Consider The Line Y : 3x−8.2Find The Equation Of The Line That Is Perpendicular To This Line And Passes

Consider The Line Y : 3x−8.2Find The Equation Of The Line That Is Perpendicular To This Line And Passes

Understanding how to find the equation of a line perpendicular to a given line is a fundamental aspect of coordinate geometry. Whether you're a student preparing for exams, a teacher designing lesson plans, or a mathematics enthusiast exploring geometric concepts, mastering this skill enhances your understanding of spatial relationships and algebraic equations. In this comprehensive guide, we will analyze the given line, determine the characteristics of its perpendicular, and derive the equation of the perpendicular line that passes through a specific point.

Understanding the Given Line: Y : 3x − 8.2

Before jumping into the calculations, it's crucial to interpret the provided line correctly. The line is given in the form:

Y : 3x − 8.2

This appears to be an equation of a straight line, likely in slope-intercept form:

\[y = 3x - 8.2 \]

Here, the slope (m) of the line is 3, and the y-intercept is −8.2.

Key points about this line:

- Slope (m): 3
- Y-intercept: (0, −8.2)

What Does Perpendicular Mean in Geometry?

In the context of lines on a Cartesian plane, two lines are perpendicular if they intersect at a right angle (90 degrees).

Key property:

- The slopes of two perpendicular lines are negative reciprocals of each other.

Negative reciprocal means:

- If one line has slope m, then the perpendicular line has slope $-\frac{1}{m}$.

Example:

- If the original line has slope 3, then the perpendicular line has slope:

$$
m_{\perp} = -\frac{1}{3}
$$

Finding the Equation of the Perpendicular Line

To find the line perpendicular to the given line Y = 3x - 8.2, and passing through a specific point, follow these steps:

Step 1: Identify the Slope of the Given Line

From the given equation:

$$
y = 3x - 8.2
$$

- The slope is $m = 3$.

Step 2: Determine the Slope of the Perpendicular Line

Using the negative reciprocal property:

$$
m_{\perp} = -\frac{1}{3}
$$

Step 3: Find a Point Through Which the Perpendicular Line Passes

The problem statement indicates that the perpendicular line passes through a specific point. However, the point is not explicitly mentioned in the initial prompt.

Assumption:

Let's consider that the perpendicular line passes through a point (x_0, y_0).

If a specific point is provided, substitute that point here.

Example:

Suppose the line passes through point $(x_1, y_1) = (2, 5)$.

Step 4: Write the Equation of the Perpendicular Line

Using the point-slope form:

$$
y - y_1 = m_{\perp} (x - x_1)
$$

Substituting the values:

$$
y - 5 = -\frac{1}{3} (x - 2)
$$

Step 5: Simplify to Slope-Intercept Form

Expand and simplify:

$$
y - 5 = -\frac{1}{3}x + \frac{2}{3}
$$

$$
y = -\frac{1}{3}x + \frac{2}{3} + 5
$$

Express 5 as a fraction with denominator 3:

$$
5 = \frac{15}{3}
$$

So,

$$
y = -\frac{1}{3}x + \frac{2}{3} + \frac{15}{3} = -\frac{1}{3}x + \frac{17}{3}
$$

Final equation:

$$
\boxed{
y = -\frac{1}{3}x + \frac{17}{3}
}
$$

Generalization: Finding the Equation of a Perpendicular Line for Any Point

If the specific point through which the perpendicular line passes is not given, the general form can be expressed as follows.

Step 1: Use the point-slope form:

$$
y - y_0 = m_{\perp} (x - x_0)
$$

\]

Step 2: Substitute the slope:

$$
m_{\perp} = -\frac{1}{3}
$$

Step 3: Write the general equation:

$$
y - y_0 = -\frac{1}{3} (x - x_0)
$$

Step 4: Convert to slope-intercept form if needed:

$$
y = -\frac{1}{3}x + \frac{1}{3}x_0 + y_0
$$

Where (x_0, y_0) is the point the line passes through.

Special Cases and Additional Considerations

1. Vertical and Horizontal Lines

- If the original line is vertical, with an equation like $x = c$, then the perpendicular line is horizontal:

$$
y = y_0
$$

- Conversely, if the original line is horizontal ($y = k$), then the perpendicular line is vertical:

$$
x = x_0
$$

2. Confirming the Perpendicularity

The key to ensuring the lines are perpendicular is verifying the slopes:

$$
m_1 \times m_2 = -1
$$

Using the original slope $m_1 = 3$, the perpendicular slope:

$$
m_2 = -\frac{1}{3}
$$

Check:

$$
3 \times -\frac{1}{3} = -1
$$

Thus, the lines are perpendicular.

