Determine All Solutions Of The Given Equation. Express Your Answer(s) Using Radian Measure. (Select All

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Solving trigonometric equations is a fundamental skill in mathematics, especially in calculus, physics, and engineering. These equations often involve sine, cosine, tangent, and other trigonometric functions, and their solutions can be infinitely many due to the periodic nature of these functions. When tasked with determining all solutions of a given equation and expressing them in radians, it is essential to follow systematic steps, understand the periodicity, and recognize the principal values and their general solutions. This article provides a comprehensive guide on how to approach such problems effectively, including detailed examples to illustrate the process.

Understanding the Basics of Trigonometric Equations

Periodic Nature of Trigonometric Functions

- The sine and cosine functions are periodic with a period of 2π.
- The tangent function has a period of π.
- Because of their periodicity, if x is a solution, then $x + 2k\pi$ (for sine and cosine) or $x + k\pi$ (for tangent), where k is any integer, is also a solution.

Principal Values and General Solutions

- The principal value is usually found within a specific interval, such as $[0, 2\pi)$ or $-\pi < x \leq \pi$.
- The general solution accounts for all possible solutions by adding the period multiplied

by an integer to the principal value.

- Expressing solutions generally involves variables like $k \in \mathbb{Z}$ to denote all integers.

Step-by-Step Approach to Solving Trigonometric Equations

Step 1: Simplify the Equation

- Use identities to rewrite the equation in a more manageable form.
- Combine like terms or factor expressions when possible.

Step 2: Isolate the Trigonometric Function

- If necessary, manipulate the equation to have a single trigonometric function on one side.
- For example, rewrite $2\sin x = 1$ as $\sin x = \frac{1}{2}$.

Step 3: Find Principal Solutions

- Use inverse trigonometric functions or known reference angles to find solutions within the principal interval.
- For $\sin x = a$, find $x = \sin^{-1}(a)$ within $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

Step 4: Determine the General Solution

- Extend the principal solutions to all solutions by adding the relevant period.
- For example, solutions for $\sin x = a$ are $x = \arcsin a + 2k\pi$ and $x = \pi - \arcsin a + 2k\pi$.

Step 5: Express All Solutions in Radians

- Write solutions explicitly, including the variable $k \in \mathbb{Z}$.
- Ensure solutions cover all possible solutions, considering the periodicity.

Common Types of Trigonometric Equations and Their Solutions

Equations Involving Sine

- Example: $\sin x = a$

- Principal solutions:
- $x = \arcsin a$
- $x = \pi - \arcsin a$

- General solutions:
- $x = \arcsin a + 2k\pi$
- $x = \pi - \arcsin a + 2k\pi$

Equations Involving Cosine

- Example: $\cos x = a$

- Principal solutions:
- $x = \arccos a$
- $x = -\arccos a + 2k\pi$ or equivalently $x = 2\pi - \arccos a$

- General solutions:
- $x = \arccos a + 2k\pi$
- $x = -\arccos a + 2k\pi$

Equations Involving Tangent

- Example: $\tan x = a$

- Principal solution:
- $x = \arctan a$

- General solution:
- $x = \arctan a + k\pi$

Illustrative Examples

Example 1: Solve $\sin x = \frac{1}{2}$ for all x in radians

1. Identify the reference angle:
   $\sin x = \frac{1}{2}$ corresponds to $x = \frac{\pi}{6}$ in the principal interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

2. Find all solutions:
   Since sine is positive, solutions are:

$$
x = \frac{\pi}{6} + 2k\pi \quad \text{and} \quad x = \pi - \frac{\pi}{6} + 2k\pi = \frac{5\pi}{6} + 2k\pi, \quad k \in \mathbb{Z}.
$$

3. Express the complete solution set:
$$
\boxed{
x = \frac{\pi}{6} + 2k\pi \quad \text{or} \quad x = \frac{5\pi}{6} + 2k\pi, \quad k \in \mathbb{Z}.
}
$$

Example 2: Solve $\cos x = -\frac{\sqrt{2}}{2}$ for all x in radians

1. Identify reference angle:
$\cos x = -\frac{\sqrt{2}}{2}$ corresponds to $x = \frac{\pi}{4}$ in the principal interval.

