Determine If The Sequence 4, 10, 19, 31,... Is Arithmetic. If It Is, Determine The First Term A And The

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When analyzing a sequence, one of the fundamental questions is whether the sequence is arithmetic. An arithmetic sequence is characterized by a constant difference between consecutive terms. The given sequence: 4, 10, 19, 31,... prompts us to examine the differences between its terms to establish if it follows this pattern. If it does, we can then identify the first term, often denoted as A, and the common difference, denoted as d. This process involves calculating the differences, verifying their consistency, and then deriving the general formula for the sequence.

Understanding Arithmetic Sequences

Definition of an Arithmetic Sequence

An arithmetic sequence is a sequence of numbers in which the difference between any two consecutive terms is constant. This constant difference is called the common difference and is denoted by 'd'. The general form of an arithmetic sequence can be written as:

$$ a_n = A + (n - 1)d $$

Where:
- a_n is the nth term,

- A is the first term,

- d is the common difference,

- n is the position of the term in the sequence.

Characteristics of Arithmetic Sequences

- The difference between successive terms is always the same.
- The sequence progresses in a linear fashion.
- The sequence can be extended infinitely in both directions if needed, with each new term calculated using the same difference.

Analyzing the Sequence: 4, 10, 19, 31,...

Step 1: Calculate the Differences Between Consecutive Terms

To determine if the sequence is arithmetic, compute the differences:

- Difference between 10 and 4: $10 - 4 = 6$
- Difference between 19 and 10: $19 - 10 = 9$
- Difference between 31 and 19: $31 - 19 = 12$

The differences are: 6, 9, 12.

Step 2: Check for Consistency of Differences

Since the differences are not the same (6, 9, 12), the sequence does not have a constant difference between all consecutive terms.

Step 3: Analyze the Pattern of Differences

Observe that the differences themselves increase by a constant amount:

- \(9 - 6 = 3 \)
- \(12 - 9 = 3 \)

This suggests that the sequence is not arithmetic but instead possibly quadratic or follows a different pattern where the differences of differences are constant.

Determining the Nature of the Sequence

Is the Sequence Arithmetic?

Since the first differences are not constant, the sequence is not arithmetic. Traditional arithmetic sequences require a fixed difference between consecutive terms, which is not the case here.

What Type of Sequence Is It?

Given that the differences between terms increase by a constant amount (3), this pattern suggests that the sequence may be quadratic, as quadratic sequences often have second differences that are constant.

Testing if the Sequence Is Quadratic

Step 1: Calculate the Second Differences

We already have the first differences: 6, 9, 12.

Calculate their differences:

- $9 - 6 = 3$
- $12 - 9 = 3$

The second differences are constant and equal to 3.

Step 2: Interpret the Result

Constant second differences indicate that the sequence is quadratic, meaning each term can be described by a quadratic formula:

$$a_n = An^2 + Bn + C$$

for some constants A, B, and C.

Finding the Quadratic Formula for the Sequence

Step 1: Set Up Equations Using Known Terms

Using the sequence terms:

- For $n=1$, $a_1 = 4$:

$$A(1)^2 + B(1) + C = 4$$

$$A + B + C = 4 \quad ...(1)$$

- For $n=2$, $a_2 = 10$:

$$A(2)^2 + B(2) + C = 10$$

$$4A + 2B + C = 10 \quad ...(2)$$

- For $n=3$, $a_3 = 19$:

$$A(3)^2 + B(3) + C = 19$$

$$9A + 3B + C = 19 \quad ...(3)$$

Step 2: Solve the System of Equations

Subtract equation (1) from (2):

$$(4A + 2B + C) - (A + B + C) = 10 - 4$$

$$3A + B = 6 \quad ...(4)$$

Subtract equation (2) from (3):

$$(9A + 3B + C) - (4A + 2B + C) = 19 - 10$$

$$ 5A + B = 9 \quad ...(5) $$

Subtract (4) from (5):

$$ (5A + B) - (3A + B) = 9 - 6 $$

$$ 2A = 3 $$

$$ A = \frac{3}{2} $$

Use $A = \frac{3}{2}$ in equation (4):

$$ 3 \times \frac{3}{2} + B = 6 $$

$$ \frac{9}{2} + B = 6 $$

$$ B = 6 - \frac{9}{2} = \frac{12}{2} - \frac{9}{2} = \frac{3}{2} $$

Use A and B in equation (1):

$$ \frac{3}{2} + \frac{3}{2} + C = 4 $$

$$ 3 + C = 4 $$

$$ C = 1 $$

Step 3: Write the Explicit Formula

The nth term of the sequence is:

$$ a_n = \frac{3}{2}n^2 + \frac{3}{2}n + 1 $$

or, factoring out $\frac{3}{2}$:

