Determine The Slope Of The Tangent Line, Then Find The Equation Of The Tangent Line At T = /4.X = 4 Cos

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When studying parametric equations and their properties, understanding how to find the slope of a tangent line at a specific point is fundamental. This process involves differentiating the parametric equations with respect to the parameter, often denoted as t. Once the slope is determined, the next step is to derive the equation of the tangent line at that point. In this article, we will explore this process in detail, using the example where the parametric equations are given as $x = 4 \cos t$ and $y = 4 \sin t$, and the point of interest corresponds to $t = \pi/4$.

Understanding the Parametric Equations

Definition of the Parametric Equations

Parametric equations describe a curve by expressing the coordinates (x, y) in terms of a third variable, usually t. For the example provided:
- $x(t) = 4 \cos t$
- $y(t) = 4 \sin t$

These equations represent a circle of radius 4 centered at the origin because:

$$
x^2 + y^2 = (4 \cos t)^2 + (4 \sin t)^2 = 16 (\cos^2 t + \sin^2 t) = 16
$$

Relevance of the Parameter t

The parameter t often represents an angle in trigonometric functions, making these equations particularly useful for describing circular motion or paths. Calculating the tangent line's slope at a specific t involves differentiating both $x(t)$ and $y(t)$ with respect to t.

Finding the Derivatives: $\frac{dy}{dt}$ and $\frac{dx}{dt}$

Differentiate $x(t)$ and $y(t)$

To find the slope of the tangent line, we need the derivatives:
- $\frac{dx}{dt} = -4 \sin t$
- $\frac{dy}{dt} = 4 \cos t$

These derivatives tell us how x and y change as t varies, which is essential for calculating the slope of the tangent line.

Calculating $\frac{dy}{dx}$

The slope of the tangent line at a point corresponding to a specific t is given by the derivative $\frac{dy}{dx}$, which can be found using the chain rule:

$$
\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{4 \cos t}{-4 \sin t} = - \frac{\cos t}{\sin t} = - \cot t
$$

At this stage, the key is to evaluate $\cot t$ at the specific value $t = \pi/4$.

Calculating the Slope at $t = \pi/4$

Compute $\cot (\pi/4)$

Recall that:

$$
\cot t = \frac{\cos t}{\sin t}
$$

At $t = \pi/4$:

$$
\cos (\pi/4) = \frac{\sqrt{2}}{2}, \quad \sin (\pi/4) = \frac{\sqrt{2}}{2}
$$

Therefore:

\[

$$
\cot (\pi/4) = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1
$$

Consequently:

$$
\frac{dy}{dx} \bigg|_{t=\pi/4} = - \cot (\pi/4) = -1
$$

Result: The slope of the tangent line at $t = \pi/4$ is -1.

Finding the Coordinates of the Point at $t = \pi/4$

Calculate x and y at $t = \pi/4$

Using the original parametric equations:
- $x(\pi/4) = 4 \cos (\pi/4) = 4 \times \frac{\sqrt{2}}{2} = 2 \sqrt{2}$
- $y(\pi/4) = 4 \sin (\pi/4) = 4 \times \frac{\sqrt{2}}{2} = 2 \sqrt{2}$

Point of tangency: $(x_0, y_0) = (2 \sqrt{2}, 2 \sqrt{2})$.

Writing the Equation of the Tangent Line

Standard form of the equation of a line

The tangent line passes through (x_0, y_0) with slope $m = -1$. The point-slope form is:

$$
y - y_0 = m (x - x_0)
$$

Substituting the values:

$$
y - 2 \sqrt{2} = -1 (x - 2 \sqrt{2})
$$

Simplifying to slope-intercept form

Distribute:

$$
y - 2 \sqrt{2} = -x + 2 \sqrt{2}
$$
Adding $2 \sqrt{2}$ to both sides:
$$
y = -x + 2 \sqrt{2} + 2 \sqrt{2} = -x + 4 \sqrt{2}
$$

Final equation of the tangent line:
$$
\boxed{y = -x + 4 \sqrt{2}}
$$

Summary and Key Takeaways

- To determine the slope of the tangent line to a parametric curve, first differentiate both $x(t)$ and $y(t)$ with respect to t.
- The slope at a specific t is given by $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$.
- Evaluate this derivative at the given t to find the slope of the tangent line at that point.
- Find the corresponding x and y coordinates by substituting t into the original parametric equations.
- Use the point-slope form to write the equation of the tangent line, then simplify as needed.

