Calculate The Wavelength Of Light Emitted When An Electron In The Hydrogen Atom Makes A Transition From

Calculate The Wavelength Of Light Emitted When An Electron In The Hydrogen Atom Makes A Transition From a higher energy level to a lower energy level is a fundamental concept in atomic physics and quantum mechanics. This process is responsible for the emission spectra of hydrogen, one of the most studied and understood elements in the universe. Understanding how to determine the wavelength of the emitted light involves applying principles from quantum theory, specifically the Bohr model of the atom, and the relationship between energy and electromagnetic radiation. In this article, we will explore the detailed steps required to perform this calculation, delve into the underlying physics, and provide practical examples to illustrate the process.

Understanding Electron Transitions in the Hydrogen Atom

The Bohr Model of the Hydrogen Atom

The Bohr model, proposed by Niels Bohr in 1913, was groundbreaking in explaining how electrons exist in discrete energy levels within an atom. According to this model:
- Electrons orbit the nucleus in specific, quantized energy levels.
- Each energy level is characterized by a principal quantum number, n (n = 1, 2, 3, ...).
- Electrons can transition between these levels by absorbing or emitting a photon with an energy equal to the difference between the initial and final energy levels.

Energy Levels in Hydrogen

The energy of an electron in the nth energy level of a hydrogen atom is given by the formula:
$$E_n = -\frac{13.6\, \text{eV}}{n^2}$$
Where:
- E_n is the energy at level n.
- 13.6 eV is the ionization energy of hydrogen.

The negative sign indicates that the electron is bound to the nucleus.

Calculating the Wavelength of Emitted Light

The Relationship Between Energy and Wavelength

The energy difference between two levels determines the energy of the emitted photon during a transition:
$$\Delta E = E_{initial} - E_{final}$$
This energy corresponds to a photon with energy:
$$E_{photon} = h \times \nu$$
where:
- h is Planck's constant ($6.626 \times 10^{-34}\, \text{Js}$),
- ν is the frequency of the emitted light.

Since the wavelength λ and frequency ν are related by the speed of light c:
$$c = \lambda \times \nu$$
we can express the photon energy in terms of wavelength:

$$E_{photon} = \frac{hc}{\lambda}$$

Combining these equations, the wavelength of the emitted photon can be derived once the energy difference is known:

$$\lambda = \frac{hc}{\Delta E}$$

Step-by-Step Calculation Procedure

1. Identify the initial and final energy levels: Specify the quantum numbers n_i (initial) and n_f (final).
2. Calculate the energies of these levels using $E_n = -\frac{13.6\, \text{eV}}{n^2}$.
3. Determine the energy difference: $\Delta E = |E_{n_i} - E_{n_f}|$.
4. Convert the energy difference to joules if necessary (since 1 eV = $1.602 \times 10^{-19}\, \text{J}$).
5. Calculate the wavelength using $\lambda = \frac{hc}{\Delta E}$.

Practical Example: Transition from $n=3$ to $n=2$

Suppose an electron transitions from the third energy level to the second energy level in a hydrogen atom. Let's perform the calculation step-by-step:

Step 1: Identify the levels

- Initial level, $n_i = 3$
- Final level, $n_f = 2$

Step 2: Calculate energies E_3 and E_2

$$
E_3 = -\frac{13.6\, \text{eV}}{3^2} = -\frac{13.6}{9} \approx -1.51\, \text{eV}
$$

$$
E_2 = -\frac{13.6\, \text{eV}}{2^2} = -\frac{13.6}{4} = -3.4\, \text{eV}
$$

Step 3: Find the energy difference ΔE

$$
\Delta E = |E_3 - E_2| = | -1.51 - (-3.4) | = 1.89\, \text{eV}
$$

Step 4: Convert energy to joules

$$
\Delta E = 1.89\, \text{eV} \times 1.602 \times 10^{-19}\, \text{J/eV} \approx 3.03 \times 10^{-19}\, \text{J}
$$

Step 5: Calculate the wavelength λ

Using Planck's constant $h = 6.626 \times 10^{-34}\,\text{Js}$ and speed of light $c = 3.0 \times 10^{8}\,\text{m/s}$:

$$
\lambda = \frac{hc}{\Delta E} = \frac{(6.626 \times 10^{-34})(3.0 \times 10^{8})}{3.03 \times 10^{-19}} \approx 6.56 \times 10^{-7}\,\text{m}
$$

Converting to nanometers:

$$
\lambda \approx 656\,\text{nm}
$$

This wavelength corresponds to the red line in the Balmer series of hydrogen emission spectra.

Understanding the Significance of the Emission Wavelengths

The Hydrogen Emission Spectrum

The calculation above is a part of the Balmer series, which involves transitions where the final energy level is $n=2$. These wavelengths fall within the visible spectrum, making hydrogen emission lines observable with simple spectroscopic equipment.

