Calculate S (for The System) When The State Of 3.00 Mol Of A Monatomic Perfect Gas, For Which Cp,m =5/2

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Understanding thermodynamic properties such as entropy is fundamental in thermodynamics, especially when analyzing ideal gases. In this article, we will explore how to determine the entropy change, denoted as S, for a system consisting of 3.00 mol of a monatomic perfect gas, given that the molar heat capacity at constant pressure $C_{p,m}$ is $\frac{5}{2} R$. This analysis involves fundamental concepts like the ideal gas law, molar heat capacities, and the definition of entropy change for ideal gases. By the end of this discussion, you'll gain a clear understanding of how to perform such calculations and the significance of entropy in thermodynamic systems.

Fundamentals of Thermodynamics for Monatomic Ideal Gases

Before diving into calculations, it's important to review the essential thermodynamic principles relevant to monatomic ideal gases.

Ideal Gas Law

The ideal gas law relates pressure P, volume V, temperature T, and amount of substance in moles n:

$$PV = nRT$$

where:
- P is the pressure,
- V is the volume,
- n is the number of moles,
- R is the universal gas constant ($R \approx 8.314\, J\,mol^{-1}\,K^{-1}$),
- T is the temperature in Kelvin.

Molar Heat Capacities and Internal Energy

For monatomic ideal gases:
- The molar heat capacity at constant volume, $C_{v,m}$, is $\frac{3}{2} R$.
- The molar heat capacity at constant pressure, $C_{p,m}$, is $C_{v,m} + R = \frac{5}{2} R$.

The internal energy U depends only on temperature for ideal gases and is given by:

$$ U = n C_{v,m} T $$

which means changes in internal energy relate directly to temperature variations.

Understanding Entropy in Thermodynamics

Entropy S is a state function that quantifies the disorder or the number of accessible microstates in a system. For ideal gases, the change in entropy between two states is well-defined and can be calculated using thermodynamic relations.

Entropy Change for an Ideal Gas

The differential form of entropy change when an ideal gas undergoes a reversible process from state 1 to state 2 is:

$$ \Delta S = n C_{p,m} \ln \frac{T_2}{T_1} - n R \ln \frac{P_2}{P_1} $$

Alternatively, expressing entropy change in terms of initial and final states with known temperatures and pressures:

$$ \Delta S = n C_{v,m} \ln \frac{T_2}{T_1} + n R \ln \frac{V_2}{V_1} $$

since for ideal gases, the relation between pressure, volume, and temperature can be used to transform between these expressions.

Note: When calculating entropy change between two states, it's often easiest to use the form involving temperatures and pressures, especially if these are specified or can be related via the ideal gas law.

Given Data and Objective

In our specific problem, the parameters are:
- Number of moles: $n = 3.00\, mol$
- Molar heat capacity at constant pressure: $C_{p,m} = \frac{5}{2} R$

The goal is to calculate the entropy S of the system at the current state, which implies determining the entropy relative to a reference state or understanding the entropy change during a process if initial and final states are specified.

Since the problem statement doesn't specify initial and final states, a common approach is to find the absolute entropy at a particular state, often relative to a standard reference point (e.g., standard temperature $T_0 = 298\,K$).

Assumption: To proceed, we will assume the task is to compute the absolute entropy of the system at a given temperature T, assuming the pressure is known or can be set to a reference value. If the pressure isn't specified, we typically choose a standard pressure (e.g., 1 atm) for comparison.

Calculating the Absolute Entropy of the Monatomic Gas

The absolute entropy of an ideal gas at a temperature T and pressure P can be calculated using the Sackur-Tetrode equation, which is derived from statistical mechanics but also aligns with classical thermodynamics:

$$S = n R \left[\ln \left(\frac{V}{n} \left(\frac{4 \pi m U}{3 n h^2} \right)^{3/2} \right) + \frac{5}{2} \right]$$

However, this form involves molecular parameters (mass m, Planck's constant h), which are not specified here. Alternatively, a more straightforward approach for molar entropy is:

$$S = C_{p,m} \ln T - R \ln P + \text{constant}$$

But since the constant depends on the reference state, it's often more practical to use tabulated data or the entropy change formula between two states.

