Consider Functions F And G.f(X) = X2 + 24x + 144g(x) = X3 216-Which Expression Is Equal To F(x) + G(x)?

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Understanding how to combine functions is a fundamental aspect of algebra and calculus. In this article, we will explore the process of adding two functions, F and G, given their definitions, and determine the resulting expression for F(x) + G(x). This involves analyzing the provided functions, performing algebraic operations, and simplifying the sum to its most basic form. Whether you're a student brushing up on algebraic concepts or someone looking to deepen your understanding of functions, this comprehensive guide will clarify the process step by step.

Understanding the Given Functions

Before we can find the sum of the functions F(x) and G(x), it is essential to understand their individual definitions clearly. The functions are provided as follows:

- $F(x) = x^2 + 24x + 144$
- $G(x) = x^3$ 216 (which appears to be a typo or a formatting issue; most likely, it is intended to be $G(x) = x^3 - 216$)

A close examination suggests that G(x) is a cubic function with a constant term. The typical notation for such functions is $G(x) = x^3 - 216$, which is a difference of cubes and can be factored further if needed.

Note: The notation "g(x) = X3 216" seems to be a typo or formatting error. Considering common mathematical conventions, we interpret it as $G(x) = x^3 - 216$.

Clarifying the Functions: F(x) and G(x)

Function F(x)

The function F(x) is a quadratic polynomial:

$F(x) = x^2 + 24x + 144$

This quadratic can sometimes be factored or rewritten in vertex form to analyze its properties, but for the purpose of adding it to G(x), we simply keep it as is.

Function G(x)

Assuming the corrected form:

$G(x) = x^3 - 216$

This is a cubic polynomial minus a constant. Recognizing that 216 is a perfect cube (6^3), G(x) can be expressed as:

$G(x) = x^3 - 6^3$

This form is useful for factoring using the difference of cubes formula.

Adding the Functions F(x) and G(x)

To find F(x) + G(x), we perform the addition term-by-term:

$F(x) + G(x) = (x^2 + 24x + 144) + (x^3 - 216)$

Rearranged, the sum becomes:

$F(x) + G(x) = x^3 + x^2 + 24x + (144 - 216)$

Simplify the constant terms:

$144 - 216 = -72$

Therefore:

$F(x) + G(x) = x^3 + x^2 + 24x - 72$

This expression is the sum of the two functions in simplified form.

Expressing the Sum in Different Forms

Having obtained the sum, it's often useful to express it in various forms for different applications, such as factoring or graphing.

Standard Polynomial Form

The sum is:

$F(x) + G(x) = x^3 + x^2 + 24x - 72$

This is a cubic polynomial in standard form.

Factoring the Sum

Factoring the sum can provide insights into its roots and behavior.

Let's attempt to factor:

$x^3 + x^2 + 24x - 72$

First, look for rational roots using Rational Root Theorem:

Possible roots are factors of -72 divided by factors of 1.

Factors of -72: ±1, ±2, ±3, ±4, ±6, ±8, ±9, ±12, ±18, ±24, ±36, ±72

Test these values:

- For x = 1:

1 + 1 + 24 - 72 = -46 (not zero)

- For x = -1:

-1 + 1 - 24 - 72 = -96 (not zero)

- For x = 2:

8 + 4 + 48 - 72 = -12 (not zero)

- For x = -2:

-8 + 4 - 48 - 72 = -124 (not zero)

- For x = 3:

27 + 9 + 72 - 72 = 36 (not zero)

- For x = -3:

-27 + 9 - 72 - 72 = -162 (not zero)

- For x = 4:

64 + 16 + 96 - 72 = 104 (not zero)

- For x = -4:

-64 + 16 - 96 - 72 = -216 (not zero)

- For x = 6:

216 + 36 + 144 - 72 = 324 (not zero)

- For x = -6:

-216 + 36 - 144 - 72 = -396 (not zero)

- For x = 8:

512 + 64 + 192 - 72 = 696 (not zero)

- For x = -8:

-512 + 64 - 192 - 72 = -712 (not zero)

- For x = 9:

729 + 81 + 216 - 72 = 954 (not zero)

- For x = -9:

-729 + 81 - 216 - 72 = -936 (not zero)

- For x = 12:

1728 + 144 + 288 - 72 = 2088 (not zero)

- For x = -12:

-1728 + 144 - 288 - 72 = -1944 (not zero)

Since none of these rational roots work, factoring over rationals is not straightforward. Alternatively, factoring can involve methods like polynomial division or numerical methods, but for many practical purposes, leaving the sum in polynomial form suffices.

Applications of the Sum of Functions

Understanding the sum of functions like F(x) and G(x) has multiple applications across mathematics and science:

- Graphing: The combined function can be graphed to analyze the overall behavior of the system.
- Calculus: Differentiating or integrating the sum can provide insights into rates of change or area under the curve.
- Problem Solving: Combining functions allows solving complex problems by breaking them down into manageable parts.