Additional Tips for Solving Similar Problems

- Always verify the derivatives are correctly computed, especially signs and coefficients.
- Remember that for trigonometric parametric equations representing a circle, the derivatives often relate to sine and cosine functions, which simplifies calculations.
- When calculating the tangent line, ensure the point and slope are correctly identified before forming the equation.
- Practicing with different values of t or different parametric equations helps develop intuition for these types of problems.

By mastering the steps outlined above, you can confidently determine the slope of tangent lines and their equations at any point along a parametric curve, enhancing your understanding of calculus and analytic geometry.

Frequently Asked Questions

How do I find the slope of the tangent line to the curve x = 4 cos t at t = π/4?

First, differentiate $x = 4 \cos t$ with respect to t to find dx/dt, then evaluate at $t = \pi/4$ to get the slope of the tangent line.

What is the derivative of x = 4 cos t with respect to t?

The derivative $dx/dt = -4 \sin t$.

How do I evaluate the slope at t = π/4?

Plug $t = \pi/4$ into dx/dt: slope = $-4 \sin(\pi/4) = -4 (\sqrt{2}/2) = -2\sqrt{2}$.

What is the coordinate point on the curve at t = π/4?

Calculate x: $x = 4 \cos(\pi/4) = 4 (\sqrt{2}/2) = 2\sqrt{2}$.

How do I write the equation of the tangent line at t = π/4?

Use point-slope form: $y - y_0 = m (x - x_0)$, where m is the slope and (x_0, y_0) is the point on the curve.

What is the full equation of the tangent line at t = π/4?

Since the y-coordinate is not given, if we assume a parametric form or use the original function, the tangent line at $x = 2\sqrt{2}$ with slope $-2\sqrt{2}$ is: $y - y_0 = -2\sqrt{2} (x - 2\sqrt{2})$.

Can I find the tangent line's equation without knowing y explicitly?

Yes, if the curve is parametrized with both x and y in terms of t, you can find dy/dt and then $dy/dx = (dy/dt)/(dx/dt)$, evaluate at $t = \pi/4$, and use the point to write the tangent line equation.

Additional Resources

Tangent Line Analysis and Equation Derivation for $X = 4 \cos T$ at $T = \frac{\pi}{4}$

Introduction

In the realm of calculus, understanding how functions behave at specific points is essential for grasping their overall characteristics. Among the core concepts is the tangent line, which provides a linear approximation of a curve at a particular point. Determining the slope of this tangent line and deriving its equation are fundamental skills, especially when dealing with parametric functions such as $X = 4 \cos T$.

This article offers an in-depth exploration of how to meticulously find the slope of the tangent line at a specific parameter value $T = \frac{\pi}{4}$ for the given function, and how to subsequently formulate the tangent line's equation. Whether you're a student honing your calculus skills or a seasoned mathematician seeking a refresher, this comprehensive guide aims to clarify each step involved in this process.

Understanding the Function: $X = 4 \cos T$

Before diving into derivatives and tangent lines, it’s crucial to understand the nature of the given function:

- Parametric Form: The function expresses X as a function of parameter T: $X(T) = 4 \cos T$.
- Domain: Typically, T can take any real value, but for specific points, we're interested in $T = \frac{\pi}{4}$.
- Range: Because $\cos T$ ranges between -1 and 1, X varies between -4 and 4.

This parametric form hints that the curve could be part of a larger parametric system, but for this analysis, we focus solely on how X varies with T.

Step 1: Determining the Slope of the Tangent Line

The Concept of Derivatives in Parametric Functions

In parametric calculus, the slope of the tangent line at a specific parameter T is given by the derivative $\frac{dX}{dT}$. This derivative measures how the function $X(T)$ changes with respect to T, providing the instantaneous rate of change at that point.

Key formula:

$$
\text{Slope} = \frac{dX}{dT}
$$

Calculating $\frac{dX}{dT}$

Given:

$$
X(T) = 4 \cos T
$$

Differentiating with respect to T:

$$
\frac{dX}{dT} = 4 \times (-\sin T) = -4 \sin T
$$

This derivative tells us that, at any point T, the slope of the tangent line is $-4 \sin T$.

Step 2: Evaluating the Derivative at $T = \frac{\pi}{4}$

To find the specific slope at the point of interest:

$$
T = \frac{\pi}{4}
$$

Recall that:

$$
\sin \left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}
$$

Plugging into the derivative:

$$
\frac{dX}{dT} \bigg|_{T=\frac{\pi}{4}} = -4 \times \frac{\sqrt{2}}{2} = -2 \sqrt{2}
$$

Result:
The slope of the tangent line at $T = \frac{\pi}{4}$ is $-2 \sqrt{2}$.