Simplified Approach:

If we consider the change in entropy from a reference state (T_0, P_0) to the current state (T, P):

$$\Delta S = n C_{p,m} \ln \frac{T}{T_0} - n R \ln \frac{P}{P_0}$$

To find the absolute entropy at the current state, we need known values of entropy at the reference state S_0, or we can express the current entropy relative to a standard state.

Step-by-Step Calculation of Entropy

Let's proceed with a typical calculation assuming:
- Reference temperature $T_0 = 298\,K$
- Reference pressure $P_0 = 1\,atm$

Suppose the system's current temperature T and pressure P are such that the system is at a known state. If only the temperature is given, and pressure is assumed to be at the reference pressure for simplicity, the entropy at this state can be calculated relative to the standard state.

Step 1: Calculate the entropy change from the reference state

$$\Delta S = n C_{p,m} \ln \frac{T}{T_0}$$

since $P = P_0$, the pressure term drops out.

Step 2: Plug in values

- $n = 3.00\, mol$
- $C_{p,m} = \frac{5}{2} R = 2.5 R$
- $R = 8.314\, J\,mol^{-1} K^{-1}$

- T is the current temperature (assumed or provided)

Suppose the temperature T is the same as the initial state, say $T = 300\,K$:

$$\Delta S = 3.00 \times 2.5 R \times \ln \frac{T}{298}$$

$$\Delta S = 3.00 \times 2.5 \times 8.314 \times \ln \left(\frac{T}{298} \right)$$

$$\Delta S \approx 3.00 \times 20.785 \times \ln \left(\frac{T}{298} \right)$$

$$\Delta S \approx 62.355 \times \ln \left(\frac{T}{298} \right) \, J/K$$

If the temperature is $T = 300\,K$:

$$\ln \left(\frac{300}{298} \right) \approx \ln(1.0067) \approx 0.0067$$

$$\Delta S \approx 62.355 \times 0.0067 \approx 0.418\, J/K$$

Thus, the entropy of the system at 300 K relative to the standard state is approximately 0.418 J/K.

Note: For absolute entropy, tabulated standard molar entropy values for monatomic gases can be used, or the Sackur-Tetrode equation can be employed if molecular parameters are known.

Summary of the Calculation Process

1. Identify known parameters:
- Number of moles n
- Molar heat capacity $C_{p,m}$
- Initial and final states (temperatures and pressures)

2. Use appropriate entropy change equations:

$$\Delta S = n C_{p,m} \ln \frac{T_2}{T_1} - n R \ln \frac{P_2}{P_1}$$

3. Calculate entropy relative to a reference state, typically using standard conditions.

4. Interpret the result:
- For a process, the entropy change indicates the disorder variation.
- For the absolute entropy, tabulated

Frequently Asked Questions

How do you calculate the entropy change (ΔS) for 3.00 mol of a monatomic perfect gas when the temperature changes from

T_1 to T_2?

The entropy change is given by $\Delta S = nR \ln(T_2/T_1)$, since for a monatomic ideal gas, the entropy change depends only on temperature and amount of substance, with R being the universal gas constant.

Given Cp,m = 5/2 R for a monatomic gas, what is the value of Cp,m in units of J/(mol·K)?

Since $R \approx 8.314$ J/(mol·K), $C_{p,m} = (5/2) \times 8.314 \approx 20.785$ J/(mol·K).

How is the specific heat at constant pressure (Cp,m) related to the calculation of entropy S for the gas?

Cp,m is used to determine the change in temperature and enthalpy during processes, and it appears in the entropy change formula when temperature varies; for isobaric processes, $\Delta S = nC_{p,m} \ln(T_2/T_1) - nR \ln(P_2/P_1)$.

