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Determine If The Points A(1,1,2), B(2,3,-2), C(3,5,-6) And D(1,-2,-2) Lie In The Same Plane. Select The

Understanding whether four points in three-dimensional space lie on the same plane is a fundamental problem in geometry, with applications spanning fields such as computer graphics, engineering, physics, and mathematics. This article provides a comprehensive guide to determine if the points A(1,1,2), B(2,3,-2), C(3,5,-6), and D(1,-2,-2) are coplanar. We will explore key concepts, step-by-step procedures, and the mathematical tools necessary to arrive at an accurate conclusion. Whether you're a student, educator, or professional, mastering this process enhances spatial reasoning and problem-solving skills.

Understanding the Concept of Coplanarity

What Does It Mean for Points to Be Coplanar?

- Points are coplanar if they all lie on the same flat surface or plane.
- In three-dimensional space, any three non-collinear points define a unique plane.
- The key question: Does the fourth point also lie on this same plane?

Significance in Geometry and Applications

- Determines structural integrity in engineering designs.
- Assists in computer graphics when rendering 3D models.
- Useful in navigation, robotics, and spatial analysis.

Mathematical Tools to Test Coplanarity

Vectors and the Scalar Triple Product

- Vectors are essential in analyzing spatial relationships.
- The scalar triple product of three vectors is used to check if a set of points are coplanar.
- Definition: For vectors u, v, and w, the scalar triple product is u · (v ×

w).
- Interpretation: The scalar triple product gives the volume of the parallelepiped formed by the three vectors. If the volume is zero, the vectors are coplanar.

Steps to Test Coplanarity Using the Scalar Triple Product

1. Choose one point as a reference point (commonly A).
2. Construct vectors from this reference point to the other points:
- AB = B - A
- AC = C - A
- AD = D - A
3. Compute the scalar triple product: AB · (AC × AD).
4. If the result is zero, points A, B, C, D are coplanar; otherwise, they are not.

Applying the Method to Points A, B, C, and D

Step 1: Write Down Coordinates and Construct Vectors

- Point A: (1, 1, 2)
- Point B: (2, 3, -2)
- Point C: (3, 5, -6)
- Point D: (1, -2, -2)

Construct vectors from A:
- AB = B - A = (2 - 1, 3 - 1, -2 - 2) = (1, 2, -4)
- AC = C - A = (3 - 1, 5 - 1, -6 - 2) = (2, 4, -8)
- AD = D - A = (1 - 1, -2 - 1, -2 - 2) = (0, -3, -4)

Step 2: Compute the Cross Product of AC and AD

- Cross product AC × AD:

\[
\begin{aligned}
AC \times AD &=
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
2 & 4 & -8 \\
0 & -3 & -4 \\
\end{vmatrix} \\
&= \mathbf{i}(4 \times -4 - (-8) \times -3) - \mathbf{j}(2 \times -4 - (-8) \times 0) + \mathbf{k}(2 \times -3 - 4 \times 0) \\
&= \mathbf{i}(-16 - 24) - \mathbf{j}(-8 - 0) + \mathbf{k}(-6 - 0) \\

&= \mathbf{i}(-40) - \mathbf{j}(-8) + \mathbf{k}(-6) \\
&= (-40, 8, -6)
\end{aligned}
\]

Step 3: Compute the Scalar Triple Product AB · (AC × AD)

- Dot product:

\[
AB \cdot (AC \times AD) = (1, 2, -4) \cdot (-40, 8, -6) = (1)(-40) + (2)(8) + (-4)(-6) = -40 + 16 + 24 = 0
\]

Interpreting the Result

- Since the scalar triple product equals zero, the four points A, B, C, and D are coplanar.
- This confirms that all four points lie on the same plane.

Additional Verification Methods

Equation of the Plane

- To further validate, find the equation of the plane passing through three points, then verify if the fourth point satisfies it.
- Procedure:
1. Use points A, B, and C to find the plane equation.
2. Plug in the coordinates of D into the plane equation.
3. If the equation holds true, D lies on the plane.

Calculating the Plane Equation

- Use the vectors AB and AC to find the normal vector n:

\[
n = AB \times AC
\]

- Cross product:

\[
\begin{aligned}

```
AB &= (1, 2, -4) \\
AC &= (2, 4, -8) \\
n &= AB \times AC \\
&= \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
1 & 2 & -4 \\
2 & 4 & -8 \\
\end{vmatrix} \\
&= \mathbf{i}(2 \times -8 - (-4) \times 4) - \mathbf{j}(1 \times -8 - (-4)
\times 2) + \mathbf{k}(1 \times 4 - 2 \times 2) \\
&= \mathbf{i}(-16 - (-16)) - \mathbf{j}(-8 - (-8)) + \mathbf{k}(4 - 4) \\
&= \mathbf{i}(0) - \mathbf{j}(0) + \mathbf{k}(0) \\
&= (0, 0, 0)
\end{aligned}
\]
```

- Since the cross product yields the zero vector, points A, B, and C are collinear, indicating they lie on a line rather than defining a unique plane. This suggests the points are collinear, and D, lying on the same line, is necessarily coplanar.

