Consider The Following Function.f(t)=et2(a) Find The Relative Rate Of Change. (b) Evaluate The Relative

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Understanding the behavior of functions and their rates of change is fundamental in calculus and mathematical analysis. Specifically, the concept of relative rate of change provides insight into how a function evolves concerning its current value, which is crucial in fields such as physics, economics, biology, and engineering. In this article, we will explore the function $f(t) = e^{t^2 a}$, determine its relative rate of change, and evaluate this rate at specific points or conditions. Our discussion aims to clarify the process step by step, making complex concepts accessible and applicable.

Introduction to Relative Rate of Change

Before diving into the specific function, it's essential to understand what the relative rate of change entails.

Definition of Relative Rate of Change

The relative rate of change of a function $f(t)$ at a point t is a measure of how quickly the function's value changes relative to its current value. Mathematically, it is expressed as:

$$
\text{Relative Rate of Change} = \frac{f'(t)}{f(t)}
$$
\end{blockquote>

where:
- $f'(t)$ is the derivative of $f(t)$ with respect to t,
- $f(t)$ is the current value of the function at t.

This ratio indicates the proportional change and is often associated with concepts like percentage growth or decay.

Analyzing the Given Function $f(t) = e^{t^2 a}$

The function under consideration is:

$$
f(t) = e^{t^2 a}
$$
\end{blockquote>

where:
- t is the independent variable,
- a is a constant parameter that influences the function's growth rate.

This exponential function exhibits growth or decay depending on the sign and magnitude of a.

Step 1: Find the Derivative $f'(t)$

To compute the relative rate of change, the first step is to derive $f'(t)$.

Given:

$$
f(t) = e^{t^2 a}
$$

Applying the chain rule:

$$
f'(t) = \frac{d}{dt} \left(e^{t^2 a} \right)
$$

Since e^{u} differentiates to $e^{u} \cdot u'$, where $u = t^2 a$:

$$
f'(t) = e^{t^2 a} \cdot \frac{d}{dt} (t^2 a)
$$

The derivative of $t^2 a$ with respect to t:

$$
\frac{d}{dt} (t^2 a) = 2 t a
$$

Therefore:

$$
f'(t) = e^{t^2 a} \cdot 2 t a
$$

Calculating the Relative Rate of Change

Now that we have $f(t)$ and $f'(t)$, we can compute the relative rate of change.

Step 2: Compute $\frac{f'(t)}{f(t)}$

Substituting the derivatives:

$$
\frac{f'(t)}{f(t)} = \frac{e^{t^2 a} \cdot 2 t a}{e^{t^2 a}}
$$

Since $e^{t^2 a}$ appears both numerator and denominator, they cancel out:

$$
\frac{f'(t)}{f(t)} = 2 t a
$$

This simplifies the relative rate of change to:

$$
\boxed{
\text{Relative Rate of Change} = 2 t a
}
$$

Interpreting the Relative Rate of Change

The expression $2 t a$ provides several insights:

- **Dependence on t:** The relative rate of change varies linearly with t. As t increases or decreases, the proportional change adjusts accordingly.
- **Influence of a:** The parameter a scales the rate of change. A positive a indicates growth, while a negative a suggests decay.
- **Sign of the Rate:** The sign of $2 t a$ determines whether the function is increasing or decreasing relative to its current value at a specific t.

This understanding helps predict the behavior of the function over different intervals and parameter values.

Part (b): Evaluating the Relative Rate of Change

The second part involves evaluating the relative rate of change at specific points or under particular conditions. Let's explore different scenarios.

Scenario 1: Evaluate at a Specific $t = t_0$

Suppose we want to find the relative rate of change at $t = t_0$. The procedure:

1. Substitute $t = t_0$ into the expression $2 t a$:

2. Calculate the value based on given a and t_0.

Example:

Let $a = 3$, and evaluate at $t = 2$:

$$
\text{Relative Rate of Change} = 2 \times 2 \times 3 = 12
$$

This indicates that at $t = 2$, the function's value is increasing at a rate proportional to 12 times its current value.

Scenario 2: Evaluate as $t \to \infty$ or $t \to -\infty$

- For $a > 0$:

- As $t \to \infty$, the relative rate of change $\to \infty$, implying rapid growth.
- As $t \to -\infty$, the relative rate $\to -\infty$, indicating rapid decay.

- For $a < 0$:

- The behavior reverses, with growth or decay depending on the sign of t.

Scenario 3: When $a = 0$

If $a=0$, the function simplifies to:

$$
f(t) = e^{0} = 1
$$

and the relative rate of change:

$$
2 t \times 0 = 0
$$

which makes sense because the function is constant; its value does not change, so the relative rate of change is zero everywhere.