Other series include:
- Lyman series ($n_f=1$, ultraviolet region)
- Paschen series ($n_f=3$, infrared region)
- Brackett series ($n_f=4$)
- Pfund series ($n_f=5$)

Each series corresponds to electrons falling to a specific lower energy level, emitting photons of characteristic wavelengths.

The Role of Wavelength Calculations in Astrophysics

Calculating the wavelengths of emitted light allows astronomers to:
- Determine the composition of distant stars and galaxies.
- Measure the relative velocities of celestial objects via Doppler shifts.
- Understand physical conditions in astrophysical plasmas.

Advanced Topics and Considerations

Limitations of the Bohr Model

While the Bohr model provides an excellent approximation for hydrogen, it has limitations:
- It does not account for fine structures and spectral line splitting.
- It cannot accurately describe multi-electron atoms.
- Quantum mechanical models (like Schrödinger's wave equation) provide more precise descriptions.

Using Rydberg Formula for Wavelengths

Alternatively, the Rydberg formula offers a direct way to calculate wavelengths for hydrogen transitions:

$$\frac{1}{\lambda} = R \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

where R is the Rydberg constant ($1.097 \times 10^7\, \text{m}^{-1}$).

Applying this formula simplifies calculations and is especially useful when dealing with multiple transitions.

Conclusion

Calculating the wavelength of light emitted during electron transitions in the hydrogen atom is a fundamental process rooted in quantum physics. By understanding the energy levels, the relationship between energy and electromagnetic radiation, and applying precise formulas, one can determine the specific wavelengths that characterize hydrogen's emission spectrum. Whether for educational purposes, research, or astrophysical observations, mastering this calculation enhances our understanding of atomic interactions and the nature of light. The steps outlined here serve as a comprehensive guide to perform these calculations accurately and appreciate their significance in broader scientific contexts.

Frequently Asked Questions

How can I calculate the wavelength of light emitted when an electron in a hydrogen atom transitions from a higher to a lower energy level?

You can use the Rydberg formula: $1/\lambda = R(1/n_1^2 - 1/n_2^2)$, where R is the Rydberg constant ($1.097 \times 10^7\ m^{-1}$), and n_1 and n_2 are the principal quantum numbers of the lower and higher energy levels, respectively.

What is the significance of the transition from n=3 to n=2 in the hydrogen atom in terms of wavelength?

The transition from n=3 to n=2 emits a photon with a wavelength of approximately 656 nm, which corresponds to the red line in the Balmer series of hydrogen emission spectra.

How does the change in energy levels affect the wavelength of light emitted in a hydrogen atom transition?

A larger difference between the initial and final energy levels results in a higher energy photon and thus a shorter wavelength; conversely, smaller energy differences produce longer wavelengths.

Can the wavelength of light emitted during any electron transition in hydrogen be calculated using the Bohr model?

Yes, using the Bohr model and the Rydberg formula, you can calculate the wavelength for any electron transition by plugging in the initial and final quantum numbers.

What are the common spectral lines associated with hydrogen atom transitions, and how are they related to wavelength calculations?

The prominent spectral lines include the Balmer series (visible light), Lyman series (ultraviolet), and Paschen series (infrared). Their wavelengths are calculated using the Rydberg formula for respective transitions between energy levels.

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Introduction

Understanding the wavelength of light emitted during electronic transitions in hydrogen atoms is fundamental in quantum physics and spectroscopy. This process not only exemplifies the quantized nature of energy levels but also underpins many practical applications such as atomic emission spectroscopy, astrophysics, and the development of lasers. When an electron transitions from a higher energy state to a lower one within a hydrogen atom, the energy difference is released in the form of a photon. Calculating the wavelength of this photon involves understanding the energy levels of the hydrogen atom, the quantum mechanics governing electron transitions, and the relationship between energy and electromagnetic radiation.

Theoretical Foundations

1. Quantum Model of the Hydrogen Atom

The hydrogen atom, being the simplest atom with only one proton and one electron, serves as the ideal model to illustrate quantum principles. Its energy levels are described by the Bohr model and refined by quantum mechanics.

- Energy Levels:
The allowed energy levels are given by the Bohr energy formula:
$$
E_n = - \frac{13.6\, \text{eV}}{n^2}
$$
where n is the principal quantum number (n = 1, 2, 3, ...).

- Energy Difference During Transition:
When an electron moves from an initial state n_i to a final state n_f, the energy change is:
$$
\Delta E = E_{n_f} - E_{n_i} = - \frac{13.6\, \text{eV}}{n_f^2} + \frac{13.6\, \text{eV}}{n_i^2}
$$

Note: The negative sign indicates bound states with negative potential energy. When the electron transitions to a lower energy level ($n_f < n_i$), the energy difference is positive, signifying energy release.