Note: The zero vector indicates the points are collinear; thus, the initial assumption about defining a plane with A, B, and C needs re-evaluation. However, in the previous step, the scalar triple product was zero, confirming coplanarity, which aligns with the fact that three collinear points are trivially coplanar.

Conclusion and Summary

- The primary method to determine if four points are coplanar involves calculating the scalar triple product of vectors derived from these points.
- In our specific case, the scalar triple product computed from points A, B, C, and D equals zero, confirming they are coplanar.
- Additional verification via the plane equation supports this conclusion.
- Recognizing the geometric relationships among points enhances understanding and supports accurate spatial analysis.

Practical Tips for Students and Professionals

- Always choose a convenient reference point when constructing vectors.
- Use the scalar triple product as a quick test for coplanarity.
- When points are collinear, they are trivially coplanar, but the method may need adaptation.
- Verify results with supplementary methods such as plane equations for confirmation.
- Use graphing tools or software for visual verification when possible.

Final Remarks

Determining whether points lie in the same plane is a vital skill in geometry, with broad applications. By mastering the use of vectors, cross products, and the scalar triple product, you can efficiently analyze spatial point configurations. The example of points A(1,1,2), B(2,3,-2), C(3,5,-6), and D(1,-2,-2) demonstrates a straightforward approach, confirming their coplanarity through calculated zero volume. Integrating mathematical rigor with visualization enhances spatial reasoning and problem-solving effectiveness in various contexts.

Keywords: coplanar points, scalar triple product, vectors in

Frequently Asked Questions

How do you determine if four points A, B, C, and D are coplanar in 3D space?

Calculate the scalar triple product of vectors AB, AC, and AD. If the scalar triple product is zero, the points are coplanar; otherwise, they are not.

What is the first step to check if points A(1,1,2), B(2,3,-2), C(3,5,-6), and D(1,-2,-2) lie in the same plane?

Find vectors AB, AC, and AD by subtracting coordinates of point A from points B, C, and D respectively.

How do you compute the vectors AB, AC, and AD for the given points?

AB = B - A = (2-1, 3-1, -2-2) = (1, 2, -4); AC = C - A = (3-1, 5-1, -6-2) = (2, 4, -8); AD = D - A = (1-1, -2-1, -2-2) = (0, -3, -4).

What is the scalar triple product and how is it used here?

The scalar triple product is given by AB · (AC × AD). If it equals zero, points are coplanar; if not, they are not.

How do you compute the cross product AC × AD for the

points?

Calculate the determinant: |i j k|; |2 4 -8|; |0 -3 -4|. The resulting vector is ((4)(-4)-(-8)(-3), - (2)(-4)-(-8)(0), (2)(-3)-4(0)).

What is the value of the scalar triple product for the given points?

First, compute AC × AD, then take the dot product with AB. If the result is zero, points are coplanar; otherwise, they are not.

What is the final conclusion if the scalar triple product is zero for points A, B, C, and D?

The points lie in the same plane, confirming they are coplanar.

Based on the calculations, do the points A(1,1,2), B(2,3,-2), C(3,5,-6), and D(1,-2,-2) lie in the same plane?

Yes, after performing the scalar triple product calculation, it is found to be zero, indicating all four points are coplanar.

Additional Resources

Determine If The Points A(1,1,2), B(2,3,-2), C(3,5,-6), And D(1,-2,-2) Lie In The Same Plane

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Introduction

In the realm of three-dimensional geometry, one fundamental question often encountered is whether a set of points in space lie on the same plane. This inquiry is not merely academic; it underpins numerous applications across engineering, computer graphics, navigation, and physics. Determining coplanarity involves a blend of analytical geometry techniques, vector calculus, and algebraic reasoning.

This article offers a comprehensive investigation into the problem: Do the points A(1,1,2), B(2,3,-2), C(3,5,-6), and D(1,-2,-2) lie in the same plane? We will explore the problem systematically, beginning with foundational concepts, followed by step-by-step calculations, and culminating in a conclusion supported by rigorous mathematical proof.

The Geometrical Framework: Understanding Coplanarity

Before delving into calculations, it is crucial to understand what it means for points to be coplanar.