Example Problems

Problem 1: Find the value of (F + G)(x) when x = 2.

Solution:

Calculate $F(2)$:

$F(2) = (2)^2 + 24(2) + 144 = 4 + 48 + 144 = 196$

Calculate $G(2)$:

$G(2) = (2)^3 - 216 = 8 - 216 = -208$

Sum:

$F(2) + G(2) = 196 + (-208) = -12$

Problem 2: Determine whether the sum function has any real roots.

Solution:

Since the sum is:

$x^3 + x^2 + 24x - 72$

and testing rational roots did not yield any rational roots, numerical methods or graphing can help identify real roots. Graphs indicate the function crosses the x-axis at some points, confirming the existence of real roots, but exact solutions may require numerical approximation.

Conclusion

Adding functions $F(x) = x^2 + 24x + 144$ and $G(x) = x^3 - 216$ results in the combined function:

$F(x) + G(x) = x^3 + x^2 + 24x - 72$

This process involves straightforward algebraic addition and simplification. Recognizing the forms of the individual functions, especially the difference of cubes in $G(x)$, can facilitate further analysis such as factoring or solving for roots. Mastering these techniques is essential for tackling more complex problems involving functions, their sums, and their behaviors.

Whether you're analyzing quadratic and cubic functions or preparing for advanced calculus, understanding how to add and manipulate functions is a vital skill. Keep practicing by applying these methods to different functions and exploring their properties to enhance your mathematical proficiency.

Frequently Asked Questions

What is the expression for F(x) given F(x) = $x^2 + 24x + 144$?

F(x) = $x^2 + 24x + 144$.

What is the expression for G(x) given G(x) = $x^3 - 216$?

G(x) = $x^3 - 216$.

How do you find F(x) + G(x)?

Add the expressions for F(x) and G(x): $(x^2 + 24x + 144) + (x^3 - 216)$.

What is the simplified form of F(x) + G(x)?

The simplified form is $x^3 + x^2 + 24x - 72$.

Which expression is equal to F(x) + G(x)?

F(x) + G(x) = $x^3 + x^2 + 24x - 72$.

Is F(x) + G(x) a polynomial? If so, what degree is it?

Yes, it is a polynomial of degree 3.

How do you verify the sum of F(x) and G(x)?

By adding their expressions algebraically and simplifying.

What is the significance of combining F(x) and G(x)?

It helps analyze the combined behavior of the two functions for different x-values.

Can F(x) + G(x) be factored further? If yes, how?

Currently, the expression $x^3 + x^2 + 24x - 72$ does not factor easily; further factorization would require advanced techniques or specific methods.

Additional Resources

Mathematics: An Expert Analysis of Combining Functions F and G

Introduction: Navigating the World of Mathematical Functions

Mathematics, often regarded as the language of science and engineering, offers a vast landscape of functions that model real-world phenomena, solve complex problems, and underpin technological advancements. Among these, algebraic functions are fundamental, providing tools for understanding relationships between variables. Today, we delve into a specific problem involving two functions, F and G, and explore how their combination can be expressed and understood.

This article aims to dissect the problem methodically, offering a comprehensive explanation suitable for students, educators, or anyone interested in the elegance and utility of algebraic functions. We will examine each function individually, analyze their structures, and then explore how to combine them into a single, simplified expression.

Understanding the Functions: Definitions and Components

Before combining F and G, it is crucial to understand their individual forms and what they represent.

Function F(x): A Quadratic Expression

Given:

$$ F(x) = x^2 + 24x + 144 $$

This is a quadratic function, characterized by an x^2 term, a linear component $24x$, and a constant 144.

- Quadratic Nature: The presence of x^2 indicates the parabola shape when graphed.
- Completing the Square: The quadratic can be rewritten to reveal its vertex form, which clarifies its geometric properties.

Let's analyze F(x) further:

$$
F(x) = x^2 + 24x + 144
$$

Notice that:

$$
x^2 + 24x + 144 = (x + 12)^2
$$

because:

$$
(x + 12)^2 = x^2 + 24x + 144
$$

Implication: F(x) is a perfect square, opening the door to easier manipulation when combining with other functions.

Function G(x): A Cubic Expression

Given:

$$ G(x) = x^3 - 216 $$

This is a cubic function, featuring an x^3 term and a constant -216.

- Cubic Nature: The x^3 term indicates the function's graph has an S-shape, with inflection points.
- Constant Term: The constant -216 suggests a shift along the y-axis.

A key insight is recognizing that 216 is a perfect cube:

$$
216 = 6^3
$$

Thus,

$$
G(x) = x^3 - 6^3
$$

This expression resembles the difference of cubes, which is a well-understood algebraic identity:

$$
a^3 - b^3 = (a - b)(a^2 + ab + b^2)
$$

Applying this identity with $a = x$ and $b = 6$:

\[

$$
G(x) = (x - 6)(x^2 + 6x + 36)
$$

This factorization can be instrumental in further simplifying the sum $F(x) + G(x)$.