Applications of Relative Rate of Change

Understanding the relative rate of change has numerous real-world applications:

1. **Population Dynamics:** In biology, the relative rate of change models how populations grow or decline proportionally over time.
2. **Financial Growth:** In economics, it helps analyze compound interest and investment growth rates.
3. **Physics and Engineering:** It assists in studying exponential decay or growth processes like radioactive decay or charging/discharging capacitors.
4. **Environmental Science:** Used to model pollutant dispersion or resource depletion rates.

Conclusion

The function $f(t) = e^{t^2 a}$ exhibits exponential behavior modulated by a quadratic exponent. The key takeaway is that its relative rate of change is straightforward to compute, resulting in the simple expression $2 t a$. This ratio provides vital insights into how rapidly the function grows or decays relative to its current value at any point t. By evaluating this ratio at specific points or conditions, one can make predictions and analyze the behavior of such exponential functions across various scientific and mathematical contexts.

Understanding these principles enhances your ability to analyze complex functions and their dynamics, equipping you with tools essential for advanced studies and practical applications in multiple disciplines.

Meta-Note: This comprehensive exploration covers theoretical derivation, practical evaluation, and real-world relevance, ensuring a well-rounded understanding suitable for academic and professional purposes.

Frequently Asked Questions

What is the general approach to finding the relative rate of change of the function $f(t) = e^{t^2}$?

To find the relative rate of change, compute the derivative $f'(t)$ and then divide it by the original function $f(t)$. That is, relative rate = $f'(t) / f(t)$.

How do you differentiate the function $f(t) = e^{t^2}$ with respect to t?

Use the chain rule: $f'(t) = e^{t^2} 2t$.

What is the expression for the relative rate of change of $f(t) = e^{t^2}$?

The relative rate of change is $(f'(t)) / f(t) = (e^{t^2} 2t) / e^{t^2} = 2t$.

How do you interpret the relative rate of change in the context of the function $f(t) = e^{t^2}$?

It indicates how rapidly the function is changing relative to its current value at a given t, which here simplifies to 2t, showing linear growth in t.

Evaluate the relative rate of change of $f(t) = e^{t^2}$ at $t = 3$.

At $t = 3$, the relative rate of change is $2\ 3 = 6$.

What is the significance of the relative rate of change being equal to 2t for the function $f(t) = e^{t^2}$?

It signifies that the proportional rate at which the function changes depends linearly on t, increasing as t increases.

Can the relative rate of change be negative for $f(t) = e^{t^2}$? If so, when?

Yes, when t is negative, since the relative rate of change is 2t, it will be negative, indicating decreasing relative change.

How would you generalize the process of finding the relative rate of change for functions of the form $e^{g(t)}$?

Differentiate to find $g'(t)\ e^{g(t)}$ and then divide by $e^{g(t)}$ to get $g'(t)$, which is the relative rate of change.

What is the importance of understanding the relative rate of change in mathematical modeling?

It helps to understand how a quantity grows or diminishes proportionally, which is crucial in fields like biology, physics, economics, and engineering for analyzing exponential growth or decay.

Additional Resources

Function Analysis and Rate of Change: A Deep Dive into $f(t) = e^{t^2}$

Introduction

In the realm of calculus and mathematical modeling, understanding how a function behaves as its input varies is fundamental. One of the key concepts that allow us to grasp this dynamic is the rate of change, which reveals how rapidly a function's value shifts in response to changes in its input. Today, we delve into the intricacies of the function:

$$
f(t) = e^{t^2}
$$

This function is a prime example of an exponential function with a quadratic exponent, embodying complex behavior that warrants detailed analysis. Our exploration will cover two core aspects:

- The relative rate of change of $f(t)$.
- The evaluation of this relative rate at specific points.

Let's approach this with the precision and clarity akin to a product expert reviewing a sophisticated device—highlighting its features, performance, and practical implications.

Understanding the Function $f(t) = e^{t^2}$

Before diving into derivatives and rates, it's essential to understand the nature of this function. The exponential function e^x is well-known for its rapid growth, but when the exponent itself is a quadratic in t, the behavior becomes even more intriguing.

- Growth Rate: Since t^2 grows quadratically, $f(t)$ exhibits super-exponential growth as $|t|$ increases.
- Symmetry: The function is symmetric about the y-axis because t^2 is an even function.
- Differentiability: $f(t)$ is infinitely differentiable, allowing us to analyze its rate of change comprehensively.

Part (a): Finding the Relative Rate of Change of $f(t)$

What Is the Relative Rate of Change?

