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Consider The Path R(t) : = (12t,8t^3/2,3t^2).Show That The Point (3,1,3/16) Lies On The Image Curve And

Understanding the behavior of parametric curves is fundamental in vector calculus and differential geometry. In this article, we explore the specific path R(t) = (12t, 8t^{3/2}, 3t^2) and demonstrate how to verify whether a given point, in this case (3, 1, 3/16), lies on the curve generated by this parametric function. We will walk through the necessary steps, perform calculations, and discuss the broader implications of this process in mathematical analysis.

Overview of the Path R(t) and Its Geometric Significance

Before delving into the verification process, it is essential to understand what the path R(t) represents geometrically.

Parametric Curve Definition

A parametric curve in three dimensions is defined by a vector function R(t), where t is a real parameter. The function specifies the coordinates (x(t), y(t), z(t)) of points along the curve as t varies over an interval.

In our case:

- x(t) = 12t
- y(t) = 8t^{3/2}
- z(t) = 3t^2

This particular curve combines polynomial and fractional power functions, producing a non-trivial three-dimensional path.

Significance of the Path

Understanding such curves is vital in multiple fields, including physics (trajectory modeling), engineering (design of paths), and pure mathematics (study of differential geometry). Verifying whether a point lies on the curve helps in tasks such as:

- Curve fitting
- Path analysis
- Geometric interpretations in multi-dimensional space

Step-by-Step Verification Process

To determine whether the point (3, 1, 3/16) is on the image of R(t), we need to find a parameter t such that R(t) equals this point.

Step 1: Equate the Components

Set the components of R(t) equal to the corresponding coordinates of the point:

1. $x(t) = 12t = 3$
2. $y(t) = 8t^{3/2} = 1$
3. $z(t) = 3t^2 = 3/16$

Step 2: Solve for t in Each Equation

Solve each equation for t:

- From x(t):
$$
12t = 3 \Rightarrow t = \frac{3}{12} = \frac{1}{4}
$$

- From y(t):
$$
8t^{3/2} = 1 \Rightarrow t^{3/2} = \frac{1}{8}
$$

- From z(t):
$$
3t^2 = \frac{3}{16} \Rightarrow t^2 = \frac{3/16}{3} = \frac{1}{16}
\Rightarrow t = \pm \frac{1}{4}
$$

Note that since t appears in different powers, we should check for consistency among these solutions.

Step 3: Verify Consistency of the Solutions

- From $x(t)$, $t = 1/4$
- From $y(t)$, $t^{3/2} = 1/8$

Calculate $t^{3/2}$ when $t = 1/4$:

$$
t^{3/2} = (t^{1/2})^3
$$

First, compute $t^{1/2}$:

$$
t^{1/2} = \sqrt{\frac{1}{4}} = \frac{1}{2}
$$

Then,

$$
t^{3/2} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}
$$

Which matches the required value.

- From $z(t)$, $t = \pm 1/4$

Since $t = 1/4$ is positive, this is consistent.

Conclusion:

All three component equations are satisfied when $t = 1/4$.

Implications and Geometric Interpretation

Having found $t = 1/4$ that satisfies all component equations, we confirm that:

$$
R\left(\frac{1}{4}\right) = (3, 1, 3/16)
$$

This shows the point $(3, 1, 3/16)$ lies on the curve defined by $R(t)$.

Geometric Significance of the Point

- The point is a specific position along the parametric path at t = 1/4.
- The path's shape can be visualized by plotting R(t) over an interval that includes t = 1/4.
- The calculations demonstrate how algebraic manipulation can verify point-curve membership.

Broader Applications and Related Concepts

Understanding how to verify whether a point lies on a curve is fundamental in multiple mathematical and applied contexts.

Applications in Physics and Engineering

- Trajectory analysis: Determining if an object passes through a certain point.
- Path planning: Ensuring a robotic arm or vehicle follows a specified path.

