# Consider The Quadratic Function F(x) = X2 5x + 12. Which Statements Are True About The Function And Its

### Consider The Quadratic Function F(x) = X2 - 5x + 12. Which Statements Are True About The Function And Its

Understanding quadratic functions is essential in algebra, as they appear frequently in various mathematical contexts and real-world applications. The quadratic function F(x) = x2 - 5x + 12 serves as an excellent example to explore fundamental properties such as its graph, roots, vertex, and symmetry. In this article, we will analyze this specific quadratic function to determine which statements about it are true, helping you deepen your comprehension of quadratic functions.

## Analyzing the Basic Form of the Quadratic Function

A quadratic function is generally expressed as F(x) = ax2 + bx + c, where a, b, and c are constants. In our case, the function is F(x) = x2 - 5x + 12, which has a = 1, b = -5, and c = 12.

## Standard Parabolic Shape

- The coefficient a = 1 is positive, indicating that the parabola opens upward.
- The graph of F(x) is a U-shaped curve that extends infinitely upwards.
- The parabola is symmetric about a vertical line known as the axis of symmetry.

## Vertex of the Parabola

- The vertex is the highest or lowest point on the parabola, depending on the direction it opens.
- Since the parabola opens upward, the vertex is the lowest point.

## Finding the Vertex of $F(x) = x^2 - 5x + 12$

The vertex of a quadratic function in standard form can be found using the formula:

xvertex = -b / (2a)

- For F(x), xvertex = -(-5) / (2 1) = 5 / 2 = 2.5

To find the y-coordinate of the vertex, substitute x = 2.5 into the function:

F(2.5) = (2.5)2 - 5(2.5) + 12 = 6.25 - 12.5 + 12 = 5.75

Therefore, the vertex is at (2.5, 5.75), which is the minimum point of the parabola.

# Determining the Roots (Zeros) of the Function

The roots of the quadratic function are the x-values where F(x) = 0, i.e., the points where the parabola intersects the x-axis.

## Using the Quadratic Formula

The quadratic formula is:

x = [-b ± √ (b² - 4ac)] / 2a

Plugging in the values:

x = [5 ± √ ((-5)² - 4 1 12)] / 2

x = [5 ± √ (25 - 48)] / 2

x = [5 ± √ (-23)] / 2

Since the discriminant (b2 - 4ac) is negative (-23), the quadratic has no real roots but two complex conjugate roots.

Implication: The parabola does not touch or cross the x-axis; it lies entirely above the x-axis, except at points with complex x-values.

# Understanding the Discriminant and Its Significance

The discriminant, D = b2 - 4ac, provides vital information about the roots and the graph of the quadratic:

- If D > 0: two distinct real roots, parabola crosses x-axis twice.
- If D = 0: one real root (double root), parabola touches x-axis at vertex.
- If D < 0: no real roots, parabola does not intersect x-axis.

For our function:

D = 25 - 48 = -23 < 0

Therefore, the quadratic has no real solutions, and the parabola does not intersect the x-axis.

# Axis of Symmetry and Its Equation

The axis of symmetry is the vertical line that passes through the vertex, dividing the parabola into mirror images.

- The equation is x = -b / (2a)
- For our function, x = 2.5

This line passes through the vertex at (2.5, 5.75).

## Y-Intercept of the Function

The y-intercept occurs when x = 0:

F(0) = 02 - 50 + 12 = 12

Thus, the parabola crosses the y-axis at (0, 12).

## Summary of True Statements About $F(x) = x^2 - 5x + 12$

Based on the analysis above, we can confirm several true statements:

- The parabola opens upward because the coefficient of $x^2$ is positive.
- The vertex of the parabola is at (2.5, 5.75), which is the minimum point.
- The parabola does not intersect the x-axis since the discriminant is negative, indicating no real roots.
- The axis of symmetry is the vertical line x = 2.5.
- The y-intercept is at (0, 12).
- Since the roots are complex, the parabola lies entirely above the x-axis, except at points with complex x-values.

## Real-World Applications of the Quadratic Function

Quadratic functions like F(x) = x2 - 5x + 12 are used in various fields for modeling and problem-solving:

### Physics and Engineering

- Describing projectile motion, where the path of an object follows a parabolic shape.
- Analyzing the trajectory of objects under gravity.

### Economics and Business

- Modeling profit and revenue functions that depend quadratically on certain variables.
- Determining maximum or minimum values such as profit maximization or cost minimization.

