Determine Whether The Differential Equation (3 X+2)+(3 Y-3) Y^{\prime}=0 Is Exact. If It Is Exact, Find

Determine Whether The Differential Equation (3 X+2)+(3 Y-3) Y^{\prime}=0 Is Exact. If It Is Exact, Find

Introduction to Differential Equations and Exactness

Understanding the nature of differential equations is fundamental in solving many problems in mathematics, physics, and engineering. One common class of differential equations is the first-order differential equations, which often take the form:

$$ M(x, y) + N(x, y) y' = 0 $$

or equivalently,

$$ \frac{dy}{dx} = -\frac{M(x, y)}{N(x, y)} $$

A differential equation is said to be exact if it can be written in the form:

$$ M(x, y) dx + N(x, y) dy = 0 $$

where M and N are continuous functions with continuous partial derivatives, and satisfy a specific condition called the exactness condition:

$$ \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} $$

In this article, we will analyze the differential equation:

$$ (3x + 2) + (3y - 3) y' = 0 $$

to determine whether it is exact and, if so, to find its solution.

Rewriting the Differential Equation in Standard Form

Before we analyze the exactness, it is helpful to express the differential equation in the standard form:

$$ M(x, y) dx + N(x, y) dy = 0 $$

Given the original equation:

$$ (3x + 2) + (3y - 3) y' = 0 $$

we can rewrite it as:

$$ (3x + 2) + (3y - 3) y' = 0 $$

which can be rearranged to:

$$ (3x + 2) dx + (3y - 3) dy = 0 $$

This form aligns perfectly with the standard form for checking exactness, where:

- $M(x, y) = 3x + 2$
- $N(x, y) = 3y - 3$

Now, we can proceed to analyze whether the differential equation is exact.

Checking for Exactness

Step 1: Determine $M(x, y)$ and $N(x, y)$

From the standard form, we identify:

- $M(x, y) = 3x + 2$
- $N(x, y) = 3y - 3$

Step 2: Compute the Partial Derivatives

Calculate:

- $\frac{\partial M}{\partial y}$
- $\frac{\partial N}{\partial x}$

Because:

$$
\frac{\partial M}{\partial y} = \frac{\partial}{\partial y} (3x + 2) = 0
$$
$$
\frac{\partial N}{\partial x} = \frac{\partial}{\partial x} (3y - 3) = 0
$$

Step 3: Compare the Partial Derivatives

Since:

$$
\frac{\partial M}{\partial y} = 0 \quad \text{and} \quad \frac{\partial N}{\partial x} = 0
$$

and they are equal, the exactness condition is satisfied:

$$
\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}
$$

Therefore, the differential equation is exact.

Finding the Solution to the Exact Differential Equation

Having established that the differential equation is exact, the next step is to find its solution.

Step 1: Integrate $M(x, y)$ with respect to x

Calculate:

$$
\Psi(x, y) = \int M(x, y) dx = \int (3x + 2) dx
$$

which yields:

$$
\Psi(x, y) = \frac{3}{2} x^2 + 2x + h(y)
$$

where $h(y)$ is an arbitrary function of y, since the integration is with respect to x.

Step 2: Differentiate $\Psi(x, y)$ with respect to y

Calculate:

$$
\frac{\partial \Psi}{\partial y} = h'(y)
$$

But, from the theory of exact equations, this derivative must equal $N(x, y)$:

$$
h'(y) = N(x, y) = 3y - 3
$$

Step 3: Integrate $h'(y)$ with respect to y

$$
h(y) = \int (3y - 3) dy = \frac{3}{2} y^2 - 3 y + C
$$

where C is an arbitrary constant.

Step 4: Write the General Solution

The potential function $\Psi(x, y)$ is:

$$
\Psi(x, y) = \frac{3}{2} x^2 + 2x + \frac{3}{2} y^2 - 3 y + C
$$

The general solution to the differential equation is given by:

$$
\boxed{
\frac{3}{2} x^2 + 2x + \frac{3}{2} y^2 - 3 y = K
}
$$

where K is an arbitrary constant, representing the family of solutions.