Mathematical Techniques

- Solving parametric equations
- Analyzing the domain of the parameter t
- Exploring the properties of the curve, such as smoothness and curvature

Related Concepts in Differential Geometry

- Arc length parameterization
- Tangent and normal vectors at a point
- Curvature and torsion analysis

Summary and Key Takeaways

In this comprehensive analysis, we:

1. Examined the parametric path R(t) = (12t, 8t^{3/2}, 3t^2).
2. Verified that the point (3, 1, 3/16) lies on the curve by solving the component equations.
3. Found that t = 1/4 satisfies all equations, confirming the point's position on the path.

4. Discussed the importance of such verification in mathematical modeling and real-world applications.

Key Points:

- Parametric curves are defined by functions that specify coordinates based on a parameter.
- To verify if a point lies on a curve, set the point's coordinates equal to the parametric equations and solve for the parameter.
- Consistency across all components confirms the point's membership in the curve's image.
- Such techniques are vital across scientific disciplines, including physics, engineering, and computer graphics.

Conclusion

The process of verifying whether a point resides on a given parametric path involves algebraic manipulation and careful solution of the component equations. In this case, we demonstrated that the point (3, 1, 3/16) indeed lies on the curve R(t) at t = 1/4. This approach exemplifies foundational methods in vector calculus and geometric analysis, fostering a deeper understanding of parametric curves and their applications. Whether modeling physical trajectories or designing complex paths, mastering these techniques is essential for students and professionals alike.

Frequently Asked Questions

How can we verify that the point (3, 1, 3/16) lies on the curve R(t) = (12t, 8t^{3/2}, 3t^2)?

To verify, we need to find a value of t such that R(t) matches the point's coordinates. Set each component equal: 12t = 3, 8t^{3/2} = 1, and 3t^2 = 3, then solve for t in each equation to see if there's a common solution.

What value of t satisfies the first component equation 12t = 3?

Solving 12t = 3 gives t = 3/12 = 1/4.

Does t = 1/4 satisfy the second component equation 8t^{3/2} = 1?

Yes. Substitute t = 1/4: 8(1/4)^{3/2} = 8 ((1/4)^{1/2})^{3} = 8 (1/2)^{3} = 8 (1/8) = 1. Therefore, t = 1/4 satisfies the second component.

Is t = 1/4 consistent with the third component equation 3t^2 = 3?

Substitute t = 1/4: 3(1/4)^2 = 3(1/16) = 3/16, which does not equal 3. Hence, t = 1/4 does not satisfy the third component.

What does the inconsistency in the third component imply about the point (3, 1, 3/16) and the curve R(t)?

It indicates that the point (3, 1, 3/16) does not lie on the image of the curve R(t), since no single t satisfies all three component equations simultaneously.

Additional Resources

Mathematics Analysis: Exploring the Path \(R(t) = (12t, \frac{8t^{3/2}}{}, 3t^2) \) and Verifying a Point on its Image Curve

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Introduction

In the realm of calculus and vector analysis, parametric curves serve as fundamental tools for visualizing and understanding complex geometric relationships. They allow us to describe the motion of particles, the shape of curves, and the behavior of functions in a multi-dimensional space. Today, we delve into a particular parametric path defined by the vector function:

\[
R(t) = \left(12t, \frac{8t^{3/2}}{}, 3t^2 \right)
\]

Our goal is to demonstrate that a specific point, \(\left(3, 1, \frac{3}{16}\right)\), lies on the image of this curve—that is, there exists some parameter \(t\) such that \(R(t) = (3, 1, 3/16) \).

This task not only illustrates the process of solving parametric equations but also highlights the importance of understanding the domain restrictions and properties of the functions involved. By the end of this analysis, you will gain insight into how to verify the position of points on parametric curves, a skill applicable across various fields like physics, engineering, and computer graphics.

Understanding the Path \(R(t) \)

The Parametric Components

The given vector function:

$$
R(t) = \left(12t,\ \frac{8t^{3/2}}{},\ 3t^2 \right)
$$

can be broken down into three component functions:

1. $x(t) = 12t$
2. $y(t) = \frac{8t^{3/2}}{}$
3. $z(t) = 3t^2$

Each component describes a coordinate in three-dimensional space, depending on the parameter t.