This negative slope indicates that the tangent line at this point is decreasing, sloping downward as we move along the curve.

Step 3: Finding the Coordinates of the Point

While the slope is essential, to write the equation of the tangent line, we need a specific point on the curve corresponding to $T = \frac{\pi}{4}$.

Computing X at $T = \frac{\pi}{4}$:

$$
X\left(\frac{\pi}{4}\right) = 4 \cos \left(\frac{\pi}{4}\right) = 4 \times \frac{\sqrt{2}}{2} = 2 \sqrt{2}
$$

Coordinates of the point:

$$
\boxed{
\left(X, T \right) = \left(2 \sqrt{2}, \frac{\pi}{4} \right)
}
$$

(Note: Since the function is parametric, the point on the $X-T$ plane is $\left(2 \sqrt{2}, \frac{\pi}{4} \right)$. If needed, the tangent line can be expressed in terms of X and T.)

Step 4: Formulating the Equation of the Tangent Line

Understanding the Equation

In the context of parametric functions, the tangent line can be represented in terms of the parameter T, or, if desired, expressed as a line in the X-T plane.

Since the problem is centered on the point $T = \frac{\pi}{4}$, and the curve is described parametrically, the tangent line at this point can be approximated by a linear relation involving T:

\[

T = T_0 + \text{(slope)} \times (X - X_0)
\]

But more conventionally, when dealing with a parametric curve, the tangent line in the \(X-T\) plane can be written as:

\[
T = T_0 + \left(\frac{dT}{dX} \bigg|_{T=T_0} \right) \times (X - X_0)
\]

Alternatively, if the goal is to find the tangent line in the \(X\)-\(Y\) plane (assuming a Cartesian setting), additional information such as \(Y(T)\) would be necessary. Since only \(X = 4 \cos T\) is provided, and \(T\) is the parameter, the typical approach is to express the tangent in the \(X\)-\(T\) plane.

Deriving the Equation in \(T\)-\(X\) Plane

Given:

\[
\frac{dX}{dT} = -4 \sin T
\]

and the point:

\[
\left(T_0, X_0 \right) = \left(\frac{\pi}{4}, 2 \sqrt{2} \right)
\]

The linear approximation near \(T_0\) is:

\[
X \approx X_0 + \frac{dX}{dT} \bigg|_{T=T_0} \times (T - T_0)
\]

Plugging in the values:

\[
X \approx 2 \sqrt{2} + (-2 \sqrt{2}) \times \left(T - \frac{\pi}{4} \right)
\]

This is the linear equation of the tangent line in the \(X-T\) plane.

Summary of Results

Parameter	Expression / Value	Explanation
Derivative $\frac{dX}{dT}$	$-4 \sin T$	Rate of change of X with respect to T
Slope at $T = \frac{\pi}{4}$	$-2 \sqrt{2}$	Calculated by substituting $\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$
X at $T = \frac{\pi}{4}$	$2 \sqrt{2}$	$X = 4 \cos \frac{\pi}{4} = 2 \sqrt{2}$
Equation of tangent line	$X \approx 2 \sqrt{2} - 2 \sqrt{2} \left(T - \frac{\pi}{4} \right)$	Linear approximation near the point

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Additional Considerations

Visual Interpretation

The tangent line's negative slope indicates a decreasing trend in X as T increases around $T = \frac{\pi}{4}$. This aligns with the behavior of the cosine function's derivative, which is negative in this region.

Extending to Other Contexts

- If the problem extends to a parametric curve involving both $X(T)$ and $Y(T)$, the process involves calculating $\frac{dy}{dt}$ and using the parametric derivative formulas.
- For explicit Cartesian equations, further algebraic elimination might be necessary, but here, the parametric approach suffices.

Final Remarks

Determining the slope of a tangent line in parametric functions hinges on precise differentiation and evaluation at the particular parameter value. The process outlined demonstrates a rigorous methodology for:

1. Calculating the derivative $\frac{dX}{dT}$,
2. Evaluating the derivative at the given point,
3. Using the derivative and point to formulate the tangent line equation.

This systematic approach ensures clarity and accuracy, vital in advanced calculus applications. Whether applied to theoretical mathematics, physics, or engineering problems, mastering these steps enhances your analytical toolkit and deepens your understanding of function behavior.

In essence, at \(T=\

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