### Architecture and Design

- Designing parabolic arches and structures that require understanding of quadratic curves.

# Conclusion: Key Takeaways About $F(x) = x^2 - 5x + 12$

In summary, the quadratic function F(x) = x2 - 5x + 12 exhibits typical properties of a parabola opening upward, with a vertex at (2.5, 5.75). Its discriminant being negative tells us there are no real roots, meaning the graph does not cross the x-axis. The axis of symmetry is at x = 2.5, and the y-intercept is at (0, 12). Understanding these properties helps in graphing the function accurately and applying it to real-world scenarios. Recognizing which statements about the function are true allows students and professionals alike to interpret quadratic models effectively and utilize their properties in various applications.

## Frequently Asked Questions

### What is the vertex of the quadratic function F(x) = x^2 - 5x + 12?

The vertex is at (2.5, 3.75).

### Is the parabola represented by F(x) = x^2 - 5x + 12 opening upwards or downwards?

It opens upwards because the coefficient of x^2 is positive.

### What is the y-intercept of the function F(x) = x^2 - 5x + 12?

The y-intercept is at (0, 12).

### Does the quadratic function F(x) = x^2 - 5x + 12 have real roots?

Yes, it has real roots at x = 3 and x = 4.

### What is the axis of symmetry for the quadratic function F(x) = x^2 - 5x + 12?

The axis of symmetry is x = 2.5.

### How many times does the graph of F(x) = x^2 - 5x + 12 cross the x-axis?

It crosses the x-axis two times at x = 3 and x = 4.

### What is the minimum value of the quadratic function F(x) = x^2 - 5x + 12?

The minimum value is 3.75 at x = 2.5.

### Can the quadratic function F(x) = x^2 - 5x + 12 be factored easily?

Yes, it factors as (x - 3)(x - 4).

### Is the quadratic function F(x) = x^2 - 5x + 12 convex or concave?

It is convex because the parabola opens upwards.

# Additional Resources

## Consider The Quadratic Function F(x) = x^2 - 5x + 12: An In-Depth Analysis of Its Properties and Significance

Quadratic functions are fundamental in the study of algebra, offering insights into the behavior of parabolic graphs, roots, and vertex forms. The function F(x) = x^2 - 5x + 12 exemplifies these properties vividly. This article provides a comprehensive, detailed exploration of this quadratic function, analyzing its key features, graphing characteristics, roots, vertex, and the implications of various statements about it. Through this analysis, we aim to clarify which assertions about the function are true, providing a robust understanding for students, educators, and enthusiasts alike.

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### Understanding the Basic Structure of the Function

#### The General Form of a Quadratic Function

Quadratic functions are generally expressed in the standard form:

$$ F(x) = ax^2 + bx + c $$

where:
- $ a \neq 0 $,
- $ b $ and $ c $ are real numbers.

In our case, the function is:

$$ F(x) = x^2 - 5x + 12 $$

which has:
- $ a = 1 $,
- $ b = -5 $,
- $ c = 12 $.

This structure indicates a parabola opening upwards (since $ a > 0 $), with specific properties determined by these coefficients.

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### Analyzing the Parabola: Key Features of F(x) = x^2 - 5x + 12

#### 1. Direction of the Parabola

Because the leading coefficient $ a = 1 $ is positive, the parabola opens upward. This means the graph has a minimum point (vertex) and extends infinitely upwards on both sides.

#### 2. Vertex of the Parabola

The vertex represents the minimum point for this upward-opening parabola. To find the vertex:

$$
x_{v} = -\frac{b}{2a}
$$

Plugging in the values:

$$
x_{v} = -\frac{-5}{2 \times 1} = \frac{5}{2} = 2.5
$$

To find the corresponding $y$-coordinate:

$$
F(2.5) = (2.5)^2 - 5 \times 2.5 + 12 = 6.25 - 12.5 + 12 = 5.75
$$

Vertex Coordinates:

$$
(2.5, 5.75)
$$

This point is the parabola's minimum.

3. Axis of Symmetry

The parabola is symmetric about the vertical line passing through the vertex:

$$
x = x_{v} = 2.5
$$

4. Roots of the Function

The roots (or zeros) of the quadratic are solutions to:

$$
x^2 - 5x + 12 = 0
$$

Using the quadratic formula:

$$
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

Plugging in the coefficients:

$$
x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 1 \times 12}}{2 \times 1} = \frac{5 \pm \sqrt{25 - 48}}{2}
$$

$$
x = \frac{5 \pm \sqrt{-23}}{2}
$$

Since the discriminant $\Delta = -23$ is negative, the quadratic has no real roots.

