Calculate The Integral Of F(x,y) 7x Over The Region D Bounded Above By Y = X(2 X) And Below By X = Y(2-

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When approaching the task of calculating a double integral such as \(\iint_D 7x \, dA\), where the region \(D\) is bounded above by the curve \(y = x(2x)\) and below by the line \(x = y(2 - ...)\), it’s essential to understand the geometry of the region and the most effective way to set up the integral. Properly identifying the boundaries and choosing the appropriate order of integration simplifies the computation process.

In this article, we'll explore step-by-step how to determine the limits of integration, visualize the region, and execute the double integral calculation for the function \(f(x, y) = 7x\). This comprehensive guide aims to enhance your understanding of multiple integration over bounded regions in the xy-plane.

Understanding the Region D and Its Boundaries

Identifying the Boundaries

The problem states that the region \(D\) is bounded above by \(y = x(2x)\) and below by \(x = y(2 - ...)\). Although the second boundary appears incomplete, for the purpose of this example, we will assume the lower boundary is given by a line or curve such as $x = y(2 - y)$ or a similar linear function, which is common in such problems. If the actual boundary differs, the approach remains similar.

Assuming the boundaries are:

- Upper boundary: $y = 2x^2$

- Lower boundary: $x = y(2 - y)$

This assumption aligns with typical forms encountered in multivariable calculus problems involving quadratic and linear boundaries.

Note: If the actual boundaries differ, adjust the functions accordingly.

Visualizing the Region D

Plotting the boundaries helps to visualize the region:

- The curve $y = 2x^2$ is a parabola opening upward.

- The line $x = y(2 - y)$ can be rewritten or plotted based on the specific form.

Creating a sketch allows for determining the intersection points, which are essential for setting the limits of integration.

Finding the Intersection Points of the Boundaries

To accurately describe the region D, locate the points where the boundary curves intersect.

Step-by-step process:

1. Set the boundary equations equal to find intersection points:

$$
y = 2x^2
$$

and

$$
x = y(2 - y)
$$

2. Substitute $y = 2x^2$ into $x = y(2 - y)$:

$$
x = (2x^2)(2 - 2x^2)
$$

3. Simplify:

$$
x = 2x^2 (2 - 2x^2) = 2x^2 \times 2 - 2x^2 \times 2x^2 = 4x^2 - 4x^4
$$

4. Rewrite as:

$$
x = 4x^2 - 4x^4
$$

5. Bring all to one side:

$$
0 = 4x^2 - 4x^4 - x
$$

6. Rearrange:

$$
4x^2 - 4x^4 - x = 0
$$

7. Rearranged:

$$
-4x^4 + 4x^2 - x = 0
$$

8. For solving, factor out common terms if possible or use numerical methods.

Note: The exact intersection points depend on the precise boundary equations. Once found, these points define the limits of integration.

Setting Up the Double Integral

Once the region D is characterized, the next step is to establish the integral's limits. The choice of order—integrating with respect to y first or x first—depends on the simplicity of the bounds.

Choosing the Order of Integration

- Integrating with respect to y first (dy dx): Useful if the boundaries are functions of x.

- Integrating with respect to x first (dx dy): Preferable if the boundaries are functions of y.

Suppose, after analysis, integrating with respect to y first is simpler:

$$
\iint_D 7x \, dA = \int_{x = x_{min}}^{x_{max}} \left(\int_{y = y_{lower}(x)}^{y_{upper}(x)} 7x \, dy \right) dx
$$

Example Limits

- x varies from x_{min} to x_{max}.

- For each x, y varies between the two boundary curves.

Performing the Integration

Inner Integral

Calculating the inner integral:

$$
\int_{y = y_{lower}(x)}^{y_{upper}(x)} 7x \, dy = 7x \times (y_{upper}(x) - y_{lower}(x))
$$

This is straightforward as $7x$ is a constant with respect to y.

Outer Integral

The outer integral becomes:

$$
\int_{x = x_{min}}^{x_{max}} 7x \times (y_{upper}(x) - y_{lower}(x)) dx
$$

Substitute the limits of x and the boundary functions to perform the integration.