Definition of Coplanarity

A set of points in three-dimensional space are coplanar if there exists a single plane that contains all of them. Equivalently, for any four points A, B, C, D, they are coplanar if and only if the volume of the tetrahedron formed by these points is zero.

This volume can be calculated using the scalar triple product:

$$
V = \frac{1}{6} |\vec{AB} \cdot (\vec{AC} \times \vec{AD})|
$$

where $\vec{AB}$, $\vec{AC}$, and $\vec{AD}$ are vectors originating from a common point, typically A.

If this volume $V = 0$, then the points are coplanar; otherwise, they are not.

Step 1: Selecting a Reference Point and Calculating Vectors

To determine coplanarity, we choose point $A(1,1,2)$ as a reference.

Calculate vectors:

- $\vec{AB} = B - A = (2 - 1, 3 - 1, -2 - 2) = (1, 2, -4)$
- $\vec{AC} = C - A = (3 - 1, 5 - 1, -6 - 2) = (2, 4, -8)$
- $\vec{AD} = D - A = (1 - 1, -2 - 1, -2 - 2) = (0, -3, -4)$

Step 2: Computing the Cross Product $\vec{AC} \times \vec{AD}$

The cross product of two vectors $\vec{u} = (u_1, u_2, u_3)$ and $\vec{v} = (v_1, v_2, v_3)$ is:

$$
\vec{u} \times \vec{v} = (u_2 v_3 - u_3 v_2, u_3 v_1 - u_1 v_3, u_1 v_2 - u_2 v_1)
$$

Compute:

\[

$$
\vec{AC} \times \vec{AD} =
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
2 & 4 & -8 \\
0 & -3 & -4 \\
\end{vmatrix}
$$

Calculations:

- i-component: $(4)(-4) - (-8)(-3) = -16 - 24 = -40$
- j-component: $-[(2)(-4) - (-8)(0)] = -[-8 - 0] = 8$
- k-component: $(2)(-3) - (4)(0) = -6 - 0 = -6$

Thus,

$$
\vec{AC} \times \vec{AD} = (-40, 8, -6)
$$

Step 3: Calculating the Scalar Triple Product

Next, compute the dot product $\vec{AB} \cdot (\vec{AC} \times \vec{AD})$:

$$
\vec{AB} \cdot (\vec{AC} \times \vec{AD}) = (1)(-40) + (2)(8) + (-4)(-6) = -40 + 16 + 24 = 0
$$

Since the scalar triple product is zero, the volume of the tetrahedron formed by points A, B, C, D is:

$$
V = \frac{1}{6} |0| = 0
$$

Conclusion: Are the Points Coplanar?

The zero value of the scalar triple product indicates that the four points $A(1,1,2), B(2,3,-2), C(3,5,-6), D(1,-2,-2)$ lie in the same plane.

Additional Verification: Geometric Interpretation

While the scalar triple product confirms coplanarity, it's instructive to interpret the result geometrically.

- Collinearity of Points B, C: The points B and C are aligned along the vector $\vec{AB}$, scaled by factors that preserve linear dependence.
- Dependence of Vectors: The vectors $\vec{AB}$, $\vec{AC}$, and $\vec{AD}$ are linearly dependent, reinforcing the conclusion.

Practical Implications and Broader Context

Understanding the coplanarity of points is vital in various fields:

- Engineering and Structural Design: Ensuring components lie in the same plane for stability.
- Computer Graphics: Rendering 3D models where surfaces are defined by coplanar points.
- Navigation and Geospatial Analysis: Determining planar regions or features.
- Mathematical Education: Teaching concepts of vectors, determinants, and spatial reasoning.

Moreover, the technique employed—scalar triple product—is a standard, robust method for such determinations, applicable to larger point sets with appropriate modifications.

Summary

- The points $A(1,1,2)$, $B(2,3,-2)$, $C(3,5,-6)$, and $D(1,-2,-2)$ are found to be coplanar.
- The key calculation involved vectors from a common point and the scalar triple product.
- The scalar triple product equaled zero, confirming coplanarity.

This investigation exemplifies the power of vector calculus in solving spatial geometry problems, offering clarity and precision that are essential for both theoretical insights and practical applications.

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Final Remarks

Determining the coplanarity of points is a foundational skill in three-dimensional geometry, blending algebraic computation with geometric intuition. Through systematic vector analysis, one can confidently assess spatial relationships, a process that underpins much of modern science and engineering.

Determine If The Points A 1 1 2 B 2 3 2 C 3 5 6 And D 1 2 2 Lie In The Same Plane Select The

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