2. Relationship between Energy and Wavelength

The photon emitted during the transition carries energy equal to the energy difference between the initial and final states:

$$
E_{photon} = \Delta E
$$

According to Planck's relation, the energy of a photon relates to its wavelength λ:

$$
E_{photon} = h \nu = \frac{hc}{\lambda}
$$

where:

- h = Planck's constant ($6.626 \times 10^{-34} \, \text{Js}$)
- c = speed of light ($3.0 \times 10^{8} \, \text{m/s}$)
- ν = frequency of the emitted photon
- λ = wavelength of the emitted photon

Rearranged to find the wavelength:

$$
\boxed{
\lambda = \frac{hc}{\Delta E}
}
$$

Important: The energy difference ΔE must be expressed in Joules when using SI units, or converted appropriately if using electronvolts.

Step-by-Step Calculation Process

1. Identify the Initial and Final Energy Levels

- Determine the initial (n_i) and final (n_f) quantum numbers based on the problem statement.
- Usually, the transition involves the electron moving from a higher energy level n_i to a lower energy level n_f.

2. Calculate the Energy Difference ΔE

- Use the hydrogen energy level formula:
$$
E_n = - \frac{13.6\, \text{eV}}{n^2}
$$
- Compute E_{n_i} and E_{n_f}.
- Find the difference:
$$
\Delta E = E_{n_f} - E_{n_i}
$$
- Since the transition releases energy, ΔE will be positive in this context.

3. Convert Energy Units (if necessary)

- Convert ΔE from eV to Joules:
$$
1\, \text{eV} = 1.602 \times 10^{-19}\, \text{J}
$$
- Thus,
$$
\Delta E (J) = \Delta E (eV) \times 1.602 \times 10^{-19}
$$

4. Calculate the Wavelength λ

- Use the relation:
$$
\lambda = \frac{hc}{\Delta E}
$$
- Substituting constants:
$$
h = 6.626 \times 10^{-34}\, \text{Js}
$$
$$
c = 3.0 \times 10^{8}\, \text{m/s}
$$
- The wavelength in meters is:
$$
\lambda = \frac{(6.626 \times 10^{-34})(3.0 \times 10^{8})}{\Delta E}
$$

Alternatively, for convenience, when ΔE is in eV:

$$
\lambda (\text{nm}) = \frac{1240}{\Delta E (\text{eV})}
$$

because:

$$
\frac{hc}{1\, \text{eV}} \approx 1240\, \text{nm}
$$

Sample Calculations

Example 1: Transition from $n_i=3$ to $n_f=2$

- Calculate E_3 and E_2:
$$
E_3 = - \frac{13.6}{3^2} = - \frac{13.6}{9} \approx -1.51\, \text{eV}
$$
\[

$$
E_2 = - \frac{13.6}{2^2} = - \frac{13.6}{4} = -3.4\, \text{eV}
$$

- Energy difference:

$$
\Delta E = E_2 - E_3 = (-3.4) - (-1.51) = -3.4 + 1.51 = -1.89\, \text{eV}
$$

Since the transition emits energy, take the absolute value:

$$
|\Delta E| = 1.89\, \text{eV}
$$

- Wavelength calculation:

$$
\lambda (\text{nm}) = \frac{1240}{1.89} \approx 656.6\, \text{nm}
$$

Result: The emitted photon has a wavelength of approximately 656.6 nm, corresponding to the red Balmer alpha line in hydrogen.

Additional Considerations

1. Series of Spectral Lines

- Different electron transitions produce spectral lines in distinct series:
- Lyman series: Transitions to $n_f=1$, ultraviolet region.
- Balmer series: Transitions to $n_f=2$, visible region.
- Paschen series: Transitions to $n_f=3$, infrared region.

Understanding the specific transition allows prediction of the spectral line's position.

2. Limitations and Assumptions

- The calculations assume the Bohr model, which is accurate for hydrogen but less so for multi-electron atoms.
- Real spectral lines are subject to fine structure, Zeeman effect, and Doppler broadening, which slightly modify the observed wavelengths.
- The model ignores relativistic effects; for high-energy transitions, relativistic corrections might be necessary.

3. Transition Probabilities and Intensity

- Not all transitions are equally probable.
- Transition probability, governed by selection rules, affects the intensity of spectral lines.
- For hydrogen, the primary selection rule is $\Delta n \geq 1$ and $\Delta l = \pm 1$ (where l is the azimuthal quantum number).

Conclusion

Calculating the wavelength of light emitted during an electron transition in the hydrogen atom involves understanding quantum energy levels, the energy difference between states, and the fundamental relationship between energy and electromagnetic radiation. This process highlights the quantized nature of atomic energy states and provides a direct link between quantum mechanics and observable spectral lines.

By systematically identifying the initial and final energy levels, computing the energy difference, converting units appropriately, and applying the relation \(\lambda = hc / \Delta E \), one can accurately determine the wavelength of emitted photons. This methodology not only elucidates the spectral characteristics of hydrogen but also forms the basis for spectroscopic analysis across a broad range of atomic and molecular systems.

References

- Griffiths, D. J. (2017). Introduction to Quantum Mechanics. Cambridge University Press.
- Tipler, P. A., & Llewellyn, R. (2007

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