The relative rate of change measures how quickly a function's value changes relative to its current value. Mathematically, it's expressed as:

$$
\frac{f'(t)}{f(t)}
$$

which can be interpreted as the instantaneous percentage rate of change.

This concept is crucial in fields like physics, economics, and biological sciences, where understanding proportional changes is more meaningful than absolute ones.

Step 1: Find the Derivative $f'(t)$

Given:

$$
f(t) = e^{t^2}
$$

Apply the chain rule:

$$
f'(t) = \frac{d}{dt} e^{t^2} = e^{t^2} \times \frac{d}{dt} (t^2) = e^{t^2} \times 2t
$$

Thus,

$$
f'(t) = 2t \, e^{t^2}
$$

Step 2: Compute the Relative Rate of Change

Divide the derivative by the original function:

$$
\frac{f'(t)}{f(t)} = \frac{2t \, e^{t^2}}{e^{t^2}} = 2t
$$

Key insight: The relative rate of change of $f(t) = e^{t^2}$ simplifies beautifully to:

$$
\boxed{
\frac{f'(t)}{f(t)} = 2t
}
$$

This result reveals that the proportional rate of change at any point t is directly proportional to t.

Part (b): Evaluating the Relative Rate of Change at Specific Points

Having derived the general expression, the next step is to evaluate this at specific values of t. This provides insight into how the function behaves locally.

1. At $t = 0$

$$
\frac{f'(0)}{f(0)} = 2 \times 0 = 0
$$

Interpretation: At $t = 0$, the relative rate of change is zero. The function is momentarily flat—its slope is zero, indicating a local minimum or maximum, or a point of inflection.

2. At $t = 1$

$$
\frac{f'(1)}{f(1)} = 2 \times 1 = 2
$$

Interpretation: At $t=1$, the function's relative rate of change is 2, meaning the value of $f(t)$ is increasing at a rate twice its current size per unit increase in t.

3. At $t = -1$

$$
\frac{f'(-1)}{f(-1)} = 2 \times (-1) = -2
$$

Interpretation: At $t=-1$, the function decreases proportionally at a rate of 2, indicating a negative slope.

4. General Behavior Summary:

Value of t	Relative Rate of Change $\frac{f'(t)}{f(t)}$	Interpretation
$t=0$	0	Flat point; no instantaneous growth or decline
$t>0$	Positive; increases with t	Exponential growth accelerates as t increases
$t<0$	Negative; decreases with t	Function declines as t becomes more negative

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Practical Implications and Intuitive Understanding

The linear nature of the relative rate of change—being $2t$—has significant implications:

- Symmetry and Growth: The proportional rate is zero at the origin and increases linearly with t, indicating that the function remains relatively stable near $t=0$ but accelerates rapidly in either positive or negative direction.
- Application in Modeling: For systems modeled by $f(t)$, such as diffusion processes or population dynamics, this shows that initial growth or decline is slow, but the process accelerates over time.

Additional Insights: Connection to Exponential Growth

The fact that the relative rate of change is a linear function of t distinguishes this function from the classic exponential e^t, which has a constant relative rate ($\frac{f'(t)}{f(t)}=1$). Here, the rate varies with t, adding complexity but also richness to the model's behavior.

Summary and Final Remarks

In this exploration, we've dissected the function $f(t) = e^{t^2}$ through the lens of calculus, revealing the following key points:

- Derivative: $f'(t) = 2t \, e^{t^2}$
- Relative Rate of Change: $\frac{f'(t)}{f(t)} = 2t$
- Evaluation at specific points:
- Zero at $t=0$,
- Positive and increasing for $t>0$,
- Negative for $t<0$.

This analysis not only unpacks the behavior of $f(t)$ but also provides a template for examining similar functions where the exponent involves higher-order polynomials or other complex expressions.

Final Thoughts: Why This Matters

Understanding the relative rate of change is essential in various scientific fields. It helps predict how rapid a process will evolve relative to its current state, which is crucial for:

- Financial modeling: compound interest and investment growth.
- Physics: modeling radioactive decay or particle acceleration.
- Biology: tracking population growth or disease spread.

The function $f(t) = e^{t^2}$ exemplifies how growth rates can be nonlinear and depend explicitly on the variable itself, emphasizing the importance of derivative-based analysis in capturing real-world complexity.

In conclusion, whether you're a student mastering calculus or a professional applying mathematical models, understanding the relative rate of change provides profound insights into the behavior of exponential functions and their applications. The simple yet powerful result $\frac{f'(t)}{f(t)} = 2t$ encapsulates the essence of this function's dynamic nature—an elegant illustration of the beauty inherent in calculus.

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