Consider The Parametric Equationsx=cos(t)sin(t);y=cos(t)+sin(t) 0t2a) Eliminate The Parameter T To Find

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Introduction

In the study of mathematics, particularly in the fields of calculus and analytic geometry, parametric equations serve as a powerful tool to describe complex curves and motions in a plane or space. They allow us to represent the coordinates of a point as functions of a parameter, often denoted as t. This approach provides flexibility in modeling various phenomena, from the trajectory of projectiles to the parametric design of curves in computer graphics.

Today, we will explore a specific set of parametric equations:

$$
x = \cos(t)\sin(t)
$$

$$
y = \cos(t) + \sin(t)
$$

Our primary goal is to eliminate the parameter t from these equations to find a direct Cartesian relationship between x and y. This process transforms the parametric form into an explicit equation involving only x and y, which can then be analyzed to understand the geometric nature of the curve.

Understanding how to eliminate parameters is a crucial skill in mathematics, as it helps visualize the curve, identify its properties, and classify its type—be it a circle, ellipse, parabola, or more complex shape. Moreover, this problem exemplifies the application of fundamental trigonometric identities and algebraic manipulation, reinforcing key mathematical concepts.

In this article, we will systematically proceed through the steps necessary to eliminate the parameter t, interpret the resulting equation, and analyze the geometric properties of the curve. We will also discuss common mistakes, provide visual insights, and highlight real-world applications where such parametric equations are relevant.

Let’s begin by examining the given parametric equations in detail.

Understanding the Given Parametric Equations

The Equations

The parametric equations are:

$$
x = \cos(t)\sin(t)
$$
$$
y = \cos(t) + \sin(t)
$$

Here, t is the parameter, which typically varies over an interval such as $[0, 2\pi]$ for trigonometric functions. These equations describe a curve traced out in the xy-plane as t varies.

Objective

Our main objective is to eliminate t to obtain an explicit relationship between x and y. This involves expressing x and y solely in terms of each other, without the parameter t.

Step-by-Step Approach to Eliminate the Parameter

Step 1: Recognize the Trigonometric Forms

Observe that both x and y involve sine and cosine functions. Notably:

$$
x = \sin(t) \cos(t)
$$
$$
y = \sin(t) + \cos(t)
$$

Since these are standard trigonometric expressions, we can leverage identities to simplify.

Step 2: Recall Relevant Trigonometric Identities

Some key identities include:

- Double Angle Identity for Sine:

$$
\sin(2t) = 2 \sin(t) \cos(t)
$$

- Sum of Sine and Cosine:

\[

$$
\sin(t) + \cos(t) = \sqrt{2} \sin \left(t + \frac{\pi}{4}\right)
$$

or equivalently,

$$
\sin(t) + \cos(t) = \pm \sqrt{2} \sin \left(t + \frac{\pi}{4}\right)
$$

These identities are instrumental in expressing the equations in a form that allows us to eliminate t.

Step 3: Express x in terms of $\sin(t) \cos(t)$

Using the double angle identity:

$$
x = \sin(t) \cos(t) = \frac{1}{2} \sin(2t)
$$

Thus,

$$
x = \frac{1}{2} \sin(2t)
$$

Step 4: Express y in terms of $\sin(t) + \cos(t)$

Recall that:

$$
y = \sin(t) + \cos(t) = \sqrt{2} \sin \left(t + \frac{\pi}{4}\right)
$$

Alternatively, we can keep it as $\sin(t) + \cos(t)$, but expressing it in terms of a single sine function simplifies the process.

Expressing Both Equations in Terms of a Single Trigonometric Function

Step 5: Use substitution to relate x and y

From the above, we have:

$$
x = \frac{1}{2} \sin(2t)
$$

$$
y = \sqrt{2} \sin \left(t + \frac{\pi}{4}\right)
$$

Our goal is to eliminate t, which appears inside the sine functions. To do this, we need to relate $\sin(2t)$ to $\sin(t + \frac{\pi}{4})$.