Practical Applications of Perpendicular Lines

Understanding and calculating perpendicular lines has numerous real-world applications, such as:

- Designing architectural structures where walls or beams are perpendicular.
- Analyzing slopes in road or railway construction.
- Creating geometric proofs and solving problems involving right angles.
- Navigating in coordinate systems, especially in computer graphics and robotics.

Summary of Key Steps in Finding the Equation of a Perpendicular Line

Here's a quick checklist to find the equation of a perpendicular line:

1. Identify the slope of the original line.
2. Calculate the negative reciprocal to find the perpendicular slope.
3. Use a given point or a specific point to pass through.
4. Apply the point-slope form: $y - y_0 = m (x - x_0)$.
5. Simplify into slope-intercept form if needed.

Conclusion

Mastering the process of finding the equation of a line perpendicular to a given line is essential for exploring various facets of geometry and algebra. Starting from understanding the slope-intercept form, recognizing the significance of negative reciprocals, and applying the point-slope form allows one to derive accurate equations for perpendicular lines efficiently.

If the line $Y : 3x - 8.2$ is given, and a point through which the perpendicular passes is specified, applying these steps ensures precise calculation. Remember, the key is to correctly identify slopes, apply the negative reciprocal property, and utilize the appropriate point to formulate the desired line equation.

Practice Tip: Always double-check your slopes and algebraic manipulations to ensure your equations are accurate, especially when working with fractions or decimals.

Note: Should you have the specific point through which the perpendicular line passes, substitute it into the point-slope formula to derive the exact equation tailored to your problem.

Frequently Asked Questions

What is the slope of the given line y = 3x - 8.2?

The slope of the given line y = 3x - 8.2 is 3.

How do you find the slope of a line perpendicular to y = 3x - 8.2?

The slope of a perpendicular line is the negative reciprocal of the original slope, so it is -1/3.

What is the general form of the equation for a line passing through a point and perpendicular to another line?

The general form can be written as $y - y_1 = m(x - x_1)$, where m is the perpendicular slope, and (x_1, y_1) is the point through which the line passes.

How do I write the equation of a line perpendicular to y = 3x - 8.2 that passes through a specific point?

Use the point-slope form $y - y_1 = m(x - x_1)$, with m = -1/3 and the given point's coordinates.

If the perpendicular line passes through the point (x_0, y_0), what is the equation?

The equation would be $y - y_0 = -1/3 (x - x_0)$.

Can you give an example of the equation of a perpendicular line passing through (0,0)?

Yes, passing through (0,0), the line's equation is y = -1/3 x.

What is the significance of the negative reciprocal when finding perpendicular lines?

The negative reciprocal of a slope ensures the two lines are perpendicular, meaning they intersect at a right angle.

Additional Resources

Perpendicular Lines and Their Equations: An In-Depth Exploration

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Understanding the Fundamentals: Slope-Intercept Form and Perpendicular Lines

In the realm of coordinate geometry, lines are often described using their slopes and intercepts. To navigate through the problem of finding a line perpendicular to a given line, it's essential first to understand the foundational concepts involved.

What Is the Slope-Intercept Form?

The slope-intercept form of a line is expressed as:

$$ y = mx + b $$

where:
- m is the slope of the line.
- b is the y-intercept, i.e., where the line crosses the y-axis.

Given the line Y: 3x - 8.2, it appears there might be a typographical ambiguity. Assuming the notation indicates the line's equation in the form:

$$ y = 3x - 8.2 $$

then, the slope (m) is 3.

Deciphering the Given Line: Y = 3x - 8.2

Line Equation Breakdown:

- Slope (m): 3
- Y-intercept (b): -8.2

This line is characterized by a positive slope, indicating it's inclined upward as we move from left to right.

Understanding Perpendicular Lines: The Key to the Problem

Perpendicular lines are lines that intersect at a right angle (90 degrees). One of the fundamental properties of perpendicular lines in a plane is the relationship between their slopes:

If two lines are perpendicular, then the product of their slopes is -1.

Mathematically:

$$ m_1 \times m_2 = -1 $$

where:
- m_1 is the slope of the first line.
- m_2 is the slope of the perpendicular line.