Implication:

- The parabola does not intersect the x-axis.
- The roots are complex conjugates:

$$
x = \frac{5 \pm i \sqrt{23}}{2}
$$

5. Discriminant and Nature of Roots

The discriminant $\Delta = b^2 - 4ac$:

$$
\Delta = 25 - 48 = -23
$$

Since $\Delta < 0$, the roots are complex, and the parabola has no real zeros.

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Graphical Interpretation and Behavior

1. Graph Shape

Since the parabola opens upward and has a vertex at $(2.5, 5.75)$, the graph is a U-shaped curve with its lowest point at that vertex.

2. Range of the Function

Because the parabola opens upward and has a minimum at $y = 5.75$, the range is:

$$
[y, \infty), \quad \text{where } y \geq 5.75
$$

This indicates the function outputs values greater than or equal to 5.75 for all real $x$.

3. Behavior at Extremes

- As $x \to \pm \infty$, $F(x) \to \infty$.
- The minimum value of the function is at the vertex's $y$-coordinate, confirming the range.

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Critical Insights and Statements About the Function

Now, considering the various possible statements about $F(x) = x^2 - 5x + 12$, we analyze which are true:

Statement 1: The quadratic has real roots.

Analysis:
False. As computed, the discriminant is negative ($-23$), so it has no real roots.

Statement 2: The parabola opens downward.

Analysis:
False. Since $a = 1 > 0$, the parabola opens upward.

Statement 3: The vertex is at $(2.5, 5.75)$.

Analysis:
True. Calculated earlier via vertex formula.

Statement 4: The function's minimum value is 5.75.

Analysis:
True. Given the vertex's $y$-coordinate.

Statement 5: The function crosses the y-axis at $(0, c)$.

Analysis:
True. When $x = 0$:

$$
F(0) = 0^2 - 5 \times 0 + 12 = 12
$$

So, it crosses the y-axis at $(0, 12)$.

Statement 6: The axis of symmetry is at $x = 2.5$.

Analysis:
True. Derived from the vertex formula.

Statement 7: The roots are complex conjugates.

Analysis:
True. Discriminant is negative, indicating complex roots.

Statement 8: The quadratic's graph is symmetric about $x = 2.5$.

Analysis:
True. This is a fundamental property of parabolas centered at their axis of symmetry.

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Deeper Mathematical Implications

1. Vertex Form of the Quadratic

Transforming to vertex form:

$$
F(x) = a(x - h)^2 + k
$$

where $(h, k)$ is the vertex.

Given:

$$
F(x) = x^2 - 5x + 12
$$

Complete the square:

$$
F(x) = (x^2 - 5x) + 12
$$

Complete the square:

$$
x^2 - 5x + \left(\frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + 12
= \left( x - \frac{5}{2} \right)^2 - \frac{25}{4} + 12
$$

Express 12 as $\frac{48}{4}$:

$$
= \left( x - \frac{5}{2} \right)^2 + \left( -\frac{25}{4} + \frac{48}{4} \right )
= \left( x - \frac{5}{2} \right)^2 + \frac{23}{4}
$$

Vertex form:

$$
F(x) = \left( x - \frac{5}{2} \right)^2 + \frac{23}{4}
$$

This confirms the vertex at $\left( \frac{5}{2}, \frac{23}{4} \right)$, aligning with previous calculations.

2. Implications of the Vertex Form

- Minimum value: $\frac{23}{4} = 5.75$, matching earlier results.
- Graph symmetry: About the line $x = \frac{5}{2}$.

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Practical Applications and Significance

Understanding the properties of this quadratic function extends beyond pure mathematics into practical domains:

- Physics: Parabolic trajectories, such as projectile motion.
- Engineering: Structural design involving parabolic arches.
- Economics: Cost and profit functions often modeled with quadratics.
- Computer Graphics: Parabolic curves in rendering and design.

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Summary of Key Takeaways

| Property | Statement | True/False | Explanation |
|---|---|---|---|
| Roots | The quadratic has real roots | False | Discriminant negative ($-23$) |
| Opening direction | Parabola opens downward | False | $a = 1 > 0$ |
| Vertex | Located at $(2.5, 5.75)$ | True | Calculated via vertex

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