Step 6: Express $\sin(2t)$ in terms of $\sin(t + \frac{\pi}{4})$

Recall the double angle formula:

$$
\sin(2t) = 2 \sin(t) \cos(t)
$$

But since $y = \sin(t) + \cos(t)$, we can express $\sin(t)$ and $\cos(t)$ in terms of y.

Deriving the Relationship Between x and y

Step 7: Express $\sin(t)$ and $\cos(t)$ in terms of y

Given:

$$
y = \sin(t) + \cos(t)
$$

We can consider the identities:

$$
(\sin(t) + \cos(t))^2 = \sin^2(t) + 2 \sin(t) \cos(t) + \cos^2(t)
$$

But since $\sin^2(t) + \cos^2(t) = 1$, this simplifies to:

$$
y^2 = 1 + 2 \sin(t) \cos(t)
$$

Recall that:

$$
x = \sin(t) \cos(t) = \frac{1}{2} \sin(2t)
$$

So,

$$
2 x = \sin(2t)
$$

Now, from the earlier identity:

$$
\sin(2t) = y^2 - 1
$$

which implies:

$$
2 x = y^2 - 1
$$

Therefore, the key relationship between x and y is:

$$
2 x = y^2 - 1
$$

or equivalently,

$$
\boxed{
y^2 = 2 x + 1
}
$$

Final Cartesian Equation and Geometric Interpretation

Step 8: Write the explicit relation

The derived relationship is:

$$
\boxed{
y^2 = 2 x + 1
}
$$

This is a quadratic equation in y and x, representing a parabola opening to the right.

Step 9: Geometric analysis

- Shape: The equation $y^2 = 2x + 1$ describes a parabola.
- Vertex: The parabola has its vertex at $(-\frac{1}{2}, 0)$.
- Axis of symmetry: The parabola opens along the x-axis, symmetric about the line $y = 0$.
- Focus and directrix: Using standard parabola properties, the focus and directrix can be computed if needed.

Step 10: Summary

The key result is: The parametric equations

$$
x = \sin(t) \cos(t)
$$
$$
y = \sin(t) + \cos(t)
$$

describe a parabola with the Cartesian equation:

$$
\boxed{
y^2 = 2x + 1
}
$$

which provides a direct relationship between x and y, eliminating the parameter t.

Additional Insights and Applications

Applications of Such Parametric Equations

Parametric equations like the ones discussed are widely used in:

- Physics: To model projectile motion and oscillations.
- Engineering: In the design of curves and mechanical linkages.
- Computer Graphics: For generating smooth curves and animations.
- Mathematics Education: To illustrate the relationship between parameterized and explicit forms.

Exploring the Curve

- Plotting the curve $y^2 = 2x + 1$ reveals a parabola opening to the right.
- The parameter t traces this parabola as it varies over its domain.

Common Mistakes to Avoid

- Confusing the identities: Make sure to correctly apply double-angle identities.
- Overlooking the domain: Remember that the parameter t varies over an interval, which affects the portion of the curve traced.
- Misinterpreting the signs: Always verify the correct signs when taking square roots or manipulating identities.

Conclusion

Eliminating the parameter from parametric equations is a fundamental technique in mathematics that enables us to understand the geometric nature of curves better. In this case

Frequently Asked Questions

How can I eliminate the parameter t from the equations x = cos(t) sin(t) and y = cos(t) + sin(t)?

You can use trigonometric identities: note that 2 cos(t) sin(t) = sin(2t). From x = cos(t) sin(t), multiply both sides by 2 to get 2x = sin(2t). Also, express y as y = cos(t) + sin(t). Using the identities, express cos(t) and sin(t) in terms of y or relate them to eliminate t. Ultimately, you'll derive an equation relating x and y without t.

What is the relationship between x and y after eliminating the parameter t?

After eliminating t, the relation between x and y is given by the equation y^2 = 1 - 2x, which describes the curve traced by the parametric equations in the xy-plane.

What is the geometric interpretation of the curve described by these parametric equations?