What is the expression for the absolute entropy S of the gas at a given temperature T and pressure P?

The absolute entropy can be calculated using $S = S^\circ + nR \ln(T/T^\circ) + nR \ln(P^\circ/P)$, where S° is the standard molar entropy at reference conditions T° and P°, and P is the pressure of the gas.

If the temperature of 3 mol of monatomic perfect gas increases from T_1 to T_2 at constant pressure, how much is the change in entropy ΔS?

The entropy change is $\Delta S = nC_{p,m} \ln(T_2/T_1)$. Substituting the values, $\Delta S = 3 \times 20.785 \times \ln(T_2/T_1)$ J/K.

Additional Resources

Calculate S (for The System) When The State Of 3.00 Mol Of A Monatomic Perfect Gas, For Which Cp,m = 5/2

When exploring the fascinating realm of thermodynamics, understanding how to accurately calculate entropy (S) for a given system is fundamental. In this detailed analysis, we focus on a classic problem: determining the entropy change of a monatomic ideal gas, specifically when 3.00 mol of the gas is involved, and the molar heat capacity at constant pressure, $C_{p,m}$, is given as $\frac{5}{2} R$.

This exploration not only provides insight into the calculation process but also underscores key thermodynamic principles, including the significance of entropy, the properties of monatomic gases, and the mathematical tools employed to evaluate thermodynamic state functions. Whether you are a student, educator, or enthusiast, this comprehensive review aims to deepen your understanding of

entropy calculations within the context of ideal gases.

Understanding the Foundations: Thermodynamics and Entropy

Before diving into calculations, it’s essential to clarify what entropy (S) represents within thermodynamics. Entropy is a state function that quantifies the degree of disorder or randomness in a system. It also measures the irreversibility of processes and is central to the second law of thermodynamics.

In the context of ideal gases, entropy changes are often calculated between two states, using thermodynamic relationships that relate temperature, volume, and pressure. The key advantage of ideal gases is that their thermodynamic properties are well-defined and follow simple relationships, making entropy calculations more straightforward compared to real gases.

System Description and Given Data

The problem involves:

- Number of moles, n: 3.00 mol
- Type of gas: Monatomic perfect (ideal) gas
- Molar heat capacity at constant pressure, $C_{p,m}$: $\frac{5}{2} R$

The molar heat capacities are crucial because they directly relate to how the system exchanges heat during temperature changes. For monatomic gases, the molar heat capacity at constant volume, $C_{v,m}$, and at constant pressure, $C_{p,m}$, are related through:

$$
C_{p,m} = C_{v,m} + R
$$

Given $C_{p,m} = \frac{5}{2} R$, we can deduce:

$$
C_{v,m} = C_{p,m} - R = \frac{5}{2} R - R = \frac{3}{2} R
$$

This aligns with the theoretical expectations for monatomic ideal gases, which typically have:

- $C_{v,m} = \frac{3}{2} R$
- $C_{p,m} = \frac{5}{2} R$

Calculating Entropy Change: The General Approach

The entropy change for an ideal gas when moving from an initial state (T_1, V_1) to a final state (T_2, V_2) is given by the fundamental thermodynamic relation:

$$
\Delta S = n C_{v,m} \ln \frac{T_2}{T_1} + n R \ln \frac{V_2}{V_1}
$$

Alternatively, if the initial and final pressures (P_1, P_2) are known instead of volumes, the relation can be expressed as:

$$
\Delta S = n C_{p,m} \ln \frac{T_2}{T_1} - n R \ln \frac{P_2}{P_1}
$$

Since the problem statement does not specify initial and final states, the most common approach is to calculate the entropy change between two states with known temperatures and volumes or pressures, or simply the absolute entropy if a reference state is specified.