Step-by-Step Calculation Example

Suppose the boundaries are:

- $y = 2x^2$ (upper boundary)

- $x = y(2 - y)$, which can be rearranged as $y = 1 \pm \sqrt{1 - x}$ depending on the form.

Assuming the region is bounded between $x = 0$ and $x = 0.5$, and y varies accordingly.

The integral becomes:

$$
\int_{x=0}^{0.5} \left(\int_{y = 2x^2}^{y_{max}(x)} 7x \, dy \right) dx
$$

Calculating the inner integral:

$$
7x \times (y_{max}(x) - 2x^2)
$$

Then, integrating with respect to x:

$$
\int_{0}^{0.5} 7x \times (y_{max}(x) - 2x^2) dx
$$

which can be computed numerically or analytically based on the explicit form of $y_{max}(x)$.

Conclusion and Tips for Calculating Double Integrals Over Bounded Regions

- Always sketch the region: Visualizing the region helps in correctly identifying the limits and choosing the optimal order of integration.

- Find intersection points: Solving the boundary equations simultaneously determines the limits of integration.

- Decide the order of integration: Choose the order that simplifies the limits and integration process.

- Express the boundaries explicitly: Write the boundary functions clearly as functions of the integration variable.

- Use symmetry when possible: Symmetrical regions can simplify the calculations.

- Double-check the limits: Ensure the limits match the region's boundaries and are consistent throughout.

- Numerical methods: When the boundaries are complex, consider numerical integration techniques for approximate solutions.

Additional Resources for Mastering Double Integrals

- Textbooks:
- "Calculus: Early Transcendentals" by James Stewart

- "Multivariable Calculus" by Ron Larson and Bruce Edwards

- Online Tools:
- Wolfram Alpha for plotting and solving equations
- Desmos graphing calculator for visualizing regions

- Practice Problems:
- Solve integrals over various regions to develop intuition
- Practice changing the order of integration for complex regions

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Final Remarks

Calculating the integral of a function like $7x$ over a region D bounded by specific curves involves understanding the geometry of the region, setting appropriate limits, and executing the integration carefully. Whether the region is bounded by quadratic curves, lines, or more complex functions, the fundamental approach remains consistent: analyze, visualize, set limits, and integrate. Mastery of these skills enhances your capability to solve a wide array of problems in multivariable calculus and related fields.

Remember: Always verify the boundary equations, intersections, and limits before performing the integration to ensure accuracy in your calculations.

Frequently Asked Questions

How do I set up the double integral to compute the integral of $f(x,y) = 7x$ over the region D bounded above by $y = x(2x)$ and below by $y = 2 - x$?

First, identify the region D in the xy-plane, where y ranges between the lower curve $y = 2 - x$ and the upper curve $y = 2x^2$. The limits for x can be found by determining the intersection points of these curves, then setting up the double integral as ∫ from $x = a$ to b ∫ from $y = 2 - x$ to $y = 2x^2$ $7x\,dy\,dx$.

What are the intersection points of $y = 2 - x$ and $y = 2x^2$ that define the bounds of integration?

To find the intersection points, set $2 - x = 2x^2$. Solving for x gives $2x^2 + x - 2 = 0$. Factoring or using the quadratic formula yields $x = 1$ and $x = -1$. Corresponding y-values are $y = 2 - 1 = 1$ and $y = 2 - (-1) = 3$. So, the intersection points are $(-1, 3)$ and $(1, 1)$.

What is the correct order of integration for calculating the double integral over region D?

Since the region D is bounded between $x = -1$ and $x = 1$, with y varying between $y = 2 - x$ (below) and $y = 2x^2$ (above), the double integral can be set up as ∫ from $x = -1$ to 1 ∫ from $y = 2 - x$ to $y = 2x^2$ $7x\,dy\,dx$.

How do I evaluate the double integral of $7x$ over the region D bounded as specified?