The curve is a parabola or a related conic section defined by the Cartesian equation derived after eliminating t, typically representing a quadratic relation between x and y.

Are there any special points or intercepts on the curve defined by these equations?

Yes, by setting t to specific values such as 0 or π/2, you can find points where the curve intersects axes. For example, at t = 0, x = 0, y = 2; at t = π/2, x = 0, y = 0, indicating points on the curve.

Can the parametric equations be used to plot the curve easily once t is eliminated?

Yes, once t is eliminated to find the Cartesian equation relating x and y, plotting becomes straightforward using standard graphing methods without needing to vary t explicitly.

What identities are useful in the process of eliminating t in these equations?

Key identities include sin(2t) = 2 sin(t) cos(t) and the Pythagorean identity sin^2(t) + cos^2(t) = 1, which help relate x and y directly and eliminate the parameter t.

Additional Resources

Consider The Parametric Equations x = cos(t) sin(t); y = cos(t) + sin(t) 0 t 2a) Eliminate The Parameter T To Find is a classic problem in the study of parametric equations, illustrating fundamental techniques in algebraic manipulation and the geometric interpretation of curves. This type of problem is central to understanding how parametric representations can be converted into Cartesian equations, providing insight into the shape and properties of the resulting curves. In this article, we will explore the process of eliminating the parameter t from the given equations, analyze the properties of the resulting Cartesian equation, and discuss the broader implications and applications of such transformations.

Understanding the Parametric Equations

The Given Equations

The parametric equations under consideration are:

$$
x = \cos(t) \sin(t)
$$

$$
y = \cos(t) + \sin(t)
$$

where t ranges over an interval, typically $0 \leq t \leq 2\pi$.

These equations describe a curve in the xy-plane as t varies. The goal is to eliminate t to find a single Cartesian equation involving only x and y.

Geometric Significance

Parametric equations like these are often used to represent curves that are difficult to express explicitly in terms of x or y. The advantage of parametric forms is that they allow for a more flexible description of curves, especially those involving trigonometric functions, which naturally lend themselves to parameterization via angles.

The Process of Eliminating the Parameter t

Step 1: Expressing $\sin(t)$ and $\cos(t)$ in terms of y

From the second equation:

$$
y = \cos(t) + \sin(t)
$$

we aim to express $\sin(t)$ and $\cos(t)$ in terms of y.

Step 2: Utilizing the Pythagorean Identity

Recall the fundamental Pythagorean identity:

$$
\sin^2(t) + \cos^2(t) = 1
$$

This identity will be crucial in eliminating t.

Step 3: Expressing $\sin(t)$ in terms of y and $\cos(t)$
From the second equation:
$$
\sin(t) = y - \cos(t)
$$
Substituting into the Pythagorean identity:
$$
\cos^2(t) + (y - \cos(t))^2 = 1
$$
Expanding:
$$
\cos^2(t) + y^2 - 2 y \cos(t) + \cos^2(t) = 1
$$
which simplifies to:
$$
2 \cos^2(t) - 2 y \cos(t) + y^2 = 1
$$

Step 4: Forming a quadratic in $\cos(t)$
Rearranged:
$$
2 \cos^2(t) - 2 y \cos(t) + (y^2 - 1) = 0
$$
Divide through by 2:
$$
\cos^2(t) - y \cos(t) + \frac{y^2 - 1}{2} = 0
$$
This quadratic in $\cos(t)$ is:
$$
\cos^2(t) - y \cos(t) + \frac{y^2 - 1}{2} = 0
$$

Solving for $\cos(t)$

Step 1: Apply the quadratic formula
$$
\cos(t) = \frac{ y \pm \sqrt{ y^2 - 2 (y^2 - 1) } }{ 2 }
$$
Calculating the discriminant:
$$
\Delta = y^2 - 2 (y^2 - 1) = y^2 - 2 y^2 + 2 = - y^2 + 2
$$
Thus:
$$
\cos(t) = \frac{ y \pm \sqrt{ - y^2 + 2 } }{ 2 }
$$

