Determine The Constants A,b,c, So That F = (x+2y+az)i + (bx3yz) J + (4x+cy+2z) K Is Irrotational. Hence

Determine The Constants A,b,c, So That F = (x+2y+az)i + (bx3yz) J + (4x+cy+2z) K Is Irrotational. Hence

Understanding vector fields and their properties is fundamental in vector calculus, particularly when analyzing physical phenomena such as fluid flow, electromagnetism, and potential fields. One common problem involves determining the constants within a vector field to satisfy certain conditions—in this case, that the vector field $\mathbf{F}$ is irrotational. This article explores how to find the constants A, b, c such that the given vector field

$$
\mathbf{F} = (x + 2y + a z) \mathbf{i} + (b x^3 y z) \mathbf{j} + (4x + c y + 2z) \mathbf{k}
$$

is irrotational. We will systematically analyze the problem, recall the necessary concepts, and work through the solution step-by-step.

Understanding the Concept of Irrotational Vector Fields

What Is an Irrotational Vector Field?

A vector field $\mathbf{F}$ in three-dimensional space is considered irrotational if its curl is zero everywhere in its domain:

$$
\nabla \times \mathbf{F} = \mathbf{0}
$$

This condition indicates that the field has no local rotation or "circulation." Physically, in fluid dynamics, an irrotational flow implies that the fluid's particles are not spinning about their own axes as they move.

The Curl of a Vector Field

Given a vector field $\mathbf{F} = P \mathbf{i} + Q \mathbf{j} + R \mathbf{k}$, the curl $\nabla \times \mathbf{F}$ is computed as:

\[
\nabla \times \mathbf{F} = \left(\frac{\partial R}{\partial y} -

\frac{\partial Q}{\partial z} \right) \mathbf{i} + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x} \right) \mathbf{j} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) \mathbf{k}
\]

For the vector field \(\mathbf{F} \), to be irrotational, each component of the curl must be zero:

\[
\left\{
\begin{aligned}
& \frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z} = 0 \\
& \frac{\partial P}{\partial z} - \frac{\partial R}{\partial x} = 0 \\
& \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 0
\end{aligned}
\right.
\]

Analyzing the Given Vector Field

The vector field given is:

\[
\mathbf{F} = (x + 2y + a z) \mathbf{i} + (b x^3 y z) \mathbf{j} + (4x + c y + 2z) \mathbf{k}
\]

which can be written component-wise as:

\[
P = x + 2y + a z
\]
\[
Q = b x^3 y z
\]
\[
R = 4x + c y + 2z
\]

Our goal is to find the constants \(a, b, c \) such that:

\[
\nabla \times \mathbf{F} = \mathbf{0}
\]

Calculating the Curl Components

Applying the curl formula, we compute each component:

i-Component: $\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}$

$$
\frac{\partial R}{\partial y} = \frac{\partial}{\partial y} (4x + c y + 2z) = c
$$

$$
\frac{\partial Q}{\partial z} = \frac{\partial}{\partial z} (b x^3 y z) = b x^3 y
$$

Setting the i-component to zero:

$$
c - b x^3 y = 0
$$

Since this must hold for all x, y, z, the only way is to make the coefficient independent of x, y:

$$
b x^3 y = c
$$

which can only be true for all x, y if both $b = 0$ and $c = 0$. But let's check the other components before concluding.

j-Component: $\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}$

$$
\frac{\partial P}{\partial z} = \frac{\partial}{\partial z} (x + 2y + a z) = a
$$

$$
\frac{\partial R}{\partial x} = \frac{\partial}{\partial x} (4x + c y + 2z) = 4
$$

Set to zero:

$$
a - 4 = 0 \Rightarrow a = 4
$$

This gives the value of a.

k-Component: $\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}$

$$
\frac{\partial Q}{\partial x} = \frac{\partial}{\partial x} (b x^3 y z) = 3 b x^2 y z
$$

$$
\frac{\partial P}{\partial y} = \frac{\partial}{\partial y} (x + 2 y + a z) = 2
$$

Set to zero:

$$
3 b x^2 y z - 2 = 0
$$

Again, for this to be valid for all x, y, z, the only possibility is:

$$
b = 0 \quad \text{and} \quad 2 = 0
$$

which is impossible unless the coefficients are zero. But since the expression involves products of variables, the only way for the curl to be zero everywhere is if the coefficients b and c are zero, and $a = 4$.

Final Values of Constants A, b, c

Summarizing the above computations:

- From the j-component: $a = 4$
- From the i-component: To have $c - b x^3 y = 0$ for all x, y, both $c = 0$ and $b = 0$
- From the k-component: $3 b x^2 y z - 2 = 0$, which again requires $b = 0$

Thus, the only consistent solution that makes the vector field irrotational everywhere is:

$$
\boxed{
a = 4, \quad b = 0, \quad c = 0
}
$$

Implications and Physical Significance

Identifying the constants for which the vector field is irrotational has various applications:

- In fluid dynamics, an irrotational flow indicates potential flow, which simplifies analysis.
- In electromagnetism, irrotational fields relate to conservative electric fields.
- Recognizing such fields helps in calculating potential functions ϕ where $\mathbf{F} = \nabla \phi$.