2. Determine solutions:
Cosine is negative in the second and third quadrants, so solutions are:
$$
x = \pi - \frac{\pi}{4} = \frac{3\pi}{4}, \quad \text{and} \quad x = \pi + \frac{\pi}{4} = \frac{5\pi}{4}.
$$

3. General solutions:
$$
x = \frac{3\pi}{4} + 2k\pi, \quad \text{and} \quad x = \frac{5\pi}{4} + 2k\pi, \quad k \in \mathbb{Z}.
$$

4. Final answer:
$$
\boxed{
x = \frac{3\pi}{4} + 2k\pi, \quad x = \frac{5\pi}{4} + 2k\pi, \quad k \in \mathbb{Z}.
}
$$

Example 3: Solve $\tan x = 1$ for all x in radians

1. Principal solution:

$\tan x = 1$ implies $x = \frac{\pi}{4}$ in $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

2. General solutions:
   Since tangent is periodic with period π,
   $$
   x = \frac{\pi}{4} + k\pi, \quad k \in \mathbb{Z}.
   $$

3. Answer:
   $$
   \boxed{
   x = \frac{\pi}{4} + k\pi, \quad k \in \mathbb{Z}.
   }
   $$

Special Cases and Additional Tips

Equations Requiring Algebraic Manipulation

- Sometimes the given equation involves multiple trigonometric functions or algebraic expressions.
- Use identities such as $\sin^2 x + \cos^2 x = 1$ or double-angle formulas to simplify.

Handling Quadratic Trigonometric Equations

- Convert equations into quadratic form:
- Example: $2\sin^2 x - \sin x - 1 = 0$.
- Solve as a quadratic in $\sin x$:
- $\sin x = \frac{1 \pm \sqrt{1 - 4 \times 2 \times (-1)}}{2 \times 2}$.

Frequently Asked Questions

How do I find all solutions to the equation sin(x) = 1/2 in radians?

The solutions are $x = \pi/6 + 2\pi k$ and $x = 5\pi/6 + 2\pi k$, where k is any integer.

What is the general solution for the equation cos(x) = -√2/2 in radians?

The solutions are $x = 3\pi/4 + 2\pi k$ and $x = 5\pi/4 + 2\pi k$, where k is any integer.

How do I determine all solutions for the equation tan(x) = 1 in radians?

The general solutions are $x = \pi/4 + \pi k$, where k is any integer.

When solving equations like sin(2x) = 0 in radians, what steps should I follow?

First, solve $\sin(2x) = 0$, which gives $2x = n\pi$, so $x = n\pi/2$, where n is any integer. Then, list all solutions accordingly.

How can I express the solutions of the equation 2cos(x) - 1 = 0 in radians?

Rearranged as $\cos(x) = 1/2$, so the solutions are $x = \pi/3 + 2\pi k$ and $x = 5\pi/3 + 2\pi k$, with k any integer.

What are the solutions for the equation sin^2(x) = 1/2 in radians?

Since $\sin^2(x) = 1/2$, then $\sin(x) = \pm\sqrt{1/2} = \pm 1/\sqrt{2} = \pm\sqrt{2}/2$. The solutions are $x = \pi/4 + 2\pi k$, $3\pi/4 + 2\pi k$, $5\pi/4 + 2\pi k$, and $7\pi/4 + 2\pi k$.

How do I approach solving equations involving multiple trigonometric functions, like sin(x) + cos(x) = 0, in radians?

Use identities or express in terms of a single function: for example, divide both sides by $\cos(x)$ (where $\cos(x) \neq 0$) to get $\tan(x) = -1$, then solve $\tan(x) = -1$, leading to $x = -\pi/4 + \pi k$.

What is the general solution for equations like sin(3x) = 0 in radians?

Solve $\sin(3x) = 0$, which gives $3x = n\pi$, so $x = n\pi/3$, where n is any integer.

Additional Resources

Determine All Solutions Of The Given Equation. Express Your Answer(s) Using Radian Measure is a fundamental task in trigonometry that often appears in advanced math courses, standardized tests, and real-world problem-solving scenarios. Understanding how to find all solutions to a trigonometric equation—not just a single solution—is crucial because these functions are periodic, meaning they repeat their values over regular intervals. Mastery of this process allows students and professionals to accurately analyze and interpret the behavior of sine, cosine, tangent, and other related functions across their entire domains.

In this guide, we will walk through a comprehensive, step-by-step approach to determine all solutions of a given trigonometric equation and how to express these solutions using radian measure. Whether you're tackling a homework problem or preparing for an exam, this structured approach will help clarify the process and ensure you don’t miss any solutions.

Understanding the Basics of Trigonometric Equations

Before diving into solving specific equations, it's important to understand some foundational concepts:

- Periodic Nature of Trig Functions:
Trigonometric functions repeat their values over specific intervals:
- Sine and cosine have a period of 2π.
- Tangent and cotangent have a period of π.

- Principal Values and General Solutions:
When solving, we often find a principal solution—the solution within a specific interval—and then extend it to find all solutions using the function's period.

- Expressing Solutions in Radians:
The radian measure is preferred because it relates directly to the unit circle, simplifying the interpretation of angles and their periodicity.

Step-by-Step Approach to Solving Trigonometric Equations

Let's outline the comprehensive method:

Step 1: Write the Equation Clearly

Identify the form of the given equation. For example:

- Basic form: $\sin x = a$
- More complex forms: $2\cos x + 1 = 0$, $\tan 3x = \sqrt{3}$

Understanding the structure guides the solving process.