Summary and Conclusion

In this detailed analysis, we examined the differential equation:

$$ (3x + 2) + (3y - 3) y' = 0 $$

and determined whether it is exact. By rewriting it in the standard form, we identified:

- $M(x, y) = 3x + 2$
- $N(x, y) = 3y - 3$

We then calculated the partial derivatives:

- $\frac{\partial M}{\partial y} = 0$
- $\frac{\partial N}{\partial x} = 0$

Since these are equal, the differential equation is exact.

To find the solution, we integrated M with respect to x, introduced an arbitrary function $h(y)$, and used the relationship with N to determine $h(y)$. The resulting general solution is:

$$
\frac{3}{2} x^2 + 2x + \frac{3}{2} y^2 - 3 y = C
$$

This implicit relation describes the family of solutions to the original differential equation.

Implications and Applications

The process of verifying exactness and solving differential equations using potential functions is a powerful technique in differential equations. Exact equations often arise in physics and engineering contexts, such as thermodynamics, where energy conservation principles lead to such equations. Recognizing the conditions for exactness simplifies solving complex differential equations and provides insight into the underlying physical or mathematical relationships.

Moreover, understanding the structure of these equations, including the significance of the functions M and N, allows mathematicians and engineers to develop more advanced solution techniques, including integrating factors for non-exact equations.

Final Remarks

- The key step in analyzing differential equations for exactness is to identify the functions M and N, and verify their mixed partial derivatives.
- When the differential equation is exact, the solution can be constructed by integrating M and matching with N to find the potential function.
- The methodology showcased here is applicable to a broad class of first-order differential equations, providing a systematic approach to their solution.

This comprehensive approach not only solves the specific differential equation at hand but also enhances the understanding of the properties and solution strategies for exact differential equations in general.

Frequently Asked Questions

How can I determine if the differential equation (3x+2) + (3y -

3) y' = 0 is exact?

To check if the differential equation is exact, rewrite it in the form $M(x,y) + N(x,y)\, y' = 0$, where M and N are functions of x and y. For the given equation, set $M = 3x + 2$ and $N = 3y - 3$. Then compute the partial derivatives $\partial M/\partial y$ and $\partial N/\partial x$. If these are equal, the differential equation is exact.

What are the partial derivatives needed to verify whether the differential equation is exact?

The partial derivatives are $\partial M/\partial y$ and $\partial N/\partial x$. For the given functions $M = 3x + 2$ and $N = 3y - 3$, we have $\partial M/\partial y = 0$ and $\partial N/\partial x = 0$. Since they are equal, the equation is exact.

Is the differential equation (3x+2)+(3y-3) y' = 0 exact?

Yes, because $\partial M/\partial y = 0$ and $\partial N/\partial x = 0$ are equal, confirming that the differential equation is exact.

How do I find the potential function if the differential equation is exact?

To find the potential function □(x,y), integrate M(x,y) with respect to x: □(x,y) = $\int M\, dx + h(y)$, where h(y) is an arbitrary function of y. Then differentiate □ with respect to y and set equal to N(x,y) to find h(y).

What is the potential function □(x,y) for the given differential equation?

Since $M = 3x + 2$, integrating with respect to x gives □(x,y) = $(3/2)x^2 + 2x + h(y)$. Differentiating with respect to y gives ∂□/∂y = $h'(y)$. Set this equal to $N = 3y - 3$, so $h'(y) = 3y - 3$. Integrating $h'(y)$ yields $h(y) = (3/2) y^2 - 3y + C$. Therefore, the potential function is □(x,y) = $(3/2) x^2 + 2x + (3/2) y^2 - 3y + C$.

How do I write the general solution to the differential equation using the potential function?

The general solution is given by setting □(x,y) = constant. For this case, it is $(3/2) x^2 + 2x + (3/2) y^2 - 3y = K$, where K is an arbitrary constant.

Can you summarize the steps to solve the given differential equation if it is exact?

Yes. First, verify the equation is exact by checking if $\partial M/\partial y = \partial N/\partial x$. Since they are equal, integrate M with respect to x to find □(x,y). Then, differentiate □ with respect to y and equate to N to find any arbitrary functions. Finally, write the general solution as □(x,y) = constant.