Establishing the Calculation Framework

To proceed effectively, assumptions or specific states are necessary. Typically, calculations are simplified by choosing a reference state where the entropy is defined as zero (or known), and then considering the change relative to that state.

Suppose the initial state is at temperature T_1 and volume V_1, and the final state at T_2, V_2. Alternatively, one might consider the process as isothermal, isobaric, or adiabatic, each influencing the calculation method.

For this problem, let's consider a general process where the temperature changes from T_1 to T_2, and the volume changes from V_1 to V_2. If the process specifics are unknown, the focus is on the general formula, which can be adapted once data is supplied.

Calculating the Absolute Entropy: Standard Method

In thermodynamics, the absolute entropy of an ideal gas at temperature T and volume V (or pressure P) relative to a standard reference state (often chosen at $T_0 = 298.15\, K$) and

$P_0 = 1\, atm$) is given by:

$$
S(T, V) = S^0 + n C_{v,m} \ln \frac{T}{T_0} + n R \ln \frac{V}{V_0}
$$

where:

- S^0 is the standard entropy at the reference state,
- V_0 is the volume at the reference state.

In many practical cases, the change in entropy ΔS between two states suffices, especially when the standard entropy S^0 is known or cancels out.

Explicit Calculation: Step-by-Step Guide

To exemplify the calculation, consider a hypothetical scenario:

- Initial state: $T_1 = 300\,K$, V_1
- Final state: $T_2 = 600\,K$, V_2

Assuming the process involves a change in temperature and volume, the entropy change becomes:

$$
\Delta S = n C_{v,m} \ln \frac{T_2}{T_1} + n R \ln \frac{V_2}{V_1}
$$

Given:

- $n = 3.00\, mol$
- $C_{v,m} = \frac{3}{2} R$
- $R = 8.314\, J/(mol \cdot K)$

Suppose the volume doubles, i.e., $V_2 / V_1 = 2$. Plugging in the values:

$$
\Delta S = 3.00 \times \frac{3}{2} R \times \ln \frac{600}{300} + 3.00 \times R \times \ln 2
$$

Calculations:

$$
\Delta S = 3.00 \times \frac{3}{2} \times 8.314 \times \ln 2 + 3.00 \times 8.314 \times \ln 2
$$

$$
\Delta S = 3.00 \times 1.5 \times 8.314 \times 0.6931 + 3.00 \times 8.314 \times 0.6931
$$

\]

\[
\Delta S = (4.5 \times 8.314 \times 0.6931) + (3.00 \times 8.314 \times 0.6931)
\]

Calculating separately:

- \(4.5 \times 8.314 = 37.413 \)
- \(37.413 \times 0.6931 \approx 25.935\, J/K \)

and

- \(3.00 \times 8.314 = 24.942 \)
- \(24.942 \times 0.6931 \approx 17.289\, J/K \)

Total entropy change:

\[
\Delta S \approx 25.935 + 17.289 = 43.224\, J/K
\]

Total entropy for 3 mol:

\[
S_{total} = n S_{per\, mol} + \text{standard entropy}
\]

If the initial entropy is known or the standard entropy \(S^0 \) is given, the absolute entropy can be calculated accordingly.

Interpreting the Results and Practical Considerations

The essential takeaways from this calculation process include:

- Process dependence: Entropy change depends on how the state variables change (temperature, volume, pressure).
- Monatomic gas properties: The specific heat capacities are well-understood, simplifying calculations.
- Standard states and references: Absolute entropy calculations require a reference point, but often only entropy differences are practically relevant.

In real-world applications, knowing the initial and final conditions allows precise entropy calculations, which are crucial for designing engines, refrigerators, and understanding natural processes.

Final Remarks: The Significance of Accurate Calculations

Calculating entropy accurately is vital in thermodynamics because it informs us about the feasibility and reversibility of processes, efficiency limits, and the fundamental nature of energy transformations. The case with a monatomic perfect gas offers a textbook example that encapsulates core concepts with straightforward mathematics

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