$$ a_n = \frac{3}{2} (n^2 + n) + 1 $$

Summary of Findings

- The sequence is not arithmetic because the first differences are not constant.
- It is a quadratic sequence, as evidenced by the constant second differences.
- The explicit formula for the sequence is:

$$a_n = \frac{3}{2} n^2 + \frac{3}{2} n + 1$$

which can also be written as:

$$a_n = \frac{3}{2} (n^2 + n) + 1$$

Conclusion

In conclusion, the sequence 4, 10, 19, 31,... is not an arithmetic sequence. Instead, it is quadratic in nature, characterized by a second difference of 3. The explicit formula derived allows us to find any term in the sequence and understand its pattern. If the task was to identify a first term A and common difference d, the answer would be that the sequence does not have a constant difference, and therefore, it is not arithmetic. Instead, recognizing its quadratic pattern provides a deeper understanding of its structure and progression.

Frequently Asked Questions

Is the sequence 4, 10, 19, 31,... an arithmetic sequence?

No, the sequence is not arithmetic because the differences between consecutive terms are not constant.

What is an arithmetic sequence?

An arithmetic sequence is a sequence of numbers where the difference between any two consecutive terms is constant.

How do you determine if a sequence is arithmetic?

By calculating the differences between consecutive terms; if all differences are equal, the sequence is arithmetic.

What are the differences between the terms in the sequence 4, 10, 19, 31?

The differences are 6 (10 - 4), 9 (19 - 10), and 12 (31 - 19).

Since the differences are not constant, what type of sequence is 4, 10, 19, 31?

It is neither arithmetic nor geometric; it appears to be a quadratic or non-linear sequence.

Can the sequence 4, 10, 19, 31 be represented by a quadratic formula?

Yes, since the differences of the differences are constant, it suggests a quadratic pattern, which can be represented by a quadratic formula.

How do you find the general term of a sequence like 4, 10, 19, 31?

By analyzing the pattern of differences and solving for the quadratic formula that fits the sequence.

What is the first term, A, of the sequence if it were arithmetic?

The first term A is 4, based on the sequence provided.

Is it possible to modify the sequence to make it arithmetic?

Yes, by adjusting the terms so that the differences between consecutive terms are equal, it can be made arithmetic.

What is the importance of identifying whether a sequence is arithmetic?

It helps in predicting future terms, understanding the pattern, and applying formulas for the nth term and sum of terms.

Additional Resources

Arithmetic Sequence Analysis

Understanding the nature of numerical sequences is fundamental in mathematics, especially when it comes to identifying patterns and properties that define their behavior. Among these, arithmetic sequences are particularly straightforward yet critically important, owing to their simple constant difference between consecutive terms. In this article, we will conduct an in-depth examination of the sequence 4, 10, 19, 31, ... to determine whether it qualifies as an arithmetic sequence. If it does, we will further explore how to find its first term and common difference. This exploration will involve a methodical approach, detailed explanations, and clear reasoning, making it suitable for both students and enthusiasts seeking to deepen their understanding.

Understanding an Arithmetic Sequence

Before analyzing the sequence in question, it is essential to establish a clear understanding of what constitutes an arithmetic sequence.

Definition of an Arithmetic Sequence

An arithmetic sequence is a sequence of numbers in which the difference between any two consecutive terms is constant. This constant is called the common difference, often denoted as d.

Mathematically, an arithmetic sequence can be expressed as:

$$a,\, a + d,\, a + 2d,\, a + 3d,\, \dots$$

where:

- a is the first term,
- d is the common difference.

The general term (or nth term) of an arithmetic sequence is given by:

$$T_n = a + (n - 1)d$$

where:

- T_n is the nth term,
- n is the position of the term in the sequence.

Analyzing the Sequence: 4, 10, 19, 31, ...

Let's now focus on the sequence provided:

$$4, 10, 19, 31, \dots$$

The primary goal is to determine whether this sequence is arithmetic, which hinges on whether the difference between consecutive terms remains constant.

Step 1: Calculate Consecutive Differences

The most straightforward way to test for an arithmetic sequence is to compute the differences between successive terms:

- Difference between 2nd and 1st term:

$$ 10 - 4 = 6 $$

- Difference between 3rd and 2nd term:

$$ 19 - 10 = 9 $$

- Difference between 4th and 3rd term:

$$ 31 - 19 = 12 $$

Observation: The differences are 6, 9, and 12. Since these are not all equal, the sequence does not seem to have a constant difference. However, to be thorough, we should analyze these differences further.

Step 2: Examine the Pattern in the Differences

The differences between terms are:

- 6
- 9
- 12

Notice that these differences themselves form an increasing sequence: 6, 9, 12.

