$
- Past inputs: $X(n-1)$, $X(n-3)$

The structure suggests a blend of recursive (feedback) and non-recursive (feedforward) components, typical in digital filters.

Homogeneous Solution

Derivation

The homogeneous part of the system is obtained by setting the input $X(n) = 0$:

$$
Y_h(n) = 0.5\, Y(n-1) - 0.3\, Y(n-2)
$$

which simplifies to the characteristic equation:

$$
Y(n) - 0.5\, Y(n-1) + 0.3\, Y(n-2) = 0
$$

Assuming solutions of the form $Y(n) = r^n$, the characteristic equation becomes:

$$
r^n - 0.5 r^{n-1} + 0.3 r^{n-2} = 0
$$

Dividing through by r^{n-2} (assuming $r \neq 0$):

$$
r^2 - 0.5 r + 0.3 = 0
$$

This quadratic equation leads to the roots:

$$
r = \frac{0.5 \pm \sqrt{(0.5)^2 - 4 \times 1 \times 0.3}}{2}
$$

Calculating the discriminant:

$$
\Delta = 0.25 - 1.2 = -0.95
$$

Since $\Delta < 0$, roots are complex conjugates:

$$
r = \frac{0.5 \pm j \sqrt{0.95}}{2}
$$

Expressed explicitly:

$$
r = \frac{0.5}{2} \pm j \frac{\sqrt{0.95}}{2} = 0.25 \pm j \times 0.4879
$$

The magnitude and phase of these roots are:

$$
|r| = \sqrt{(0.25)^2 + (0.4879)^2} \approx \sqrt{0.0625 + 0.238} \approx \sqrt{0.3005} \approx 0.548
$$

$$
\angle r = \arctan\left(\frac{0.4879}{0.25}\right) \approx \arctan(1.951) \approx 62.4^\circ
$$

Homogeneous Solution Expression

The general homogeneous solution is:

$$
Y_h(n) = A (0.548)^n \cos(62.4^\circ n) + B (0.548)^n \sin(62.4^\circ n)
$$

where A and B are constants determined by initial conditions.

Particular Solution and System Response

Input-Dependent Terms

The non-homogeneous part involves inputs $X(n-1)$ and $X(n-3)$. To find the particular solution, consider the nature of $X(n)$. Since the input function is unspecified, the analysis proceeds generally, using the Z-transform approach to derive the system’s transfer function.

Transfer Function Analysis

Z-Transform of the Difference Equation

Applying the Z-transform (assuming zero initial conditions):

$$
Y(z) = 0.5 z^{-1} Y(z) - 0.3 z^{-2} Y(z) + 2 z^{-1} X(z) + z^{-3} X(z)
$$

Rearranged:

$$
Y(z) [1 - 0.5 z^{-1} + 0.3 z^{-2}] = [2 z^{-1} + z^{-3}] X(z)
$$

Expressed as:

$$
H(z) = \frac{Y(z)}{X(z)} = \frac{2 z^{-1} + z^{-3}}{1 - 0.5 z^{-1} + 0.3 z^{-2}}
$$

Multiply numerator and denominator by z^3 to clear negative exponents:

$$
H(z) = \frac{2 z^{2} + 1}{z^{3} - 0.5 z^{2} + 0.3 z}
$$

or

$$
H(z) = \frac{2 z^{2} + 1}{z (z^{2} - 0.5 z + 0.3)}
$$

This transfer function encapsulates the system's frequency response and stability characteristics.

Stability and Pole Analysis

Pole Locations

The system's stability hinges on the roots of the denominator:

$$
z^{2} - 0.5 z + 0.3 = 0
$$

Discriminant:

$$
\Delta_z = (0.5)^2 - 4 \times 1 \times 0.3 = 0.25 - 1.2 = -0.95
$$

Roots:

$$
z = \frac{0.5 \pm j \sqrt{0.95}}{2} \approx 0.25 \pm j 0.4879
$$

Magnitude:

$$
|z| = \sqrt{(0.25)^2 + (0.4879)^2} \approx 0.548
$$

Since these pole magnitudes are less than 1, the system is BIBO (Bounded Input, Bounded Output) stable.

Frequency Response and System Characteristics

Magnitude and Phase Response

The transfer function $H(z)$ evaluated on the unit circle $z = e^{j \omega}$:

$$
H(e^{j \omega}) = \frac{2 e^{2 j \omega} + 1}{j e^{j \omega} (e^{2 j \omega} - 0.5 e^{j \omega} + 0.3)}
$$

This expression can be used to analyze the frequency response, revealing resonant frequencies, filtering characteristics, and phase shifts.

Implications and Practical Considerations

Filter Design and Implementation

- The roots' magnitudes suggest a stable filter with a decaying response over time.
- The complex conjugate poles indicate oscillatory behavior with a damping factor aligned with the pole magnitude (~0.548).
- The zeros influence the frequency nulls or peaks, depending on their location in the Z-plane.

System Response to Typical Inputs

- For step inputs, the transient response will decay exponentially, modulated by oscillations.
- For sinusoidal inputs, the frequency response determines amplitude scaling and phase shift.

Conclusion

The comprehensive analysis of the discrete-time system:

$$
Y(n) = 0.5\, Y(n-1) - 0.3\, Y(n-2) + 2\, X(n-1) + X(n-3)
$$

reveals a stable, second-order system characterized by complex conjugate poles within the unit circle. Its transfer function indicates that the system acts as a filter with specific frequency-dependent gain and phase properties.

This detailed investigation underscores the importance of pole-zero analysis, stability criteria, and transfer function derivation in understanding and designing discrete-time systems. Such insights are vital for engineers and scientists working in digital signal processing, control systems, and related fields to optimize system performance and predict behavior under various input conditions.

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Note: The above analysis assumes ideal conditions and zero initial states; real-world applications may require additional considerations such as initial condition effects, quantization, and non-linearities.

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