What is the explicit solution to the differential equation

$(3x+2)+(3y-3) y' = 0$?

The explicit solution is $(3/2) x^2 + 2x + (3/2) y^2 - 3y = K$, where K is an arbitrary constant.

Are there any special considerations when solving an exact differential equation like this one?

Yes. Ensure the equation is indeed exact by verifying the partial derivatives. If not exact, consider multiplying by an integrating factor. For this problem, since it is exact, directly find the potential function as described.

Additional Resources

Determine Whether The Differential Equation $(3X + 2) + (3Y - 3)Y' = 0$ Is Exact. If It Is Exact, Find

---

Introduction

Differential equations are fundamental in modeling various phenomena across physics, engineering, biology, and other sciences. Among the many types of differential equations, exact differential equations hold a special place because they can often be solved straightforwardly once their exactness is established. This review provides a comprehensive exploration of the process for determining whether a given differential equation is exact, specifically analyzing the form:

$$
(3X + 2) + (3Y - 3)Y' = 0
$$

and, if it is exact, guiding through the steps to find its solution.

Understanding the Differential Equation

The first step in analyzing the given differential equation is to rewrite it in a standard form. The equation:

$$
(3X + 2) + (3Y - 3)Y' = 0
$$

can be expressed as:

$$
(3X + 2) + (3Y - 3) \frac{dY}{dX} = 0
$$

Rearranged, this becomes:

$$
(3Y - 3) \frac{dY}{dX} = - (3X + 2)
$$

or equivalently,

$$
(3Y - 3) dY = - (3X + 2) dX
$$

which suggests that the differential equation can be viewed as an implicit relationship between X and Y. To analyze its exactness, the standard form of an exact differential equation is:

$$
M(X, Y) + N(X, Y) \frac{dY}{dX} = 0
$$

or

$$
M(X, Y) dX + N(X, Y) dY = 0
$$

Expressing the Equation in Standard Exact Form

Let's identify M and N:

- $M(X, Y) = 3X + 2$
- $N(X, Y) = 3Y - 3$

Thus, the differential form becomes:

$$
(3X + 2) dX + (3Y - 3) dY = 0
$$

This is the form suitable for checking exactness.

What Does It Mean for a Differential Equation to Be Exact?

A differential equation in the form:

$$
M(X, Y) dX + N(X, Y) dY = 0
$$

is exact if there exists a function $\Psi(X, Y)$ such that:

$$
\frac{\partial \Psi}{\partial X} = M(X, Y), \quad \frac{\partial \Psi}{\partial Y} = N(X, Y)
$$

In other words, the differential form:

$$
M dX + N dY
$$

is exact if:

$$
\frac{\partial M}{\partial Y} = \frac{\partial N}{\partial X}
$$

This is the necessary and sufficient condition for exactness.

Step-by-Step Process to Determine Exactness

1. Compute $\frac{\partial M}{\partial Y}$ and $\frac{\partial N}{\partial X}$

Given:

- $M(X, Y) = 3X + 2$

- $N(X, Y) = 3Y - 3$

Calculate:

$$
\frac{\partial M}{\partial Y} = 0
$$

$$
\frac{\partial N}{\partial X} = 0
$$

Since both derivatives are zero, they are equal:

$$
\frac{\partial M}{\partial Y} = \frac{\partial N}{\partial X} = 0
$$

Conclusion: The differential equation is exact because these derivatives are equal.

---

4. Finding the Potential Function $\Psi(X, Y)$

Since the equation is exact, our goal is to find $\Psi(X, Y)$ such that:

$$
\frac{\partial \Psi}{\partial X} = M(X, Y) = 3X + 2
$$

$$
\frac{\partial \Psi}{\partial Y} = N(X, Y) = 3Y - 3
$$

---

5. Integrate $M(X,Y)$ with respect to X

Integrate:

$$
\Psi(X, Y) = \int (3X + 2) dX
$$

$$
\Psi(X, Y) = \frac{3}{2} X^2 + 2X + h(Y)
$$

where $h(Y)$ is an arbitrary function of Y (since the partial derivative with respect to X might have omitted terms independent of X).