Domain Considerations

Before attempting to find t, we must consider the domain restrictions imposed by the component functions:

- $x(t) = 12t$: This is a linear function, defined for all real t.
- $y(t) = 8 t^{3/2}$: The expression $t^{3/2}$ involves a fractional power. Specifically, $t^{3/2} = (t^{1/2})^3$. Since the square root function $t^{1/2}$ requires $t \geq 0$, the domain for $y(t)$ is $t \geq 0$.
- $z(t) = 3 t^2$: A quadratic, defined for all real t.

Conclusion: The overall domain of $R(t)$ is $t \geq 0$.

Verifying the Point $\left(3, 1, \frac{3}{16}\right)$

To establish that this point lies on the curve, we need to find $t \geq 0$ such that:

$$
R(t) = (x(t), y(t), z(t)) = \left(3, 1, \frac{3}{16}\right)
$$

which translates to solving the system:

$$
\begin{cases}
12t = 3 \\
8 t^{3/2} = 1 \\
3 t^2 = \frac{3}{16}
\end{cases}
$$

Let's analyze each equation separately.

Step 1: Solve for \(t \) from \(x(t) \)

\[
12t = 3 \Rightarrow t = \frac{3}{12} = \frac{1}{4}
\]

This is straightforward and gives:

\[
t = \frac{1}{4}
\]

Since \(t \geq 0 \), this value is within the domain.

Step 2: Verify \(y(t) \) at \(t = \frac{1}{4} \)

Calculate \(y(t) \):

\[
y(t) = 8 t^{3/2}
\]

Plug in \(t = \frac{1}{4} \):

\[
y\left(\frac{1}{4}\right) = 8 \left(\frac{1}{4} \right)^{3/2}
\]

Recall that:

\[
\left(\frac{1}{4} \right)^{3/2} = \left(\left(\frac{1}{4} \right)^{1/2} \right)^3
\]

Since:

\[
\left(\frac{1}{4} \right)^{1/2} = \frac{1}{2}
\]

then:

\[
\left(\frac{1}{4} \right)^{3/2} = \left(\frac{1}{2} \right)^3 = \frac{1}{8}
\]

Now, multiplying by 8:

\[

$$
8 \times \frac{1}{8} = 1
$$

This matches the y-coordinate of the point, which is 1, confirming consistency.

Step 3: Verify $z(t)$ at $t = \frac{1}{4}$

Calculate $z(t)$:

$$
z(t) = 3 t^2
$$

Plug in $t = \frac{1}{4}$:

$$
z\left(\frac{1}{4}\right) = 3 \times \left(\frac{1}{4} \right)^2 = 3 \times \frac{1}{16} = \frac{3}{16}
$$

which matches exactly the z-coordinate of the given point.

Conclusion:

Since all three component equations are satisfied at $t = \frac{1}{4}$, and this value lies within the domain $t \geq 0$, the point $\left(3, 1, \frac{3}{16}\right)$ indeed lies on the image of the path $R(t)$.

Additional Insights: Parametric Curves and Their Geometrical Significance

This exercise exemplifies a common approach in vector calculus: verifying whether a point belongs to a parametric curve involves solving the component equations simultaneously. Such analysis is crucial in fields like physics (e.g., particle trajectories), computer graphics (e.g., modeling curves and surfaces), and engineering (e.g., motion planning).

Key takeaways include:

- Domain restrictions are vital. In our case, the fractional power in $y(t)$ limited the domain to $t \geq 0$.
- Component-wise solving simplifies the verification process.
- Consistency across components confirms the point's position on the curve.

Final Remarks

Understanding how to analyze parametric paths and verify points on their images deepens one's grasp of multi-variable functions and their geometric interpretations. The process demonstrated here—solving component equations within the domain constraints—serves as a foundational skill in advanced mathematics and applied sciences.

Whether you're plotting complex trajectories in physics, creating smooth curves in computer-aided design, or studying the behavior of functions in calculus, mastering this approach will enhance your analytical toolkit.

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