First, integrate $7x$ with respect to y: ∫ (from $y=2 - x$ to $y=2x^2$) $7x\,dy = 7x\,[y]$ from $2 - x$ to $2x^2 = 7x\,(2x^2 - (2 - x)) = 7x\,(2x^2 - 2 + x)$. Then, integrate the resulting expression with respect to x from -1 to 1 to find the total value.

What is the final value of the integral after performing the calculations?

Calculating the integral, ∫ from -1 to 1 of $7x (2x^2 - 2 + x) dx$, simplifies to zero because the integrand is an odd function over a symmetric interval. Therefore, the value of the integral is 0.

Additional Resources

Calculating the integral of the function 7x over a complex region D bounded by curved and linear boundaries is a fundamental task in multivariable calculus, with applications spanning physics, engineering, and mathematical modeling. This analytical process involves understanding the geometric shape of the region, choosing appropriate coordinate systems, and applying integral calculus techniques to evaluate the integral accurately. In this article, we delve into the detailed steps necessary to compute the integral of the function F(x, y) = 7x over a region D bounded above by the curve y = x(2x) and below by the line x = y(2-), a problem that exemplifies the intricate interplay between geometry and calculus.

Understanding the Geometric Region D

Identifying the Boundary Curves and Lines

The first step in solving the problem involves analyzing the boundaries that define the region D. The region is specified as being bounded above by the curve y = x(2x) and below by a line described by x = y(2-). To proceed, it's crucial to interpret these boundaries clearly:

- Upper Boundary: y = x(2x) simplifies to $y = 2x^2$. This is a parabola opening upward with its vertex at the origin.
- Lower Boundary: x = y(2-). The notation here suggests a linear relationship involving y, but it appears

incomplete or potentially miswritten. Assuming the intended boundary is $x = y(2 - y)$, or perhaps $x = y(2 - y)$, which would define a different region. Alternatively, if the boundary is $x = y(2 - y)$, it becomes a quadratic in y, which is more consistent with typical problem structures.

Note: Since the original boundary line appears incomplete ("x = y(2-"), which is likely a typo or partial expression, we will assume the intended boundary is $x = y(2 - y)$.

Assumption: The lower boundary is $x = y(2 - y)$, a quadratic function in y, opening downward with roots at $y=0$ and $y=2$.

Summary of Boundaries:

- Upper boundary: $y = 2x^2$ (parabola)
- Lower boundary: $x = y(2 - y)$ (quadratic in y)

This interpretation aligns with common problems involving bounded regions between a parabola and a quadratic curve.

Visualizing the Region D

Plotting these boundaries helps in understanding the region:

- The parabola $y = 2x^2$ opens upward, passing through the origin, symmetric about the y-axis.
- The curve $x = y(2 - y)$ is a downward-opening parabola in y with roots at $y=0$ and $y=2$, describing the boundary in the y-direction.

The intersection points of these two boundaries determine the limits of integration:

- Set $y = 2x^2$
- Set $x = y(2 - y)$

Substituting y from the first into the second:

$$x = (2x^2)(2 - 2x^2)$$

This leads to a polynomial equation in x, which can be solved to find the intersection points.

Determining the Limits of Integration

Finding the Intersection Points

To find the intersection points of the boundaries, we set:

1. $y = 2x^2$ (upper boundary)
2. $x = y(2 - y)$ (lower boundary)

Expressing both in terms of y:

- From the first: $y = 2x^2 \Rightarrow x = \pm\sqrt{(y/2)}$

- From the second: $x = y(2 - y)$

At the intersection points, these must be equal:

$$y(2 - y) = \pm\sqrt{(y/2)}$$

Since x appears explicitly in the boundary equations, it's often easier to eliminate one variable to find the y-values where the boundaries intersect.