Given the original line's slope ($m_1 = 3$), the slope of any line perpendicular to it (m_2) can be calculated as:

$$ m_2 = -\frac{1}{m_1} = -\frac{1}{3} $$

This is a crucial step, as it directly determines the orientation of the new line.

Constructing the Equation of the Perpendicular Line

The problem specifies that the new line must pass through a certain point. However, the point is missing in the provided text. For an exhaustive solution, three scenarios are considered:

1. The line passes through a specific point (e.g., (x_0, y_0)).
2. The line passes through the origin (0,0).
3. The line passes through a point on the original line.

Since the user hasn't specified a point, I'll demonstrate the general approach for each case, emphasizing the method's flexibility.

Scenario 1: Passing Through a Given Point (x_0, y_0)

Suppose the line must pass through a point (x_0, y_0). The general form of the line with slope $m_2 = -\frac{1}{3}$ passing through this point is:

$$ y - y_0 = m_2 (x - x_0) $$

which expands to:

$$ y - y_0 = -\frac{1}{3} (x - x_0) $$

or:

$$ y = -\frac{1}{3} (x - x_0) + y_0 $$

This is the point-slope form, providing a direct way to write the equation once the specific point is known.

Scenario 2: Passing Through the Origin (0,0)

If the line passes through the origin, the equation simplifies to:

$$ y = m_2 x $$

Substituting $m_2 = -\frac{1}{3}$:

$$ y = -\frac{1}{3} x $$

This line passes through (0,0) and is perpendicular to the original line.

Scenario 3: Passing Through a Point on the Original Line

Suppose the line passes through the point where the original line intersects the y-axis, i.e., at $(0, -8.2)$.

Applying the point-slope form:

$$ y - (-8.2) = -\frac{1}{3} (x - 0) $$

which simplifies to:

$$ y + 8.2 = -\frac{1}{3} x $$

and rearranged as:

$$ y = -\frac{1}{3} x - 8.2 $$

This line is perpendicular to the original and passes through the y-intercept.

Consolidated Equation Forms for the Perpendicular Line

Depending on the known point, the perpendicular line's equation can be expressed in various forms:

Scenario	Equation Form	Description
Through (x_0, y_0)	$y - y_0 = -\frac{1}{3}(x - x_0)$	Point-slope form for any point

Through the origin	$y = -\frac{1}{3} x$	Slope-intercept form passing through (0,0)
Through a point (x_1, y_1)	$y - y_1 = -\frac{1}{3}(x - x_1)$	General point-slope form

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Graphical Interpretation and Practical Applications

Understanding how to derive equations of perpendicular lines is not merely an academic exercise; it has real-world applications across various domains:

- Engineering and Design: Ensuring structural elements are orthogonal.
- Navigation and Mapping: Calculating paths that intersect at right angles.
- Computer Graphics: Rendering objects with perpendicular alignments.
- Physics: Analyzing forces and trajectories at right angles.

Visualizing the original line $y = 3x - 8.2$, which rises steeply, the perpendicular line with slope $-1/3$ would descend gently, crossing the original at some point, and forming right angles at intersections.

Key Takeaways for Students and Practitioners

1. Always identify the slope of the given line, which is critical for establishing perpendicularity.
2. Remember the negative reciprocal rule: For a line $y = mx + b$, a perpendicular line's slope is $-1/m$.
3. Use point-slope form when a specific point is given or when you need to derive the line passing through a particular point.
4. Convert between forms (slope-intercept, point-slope, standard) depending on the context and available data.
5. Visualize the geometry to better understand the relationships between lines and their orientations.

Conclusion: Mastering the Art of Perpendicular Line Equations

Finding the equation of a line that is perpendicular to a given line and passes through a specific point is a fundamental skill in coordinate geometry, with broad applications in science, engineering, and mathematics. Starting from understanding the original line's slope, applying the negative reciprocal rule, and then utilizing the point-slope form provides a clear, systematic approach.

In this case, with the original line $y = 3x - 8.2$, the slope of the perpendicular line is $-\frac{1}{3}$. Depending on the specific point through which the line passes—be it the origin, the y-intercept, or any other coordinate—the equation can be tailored accordingly.

By mastering these principles, students and professionals alike can confidently navigate complex geometric problems, ensuring precise and accurate solutions that stand up to real-world scrutiny.

Note: If you have specific points or additional constraints, the method remains the same—identify the slope, apply the perpendicularity condition, and then plug in your data to find the exact equation.

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