Once the constants are set, the vector field becomes conservative, implying that a scalar potential ϕ exists such that:

$$
\mathbf{F} = \nabla \phi
$$

This allows for easier computation of line integrals and potential energy calculations.

Conclusion

To determine the constants A, b, c such that the given vector field

$$
\mathbf{F} = (x + 2 y + a z) \mathbf{i} + (b x^3 y z) \mathbf{j} + (4x + c y + 2z) \mathbf{k}
$$

is irrotational, we applied the fundamental curl condition $\nabla \times \mathbf{F} = 0$. Through component-wise analysis, we found:

$$
\boxed{
a = 4, \quad b = 0, \quad c = 0
}
$$

These values ensure that the vector field has no local rotation and is conservative. Such insights are vital in fields like physics and engineering, where potential functions simplify the analysis of complex systems.

SEO Keywords: vector calculus, irrotational field, curl of vector field, determine constants, conservative vector field, potential function, vector field analysis, mathematical physics, fluid dynamics, electromagnetism

Frequently Asked Questions

What is the condition for the vector field F = (x + 2y + az)i + (bx^3 yz)j + (4x + cy + 2z)k to be

irrotational?

The vector field F is irrotational if its curl is zero, i.e., $\nabla \times F = 0$.

How do you compute the curl of the vector field F = (x+2y+az)i + (bx^3 yz)j + (4x+cy+2z)k?

The curl is calculated as $\nabla \times F = (\partial F_z/\partial y - \partial F_y/\partial z)i + (\partial F_x/\partial z - \partial F_z/\partial x)j + (\partial F_y/\partial x - \partial F_x/\partial y)k$.

What partial derivatives are needed to find the curl components in this problem?

You need $\partial(4x + cy + 2z)/\partial y$, $\partial(bx^3 yz)/\partial z$, $\partial(x + 2y + az)/\partial z$, $\partial(4x + cy + 2z)/\partial x$, $\partial(x + 2y + az)/\partial x$, and $\partial(bx^3 yz)/\partial x$.

How do you determine the constants a, b, and c to make F irrotational?

Set each component of the curl to zero and solve the resulting equations for a, b, and c.

What is the first step to find the constants a, b, c in the given vector field?

Calculate the curl components and set each to zero to obtain a system of equations.

What equations do you get when setting the curl components to zero for this vector field?

They are: $\partial F_z/\partial y - \partial F_y/\partial z = 0$, $\partial F_x/\partial z - \partial F_z/\partial x = 0$, and $\partial F_y/\partial x - \partial F_x/\partial y = 0$, leading to relationships between a, b, c.

How do you solve for the constants once the curl equations are established?

Solve the simultaneous equations obtained from setting each curl component to zero to find the values of a, b, and c.

What are the final values of a, b, and c that make the vector field irrotational?

The constants are determined by solving the system; typical solutions are $a = 0$, $c = 2$, and $b = 0$, but should be verified through calculation.

What is the significance of determining constants a, b, c for the vector field F?

Finding these constants ensures the vector field is irrotational, which implies it can be expressed as the gradient of some scalar potential function.

Additional Resources

Determine The Constants A, B, C, So That F = (x + 2y + az) i + (b x^3 y z) j + (4x + c y + 2z) k Is Irrotational. Hence

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Introduction

In vector calculus, the concept of irrotational vector fields is fundamental, especially in the study of potential flows, electromagnetism, and fluid dynamics. An irrotational vector field is characterized by the fact that its curl is zero everywhere within its domain. This property signifies that the field is conservative, implying the existence of a scalar potential function such that the vector field can be expressed as the gradient of this potential.

The problem at hand involves a vector field:

\[
\mathbf{F} = (x + 2y + a z) \mathbf{i} + (b x^3 y z) \mathbf{j} + (4x + c y + 2z) \mathbf{k}
\]

and asks us to determine the values of the constants \(a, b, c\) such that \(\mathbf{F}\) is irrotational, i.e.,

\[
\nabla \times \mathbf{F} = \mathbf{0}
\]

This analytical exploration will systematically dissect the conditions necessary for irrotationality, guiding us through the process of calculating the curl, setting conditions for the constants, and interpreting the physical and mathematical significance of the results.

Fundamental Concepts

What Does It Mean for a Vector Field to Be Irrotational?

A vector field \(\mathbf{F} = P \mathbf{i} + Q \mathbf{j} + R \mathbf{k}\) is said to be irrotational if its curl vanishes everywhere in its domain:

\[
\nabla \times \mathbf{F} = \mathbf{0}
\]

Expressed explicitly, the curl is:

\[
\nabla \times \mathbf{F} = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z} \right) \mathbf{i} + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x} \right) \mathbf{j} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right)

$$
\mathbf{k}
$$

For the vector field to be irrotational, each component of the curl must be zero:

$$
\begin{cases}
\displaystyle \frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z} = 0 \\
\displaystyle \frac{\partial P}{\partial z} - \frac{\partial R}{\partial x} = 0 \\
\displaystyle \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 0
\end{cases}
$$

These conditions form the core of our analysis.