Approach:

- Substitute x from the second boundary into the first:

Given $x = y(2 - y)$, then:

- $y = 2[x]^2$

Plug in x:

- $y = 2[y(2 - y)]^2$

Simplify:

- $y = 2 [y^2 (2 - y)^2]$

- $y = 2 y^2 (2 - y)^2$

Divide both sides by y (assuming $y \neq 0$):

- $1 = 2 y (2 - y)^2$

This yields an equation:

- $1 = 2 y (2 - y)^2$

Now, expand $(2 - y)^2$:

$(2 - y)^2 = 4 - 4 y + y^2$

So,

$1 = 2 y (4 - 4 y + y^2)$

Expand the right side:

$1 = 2 y 4 - 2 y 4 y + 2 y y^2$

$1 = 8 y - 8 y^2 + 2 y^3$

Bring all to one side:

$2 y^3 - 8 y^2 + 8 y - 1 = 0$

This is a cubic equation in y:

$2 y^3 - 8 y^2 + 8 y - 1 = 0$

Dividing through by 2:

$y^3 - 4 y^2 + 4 y - 0.5 = 0$

This cubic can be solved numerically or analytically to find the approximate y-values at the intersection points.

Numerical Solution:

Using methods like Newton-Raphson or synthetic division, approximate solutions for y are obtained, leading to corresponding x-values via $x = y(2 - y)$.

Summary:

- The intersection points are located at specific y-values between 0 and 2.
- These points define the limits for y in the integral.

Establishing the Region in Coordinates

Given the above, the region D can be described as:

- In the y-direction: from y = 0 to y = y_intersect (the larger root).
- For each y, x varies between the left boundary x = y(2 - y) and the right boundary x = √(y/2), considering the positive root (since x should be real and positive within the region).

Alternatively, if the region is better suited for y as the outer variable and x as the inner variable, the limits are:

- y from 0 to y_max (intersect y-value)
- For each y, x from x = y(2 - y) to x = √(y/2)

This setup allows the integral to be expressed with clear bounds.

Choosing the Appropriate Coordinate System

Rectangular Coordinates

In the Cartesian (x, y) coordinate system, the limits are complex but manageable, especially once the intersection points are calculated.

Advantages:

- Direct interpretation of boundaries.
- Simple to set up the integral once limits are established.

Disadvantages:

- Boundaries involve quadratic and parabola equations.
- Potential for complicated limits if the curves intersect multiple times.

Change of Variables: Polar or Other Coordinates

Given the shape involves parabolas centered at the origin and quadratic curves, transforming to polar coordinates might simplify the boundaries:

- $x = r \cos \theta$
- $y = r \sin \theta$

However, the boundaries are not naturally expressed in polar form, so this might complicate the integral unless symmetry or specific angular bounds are exploited.

Conclusion:

For this particular problem, sticking with Cartesian coordinates and carefully calculating the limits is more straightforward.

Calculating the Integral

Formulating the Integral

The goal is to compute:

$$
\iint_D 7x \, dA
$$

Expressed as an iterated integral, with limits determined from earlier analysis:

$$
\int_{y=0}^{y=y_{max}} \int_{x = y(2 - y)}^{x = \sqrt{\frac{y}{2}}} 7x \, dx \, dy
$$

Alternatively, depending on the region's orientation, the order of integration may be reversed.

Evaluating the Inner Integral

The inner integral with respect to x:

$$
\int_{x = y(2 - y)}^{x = \sqrt{\frac{y}{2}}} 7x \, dx
$$

Compute:

$$
7 \int_{x = y(2 - y)}^{\sqrt{\frac{y}{2}}} x \, dx = 7 \left[\frac{x^2}{2} \right]_{x=y(2 - y)}^{x= \sqrt{\frac{y}{2}}}
$$

Plugging in the limits:

\[

7 \left(\frac{1}{2} \left(\frac{y}{2}\right) - \frac{1}{2} \left(y(2 - y) \right)^2 \right)

\]

Simplify:

\[

\frac{7}{2} \left(\frac{y}{2} - \left(y(2 - y) \right)^2 \right)

\]

Note that:

\[

\left(y(2 - y) \right)^2 = y^2 (2 - y)^2 = y^2 (4 - 4 y + y^2) = 4 y^2 - 4

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