Step 2: Expressing x in terms of y
Recall that:

$$
x = \cos(t) \sin(t)
$$

From earlier:

$$
\sin(t) = y - \cos(t)
$$

So:

$$
x = \cos(t) (y - \cos(t)) = y \cos(t) - \cos^2(t)
$$

Step 3: Expressing $\cos^2(t)$ in terms of y

From the quadratic solution, $\cos(t)$ is known in terms of y. Additionally, the identity:

$$
\cos^2(t) = \left(\frac{ y \pm \sqrt{ - y^2 + 2 } }{ 2 } \right)^2
$$

can be utilized to express x directly in terms of y.

Deriving the Cartesian Equation

Simplification Strategy

Instead of substituting back the complicated expression for $\cos(t)$, a more straightforward approach involves combining the original equations differently.

Alternative Approach: Use sum and product identities

Given:

$$
x = \cos(t) \sin(t)
$$

and:

$$
y = \cos(t) + \sin(t)
$$

Recall the double-angle identities:

$$
\sin(2t) = 2 \sin(t) \cos(t)
$$

$$
\cos(2t) = \cos^2(t) - \sin^2(t)
$$

From the first parametric equation:

$$
x = \frac{1}{2} \sin(2t)
$$

which means:

$$
\sin(2t) = 2x
$$

Similarly, from the second:

$$
y = \cos(t) + \sin(t)
$$

Squaring both sides:

$$
y^2 = (\cos(t) + \sin(t))^2 = \cos^2(t) + 2 \sin(t) \cos(t) + \sin^2(t)
$$

$$
= (\cos^2(t) + \sin^2(t)) + 2 \sin(t) \cos(t) = 1 + 2 x
$$

Thus:

$$
y^2 = 1 + 2 x
$$

Final Cartesian Equation

Rearranged:

$$
2 x = y^2 - 1
$$

or equivalently:

$$
x = \frac{ y^2 - 1 }{ 2 }
$$

This is the Cartesian form of the curve.

Geometric Interpretation of the Result

The equation:

$$
x = \frac{ y^2 - 1 }{ 2 }
$$

represents a parabola opening along the x-axis. Its vertex is at $y = 0$, with the parabola symmetric about the y-axis.

Features:
- Shape: Parabola
- Axis of symmetry: y-axis
- Vertex: At $(-\frac{1}{2}, 0)$
- Range of y: Since $\sin(2t)$ varies between -1 and 1, $2x$ varies between -1 and 1, corresponding to $y^2 - 1$ between -2 and 0. Therefore, y^2 between -1 and 1, which implies y between -1 and 1.

Validity Domain:
The original parametric equations are valid where the square root $\sqrt{ - y^2 + 2 }$ is real, i.e., when:

$$
- y^2 + 2 \ge 0 \implies y^2 \le 2
$$

which aligns with the earlier observations.

Broader Implications and Applications

Techniques Demonstrated
- Elimination of Parameters: Using identities to convert parametric equations into Cartesian form.
- Utilization of Trigonometric Identities: Double-angle formulas simplify the process significantly.
- Understanding Curve Geometry: The Cartesian equation reveals the shape and symmetry of the curve.

Practical Applications
- Physics: Describing particle trajectories where parameters represent time.
- Engineering: Signal processing where waveforms are described parametrically.
- Computer Graphics: Rendering curves based on parametric equations and converting them for plotting.

Advantages & Disadvantages
Pros:
- Simplifies the analysis of curves.
- Facilitates plotting and understanding geometric properties.
- Helps in solving equations involving constraints.

Cons:
- Sometimes the algebra becomes complex, especially with more complicated parametrizations.
- May introduce extraneous solutions; careful analysis is needed to validate the curve.

Summary and Final Remarks

The problem of eliminating the parameter t from the given parametric equations:

$$
x = \cos(t) \sin(t), \quad y = \cos(t) + \sin(t)
$$

demonstrates the power of trigonometric identities and algebraic manipulation in deriving Cartesian equations from param

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