---

6. Differentiate $\Psi(X,Y)$ with respect to Y and compare to $N(X,Y)$

$$
\frac{\partial \Psi}{\partial Y} = h'(Y)
$$

But from earlier, we know:

$$
\frac{\partial \Psi}{\partial Y} = N(X, Y) = 3Y - 3
$$

So:

$$
h'(Y) = 3Y - 3
$$

Integrate:

$$
h(Y) = \frac{3}{2} Y^2 - 3Y + C
$$

\]

where C is an arbitrary constant.

---

7. Write the General Solution

Combining the parts, the potential function becomes:

$$
\Psi(X, Y) = \frac{3}{2} X^2 + 2X + \frac{3}{2} Y^2 - 3Y + C
$$

The general solution to the differential equation is obtained by setting $\Psi(X, Y) = \text{constant}$:

$$
\boxed{
\frac{3}{2} X^2 + 2X + \frac{3}{2} Y^2 - 3Y = K
}
$$

where K is an arbitrary constant.

Summary of the Solution Process

Step	Description	Result
1	Rewrite the differential equation in $M dX + N dY = 0$ form	$(3X + 2) dX + (3Y - 3) dY = 0$
2	Calculate $\partial M / \partial Y$ and $\partial N / \partial X$	Both are 0, indicating the equation is exact
3	Find potential function $\Psi(X, Y)$	$\Psi = \frac{3}{2} X^2 + 2X + \frac{3}{2} Y^2 - 3Y + C$
4	Write the general solution	$\frac{3}{2} X^2 + 2X + \frac{3}{2} Y^2 - 3Y = K$

Extended Discussion: When Is a Differential Equation Not Exact?

While the specific equation in question is exact, many differential equations are not. When they are not, the following strategies are often employed:

- Finding an Integrating Factor: An auxiliary function that, when multiplied with the original differential equation, makes it exact.
- Transforming Variables: Substituting variables to convert the differential equation into an exact or more manageable form.
- Using Approximate or Numerical Methods: When an explicit solution cannot be obtained analytically.

Recognizing Exactness in Practice

In practice, recognizing whether an equation is exact involves:

- Checking the derivatives $\partial M / \partial Y$ and $\partial N / \partial X$.
- If they are equal, the equation is exact.
- If not, considering integrating factors that depend only on X, only on Y, or both.

Significance and Applications of Exact Differential Equations

Exact differential equations are significant because:

- They often arise in thermodynamics, fluid mechanics, and electromagnetism, where potential functions represent physically meaningful quantities like energy, potential, or flux.
- They provide a systematic method for solving complex relationships between variables.
- They connect to the concept of conservative fields where potential functions exist.

Final Remarks

The differential equation:

$$
(3X + 2) + (3Y - 3)Y' = 0
$$

is exact, as verified through the partial derivatives. The process involved identifying M and N, verifying the exactness condition, integrating to find the potential function, and then expressing the general solution.

This detailed analysis showcases the power and elegance of the method of exact differential equations, emphasizing the importance of understanding derivatives, potential functions, and the conditions for exactness. Mastery of these concepts equips students and practitioners with a robust toolset for tackling a wide variety of differential equations encountered in theoretical and applied contexts.

References for Further Study

- Boyce, W. E., & DiPrima, R. C. (2017). Elementary Differential Equations and Boundary Value Problems. Wiley.
- Zill, D. G. (2010).

[Determine Whether The Differential Equation 3 X 2 3 Y 3 Y Prime 0 Is Exact If It Is Exact Find]

Determine Whether The Differential Equation 3 X 2 3 Y 3 Y Prime 0 Is Exact If It Is Exact Find

Related Articles

- Create A Chart Containing The Three Forms Of Business Organizations: Proprietorships, Partnerships, And
- Employees At Livingsocial Escapes Who Are Motivated By Excitement About The Trips They Are Able To Take
- Find The Number Of Units That Must Be Produced And Sold In Order To Yield The Maximum Profit, Given The

Back to Home