Let's look at the differences between these differences:

- 9 - 6 = 3
- 12 - 9 = 3

Since the second differences are constant (both equal to 3), this indicates that the sequence is not arithmetic but is instead quadratic in nature.

Understanding the Nature of the Sequence: Is it Arithmetic?

Based on the calculations above, the sequence 4, 10, 19, 31, ... does not qualify as an arithmetic sequence because the differences between consecutive terms are not constant. Instead, the sequence exhibits second differences that are constant, suggesting it is a quadratic sequence.

Summary:

- The sequence's first differences: 6, 9, 12
- The second differences: 3, 3

Since the second differences are constant, it indicates the sequence follows a quadratic pattern, not an arithmetic one.

Implications of the Sequence's Nature

Given that the sequence is quadratic, the general term can be represented as:

$$T_n = An^2 + Bn + C$$

where:

- A, B, and C are constants to be determined.

This form accounts for the constant second differences because the second difference of a quadratic sequence is always constant and equal to 2A.

Step 3: Find the Quadratic Formula for the Sequence

Using the known terms, we can set up equations to find A, B, and C.

- For $n = 1$, $T_1 = 4$:

$$A(1)^2 + B(1) + C = 4$$

$$A + B + C = 4 \quad \text{(Equation 1)}$$

- For $n = 2$, $T_2 = 10$:

$$A(2)^2 + B(2) + C = 10$$

$$4A + 2B + C = 10 \quad \text{(Equation 2)}$$

- For $n = 3$, $T_3 = 19$:

$$A(3)^2 + B(3) + C = 19$$

$$9A + 3B + C = 19 \quad \text{(Equation 3)}$$

Step 4: Solve the System of Equations

Subtract Equation 1 from Equation 2:

$$(4A + 2B + C) - (A + B + C) = 10 - 4$$

$$3A + B = 6 \quad \text{(Equation 4)}$$

Subtract Equation 2 from Equation 3:

$$ (9A + 3B + C) - (4A + 2B + C) = 19 - 10 $$

$$ 5A + B = 9 \quad \text{(Equation 5)} $$

Now subtract Equation 4 from Equation 5:

$$ (5A + B) - (3A + B) = 9 - 6 $$

$$ 2A = 3 $$

$$ A = \frac{3}{2} $$

Substitute $A = \frac{3}{2}$ into Equation 4:

$$ 3 \times \frac{3}{2} + B = 6 $$

$$ \frac{9}{2} + B = 6 $$

$$ B = 6 - \frac{9}{2} = \frac{12}{2} - \frac{9}{2} = \frac{3}{2} $$

Finally, substitute A and B into Equation 1:

$$ \frac{3}{2} + \frac{3}{2} + C = 4 $$

$$ 3 + C = 4 $$

$$ C = 1 $$

Formulating the General Term

With these values, the nth term of the sequence is:

$$ T_n = \frac{3}{2} n^2 + \frac{3}{2} n + 1 $$

or, factoring out $\frac{3}{2}$:

$$ T_n = \frac{3}{2} (n^2 + n) + 1 $$

This quadratic formula accurately models the sequence's terms.

Summary and Final Insights

Is the sequence 4, 10, 19, 31, ... arithmetic?

No. The differences between consecutive terms are not constant, but the second differences are. This indicates the sequence is quadratic, not arithmetic.

What is the general term of the sequence?

$$T_n = \frac{3}{2} n^2 + \frac{3}{2} n + 1$$

What does this mean?

- The sequence grows quadratically, with the terms increasing at an accelerating rate.
- The second difference being constant at 3 aligns with the quadratic nature, since for a quadratic sequence, the second difference is always $2A$. Here, $2A = 3 \Rightarrow A = \frac{3}{2}$, consistent with our calculation.

Can this sequence be considered arithmetic?

No. Because an arithmetic sequence requires a constant first difference, which this sequence does not have.

Final note:

While some might initially suspect the sequence could be arithmetic due to its increasing nature, a detailed analysis reveals its quadratic growth pattern. Recognizing the second difference as a hallmark of quadratic sequences is key to understanding its true nature.

Conclusion

In this comprehensive analysis, we've demonstrated that the sequence 4, 10, 19, 31,... is not arithmetic but quadratic in nature. By calculating successive differences and solving for the quadratic formula, we've established a clear mathematical model for the sequence's terms. This process underscores the importance of examining differences in sequences to identify their underlying patterns, whether linear, quadratic, or more complex.

Understanding these distinctions equips students, educators, and enthusiasts with the tools to analyze and categorize sequences effectively, fostering deeper mathematical insight. Whether you are tackling similar problems or exploring the properties of various sequences, the methodical approach exemplified here serves as a robust framework for mathematical investigation.

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