Step 2: Isolate the Trigonometric Function

Manipulate the equation into a standard form where the trigonometric function is isolated:

- For example, if $2\cos x + 1 = 0$, then $\cos x = -\frac{1}{2}$.

Step 3: Find the Principal Solution(s)

Determine the solutions within a principal interval, typically:

- $x \in [0, 2\pi)$ for sine and cosine.
- $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2} \right)$ for tangent, depending on the context.

Use inverse functions or known values to find solutions:

- Using Inverse Functions:
$x = \arcsin a$, $x = \arccos a$, $x = \arctan a$.

- Using Known Values and Unit Circle:
Recall common angles (e.g., $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$) and their sine, cosine, and tangent values.

Example:
Solve $\sin x = \frac{1}{2}$:

- Principal solutions:
$x = \arcsin \frac{1}{2} = \frac{\pi}{6}$ (since $\sin \frac{\pi}{6} = \frac{1}{2}$).

- Additional solutions within the principal interval:
$x = \frac{5\pi}{6}$, because sine is positive in the first and second quadrants.

Step 4: Find All Solutions Using Periodicity

Once the principal solutions are identified, generate all solutions by adding integer multiples of the function's period:

- For $\sin x = a$, solutions are:

$x = \text{principal solutions} + 2\pi n$, where $n \in \mathbb{Z}$.

- For $\cos x = a$:

$x = \pm \arccos a + 2\pi n$.

- For $\tan x = a$:

$x = \arctan a + \pi n$.

Note:
Be sure to include all solutions that satisfy the original equation, considering the symmetry and periodicity.

Step 5: Express All Solutions in Radian Measure

Write the solutions explicitly, incorporating the period:

Example (continued):
Solutions to $\sin x = \frac{1}{2}$:

$$
x = \frac{\pi}{6} + 2\pi n \quad \text{and} \quad x = \frac{5\pi}{6} + 2\pi n, \quad n \in \mathbb{Z}.
$$

Similarly, solutions to equations involving cosine or tangent will follow their periodicity formulas.

Special Cases and Tips

- Equations with Multiple Angles:
Equations like $\sin 2x = \frac{\sqrt{3}}{2}$ require substitution and solving for the inner function, then expanding solutions.

- Quadratic Trigonometric Equations:
Equations like $\sin^2 x + \sin x - 2 = 0$ can be tackled like algebraic quadratics, setting $y = \sin x$.

- Domain Restrictions:
Always check whether solutions fall within the specified domain or the principal interval.

- Use of Trigonometric Identities:
Simplify complex equations using identities such as:

$$
\sin^2 x + \cos^2 x = 1,
$$

or double-angle formulas.

Worked Example: Solving a Complete Equation

Problem:
Determine all solutions of the equation:

\[
2 \sin x \cos x = \frac{\sqrt{3}}{2}
\]

Step 1: Recognize the identity:
\(2 \sin x \cos x = \sin 2x \).

Step 2: Rewrite the equation:
\[
\sin 2x = \frac{\sqrt{3}}{2}
\]

Step 3: Find principal solutions for \(2x \):

\[
2x = \arcsin \left(\frac{\sqrt{3}}{2} \right) = \frac{\pi}{3}
\]

and

\[
2x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}
\]

since \(\sin \theta = \frac{\sqrt{3}}{2} \) at \(\theta = \frac{\pi}{3} \) and \(\theta = \frac{2\pi}{3} \).

Step 4: Find all solutions for \(2x \):

\[
2x = \frac{\pi}{3} + 2 \pi n \quad \text{and} \quad 2x = \frac{2\pi}{3} + 2 \pi n, \quad n \in \mathbb{Z}
\]

Step 5: Solve for \(x \):

\[
x = \frac{\pi}{6} + \pi n \quad \text{and} \quad x = \frac{\pi}{3} + \pi n, \quad n \in \mathbb{Z}
\]

Final Answer:

\[
\boxed{

x = \frac{\pi}{6} + \pi n \quad \text{or} \quad x = \frac{\pi}{3} + \pi n, \quad n \in \mathbb{Z}
}
\]

Expressed in radian measure, these are the solutions valid for all integers \(n \).

Summary and Final Tips

- Always start by simplifying the equation to a basic trigonometric form.
- Find the principal solutions within the primary interval using inverse functions or known angles.
- Generate all solutions by adding multiples of the relevant period (\(2\pi \) for sine and cosine, \(\pi \) for tangent).
- Express your solutions explicitly, ensuring they are in radian measure.
- Verify solutions within the context or specified domain, if any are provided.

By following this structured approach, you can confidently determine all solutions of the given equation and express your answers in radian measure. Practice with various equations to develop intuition, and always double-check your solutions for completeness.

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