Combining the Functions: Deriving $F(x) + G(x)$

The core task is to compute and simplify the expression:

$$
F(x) + G(x)
$$

Given the earlier expressions, this becomes:

$$
(x + 12)^2 + (x^3 - 216)
$$

or, more explicitly:

$$
F(x) + G(x) = (x + 12)^2 + x^3 - 216
$$

Let's proceed step-by-step through the algebraic combination.

Step 1: Expand the Square

Expand $(x + 12)^2$:

$$
(x + 12)^2 = x^2 + 2 \times 12 \times x + 12^2 = x^2 + 24x + 144
$$

Note: This matches the original form of F(x), confirming our earlier identification.

Step 2: Write the Sum Explicitly

Now, combine the expanded form with $G(x)$:

$$
F(x) + G(x) = x^2 + 24x + 144 + x^3 - 216
$$

Rearranged for clarity:

$$
F(x) + G(x) = x^3 + x^2 + 24x + (144 - 216)
$$

Simplify constants:

$$
144 - 216 = -72
$$

Thus, the combined expression:

$$
F(x) + G(x) = x^3 + x^2 + 24x - 72
$$

This is a cubic polynomial, which can be analyzed further for roots, factorization, or applications.

Factoring the Resulting Expression: Is There a Simpler Form?

Given the combined polynomial:

$$
x^3 + x^2 + 24x - 72
$$

Let's explore whether it can be factored further, which can reveal roots or simpler forms.

Step 1: Rational Root Theorem

The Rational Root Theorem suggests that any rational root p/q (in lowest terms) of the polynomial must satisfy:

- p divides the constant term -72
- q divides the leading coefficient 1

Possible roots are divisors of -72:

$$
\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 9, \pm 12, \pm 18, \pm 24, \pm 36, \pm 72
$$

Test these candidates by substituting into the polynomial.

Step 2: Testing Potential Roots

Let's test $x = 3$:

$$
3^3 + 3^2 + 24 \times 3 - 72 = 27 + 9 + 72 - 72 = 36
$$

Not zero; discard.

Test $x = -3$:

$$
(-3)^3 + (-3)^2 + 24 \times (-3) - 72 = -27 + 9 - 72 - 72 = -162
$$

No.

Test $x = 2$:

$$
8 + 4 + 48 - 72 = -12
$$

No.

Test $x = -2$:

$$
-8 + 4 - 48 - 72 = -124
$$

No.

Test $x = 1$:

$$
1 + 1 + 24 - 72 = -46
$$

No.

Test $x = -1$:

$$
-1 + 1 - 24 - 72 = -96
$$

No.

Test $x = 6$:

$$
216 + 36 + 144 - 72 = 324
$$

No.

Test $x = -6$:

$$
-216 + 36 - 144 - 72 = -396
$$

No.

Try $x = 4$:

$$
64 + 16 + 96 - 72 = 104
$$

No.

Try $x = -4$:

$$
-64 + 16 - 96 - 72 = -216
$$

No.

Try $x = 12$:

$$
1728 + 144 + 288 - 72 = 2088
$$

No.

Similarly, testing other divisors yields no roots.

Conclusion: No Rational Roots

Since none of the rational candidates satisfy the polynomial, the cubic doesn't factor nicely over the rationals. It may have irrational or complex roots, but for our purposes, the simplified form:

$$
\boxed{F(x) + G(x) = x^3 + x^2 + 24x - 72}
$$

is the most straightforward expression.

Implications and Applications of the Result

Having derived the sum of the functions, it's essential to understand its significance.

- Graphical Interpretation: The combined polynomial represents the sum of a quadratic and a cubic function, resulting in a cubic polynomial that can be graphed to observe its behavior across different x-values.

- Roots and Intercepts: Although explicit roots are elusive via rational factoring, numerical methods or graphing tools can approximate where the function crosses the x-axis, which is crucial in applications such as physics or economics where these functions might model real phenomena.

- Further Factorization: If needed, advanced algebraic techniques or computational tools could approximate roots or factor over complex numbers.

Summary: Key Takeaways

- The quadratic function $F(x)$ can be expressed as a perfect square:

$$
F(x) = (x + 12)^2
$$

- The cubic function $G(x)$ is a difference of cubes:

$$
G(x) = x^3 - 6^3 = (x - 6)(x^2 + 6x + 36)
$$

- Combining \(F(x) \) and \(G(x) \):

\[
F(x)

Consider Functions F And G F X X2 24x 144g X X3 216 Which Expression Is Equal To F X G X

Consider Functions F And G F X X2 24x 144g X X3 216 Which Expression Is Equal To F X G X

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