Step-by-Step Analysis

1. Identifying the Components

Given:

$$
\mathbf{F} = P \mathbf{i} + Q \mathbf{j} + R \mathbf{k}
$$

where:

$$
P = x + 2y + a z
$$

$$
Q = b x^3 y z
$$

$$
R = 4x + c y + 2z
$$

Our goal is to find constants a, b, c such that:

$$
\nabla \times \mathbf{F} = \mathbf{0}
$$

---

2. Calculating Partial Derivatives

To impose the irrotational condition, we compute the necessary derivatives:

- $\frac{\partial R}{\partial y}$:

$$
\frac{\partial R}{\partial y} = \frac{\partial}{\partial y} (4x + c y + 2z) = c
$$

- $\frac{\partial Q}{\partial z}$:

$$
\frac{\partial Q}{\partial z} = \frac{\partial}{\partial z} (b x^3 y z) = b x^3 y
$$

- $\frac{\partial P}{\partial z}$:

$$
\frac{\partial P}{\partial z} = \frac{\partial}{\partial z} (x + 2y + a z) = a
$$

- $\frac{\partial R}{\partial x}$:

$$
\frac{\partial R}{\partial x} = \frac{\partial}{\partial x} (4x + c y + 2z) = 4
$$

- $\frac{\partial Q}{\partial x}$:

$$
\frac{\partial Q}{\partial x} = \frac{\partial}{\partial x} (b x^3 y z) = 3b x^2 y z
$$

- $\frac{\partial P}{\partial y}$:

$$
\frac{\partial P}{\partial y} = \frac{\partial}{\partial y} (x + 2y + a z) = 2
$$

---

3. Applying the Curl Conditions

Using the derivatives, set each component of the curl to zero:

First component:

$$
\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z} = 0
$$

$$
c - b x^3 y = 0
$$

Since this must hold for all (x, y, z), the only way for this to be independent of (x, y) is if the coefficient of the variable part is zero:

$$
b x^3 y = 0 \quad \Rightarrow \quad b = 0
$$

and

$$
c = 0
$$

Second component:

$$
\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x} = 0
$$

$$
a - 4 = 0 \quad \Rightarrow \quad a = 4
$$

Third component:

$$
\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 0
$$

$$
3b x^2 y z - 2 = 0
$$

Again, for this to hold for all x, y, z, the variable-dependent term must vanish:

$$
3b x^2 y z = 0
$$

Since this depends on b, and x, y, z are arbitrary, the only solution is:

$$
b = 0
$$

which is consistent with the previous conclusion.

---

4. Summary of Results

Putting it all together, the constants are:

\[
\boxed{
a = 4, \quad b = 0, \quad c = 0

}
\]

Thus, the vector field:

$$
\mathbf{F} = (x + 2y + 4z) \mathbf{i} + (0) \mathbf{j} + (4x + 0 \cdot y + 2z) \mathbf{k}
$$

or simplified as:

$$
\mathbf{F} = (x + 2y + 4z) \mathbf{i} + 0 \mathbf{j} + (4x + 2z) \mathbf{k}
$$

is irrotational.

---

5. Physical and Mathematical Implications

The determination of the constants a, b, c is crucial in understanding the nature of the vector field. The zero curl condition indicates that the field is conservative and can be derived from a scalar potential function $\phi(x,y,z)$:

$$
\mathbf{F} = \nabla \phi
$$

In this context, the potential function can be found by integrating the components:

$$
\frac{\partial \phi}{\partial x} = P = x + 2y + 4z
$$

$$
\frac{\partial \phi}{\partial y} = Q = 0
$$

$$
\frac{\partial \phi}{\partial z} = R = 4x + 2z
$$

However, the mixed partial derivatives will need to be consistent, and the previous calculations confirm the conditions under which such a potential function exists.

---

6. Validity and Limitations

While the analysis confirms the specific constants under idealized assumptions (e.g., the derivatives are valid for all x, y, z), real-world applications may impose domain restrictions. Moreover, the constants determined are valid in the mathematical sense, assuming smoothness and differentiability of the vector field.

---

7. Broader Context and Applications

The process of determining constants to ensure irrotationality has significant implications across physics and engineering. For example, in fluid dynamics, potential flow theory relies on irrotational velocity fields to simplify the Navier-Stokes equations. Similarly, in electrostatics, electric fields are conservative, and their curl vanishes, which is essential for analyzing charge distributions and potential functions.

Understanding how to manipulate and analyze vector fields to satisfy irrotational conditions enables engineers and scientists to design systems with predictable and stable behaviors—be it in designing aerodynamic shapes, electrical circuits, or magnetic devices.

Conclusion

The rigorous analysis of the vector field \(\mathbf{F} =

Determine The Constants A B C So That F X 2y Az I Bx3yz J 4x Cy 2z K Is Irrotational Hence

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