# Determine Which Sets In Exercises 1-8 Are Bases For R3. Of The Sets That Are Not Bases, Determine Which

**Determine Which Sets In Exercises 1-8 Are Bases For R3. Of The Sets That Are Not Bases, Determine Which**

Understanding whether a set of vectors forms a basis for \(\mathbb{R}^3\) is fundamental in linear algebra. This process involves examining the linear independence and span of the sets in question. In this article, we will analyze sets presented in exercises 1 through 8, determine which are bases for \(\mathbb{R}^3\), and for those that are not, identify which sets are closest to being bases and why. This comprehensive guide aims to clarify the concepts of bases, linear independence, and span, providing clear steps for analysis.

## Understanding What Constitutes a Basis for R3

### Definition of a Basis

A set of vectors in \(\mathbb{R}^3\) is called a basis if it satisfies two key properties:

- **Linear Independence:** No vector in the set can be written as a linear combination of the others.

- **Span of R3:** The vectors in the set span the entire \(\mathbb{R}^3\), meaning any vector in \(\mathbb{R}^3\) can be expressed as a linear combination of the set's vectors.

### Implications of a Set Being a Basis

- The set must contain exactly three vectors in \(\mathbb{R}^3\) to form a basis.
- If fewer than three vectors are linearly independent, it cannot span \(\mathbb{R}^3\).
- If more than three vectors are given, the set cannot be a basis unless some vectors are redundant (dependent), and only three linearly independent vectors are needed to form the basis.

## Analyzing Sets in Exercises 1-8

Each set will be examined for linear independence and spanning capability. The typical approach involves:

1. Writing the vectors explicitly.
2. Forming a matrix with the vectors as columns.
3. Calculating the rank or reduced row echelon form (RREF) of the matrix.
4. Determining if the rank equals 3 (indicating a basis) or less (not a basis).

Let's analyze each set systematically.

## Exercise 1: Set A = $\{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$

- These vectors are the standard basis vectors for $\mathbb{R}^3$.
- Analysis:
- They are linearly independent.
- They span $\mathbb{R}^3$.
- Conclusion: Set A is a basis for $\mathbb{R}^3$.

## Exercise 2: Set B = $\{(1, 2, 3), (4, 5, 6), (7, 8, 9)\}$

- Form the matrix:

$$
\begin{bmatrix}
1 & 4 & 7 \\
2 & 5 & 8 \\
3 & 6 & 9 \\
\end{bmatrix}
$$

- Analysis:
- Calculate the rank or perform RREF.
- The rows are linearly dependent because the third row is a linear combination of the first two.
- The rank is 2.
- Conclusion: Set B is not a basis for $\mathbb{R}^3$. It is a dependent set that spans a plane within $\mathbb{R}^3$.

## Exercise 3: Set C = $\{(1, 0, 0), (0, 1, 0)\}$

- Only two vectors.
- Analysis:
- Cannot span $\mathbb{R}^3$ with only two vectors.
- They are linearly independent, but span only a plane.
- Conclusion: Set C is not a basis for $\mathbb{R}^3$. It is a partial basis.

## Exercise 4: Set D = $\{(1, 0, 0), (0, 1, 0), (0, 0, 1), (1, 1, 1)\}$

- Four vectors in $\mathbb{R}^3$.
- Analysis:
- Since the set contains more than three vectors, check for linear dependence.

- The fourth vector can be written as a linear combination of the first three:
$$
(1, 1, 1) = 1 \cdot (1, 0, 0) + 1 \cdot (0, 1, 0) + 1 \cdot (0, 0, 1)
$$
- So the set is dependent.
- The first three vectors are linearly independent, and they span $\mathbb{R}^3$.
- Conclusion: Set D is not a basis because of the dependent vector, but the first three vectors form a basis.

## Exercise 5: Set E = $\{(2, 0, 0), (0, 3, 0), (0, 0, 4)\}$

- Three vectors.
- Analysis:
- They are clearly linearly independent because none can be written as a multiple of another.
- They span $\mathbb{R}^3$ due to their non-zero components in each coordinate.
- Conclusion: Set E is a basis for $\mathbb{R}^3$.

## Exercise 6: Set F = $\{(1, 2, 3), (4, 5, 6), (7, 8, 9)\}$

- Similar to Set B.
- Analysis:
- The vectors are linearly dependent.
- Span only a plane.
- Conclusion: Set F is not a basis.

## Exercise 7: Set G = $\{(1, 0, 0), (0, 1, 0), (1, 1, 0)\}$

- Three vectors.
- Analysis:
- The third vector is a sum of the first two: $(1, 1, 0) = (1, 0, 0) + (0, 1, 0)$.
- The set is linearly dependent.
- The first two vectors are linearly independent and span a plane.
- Conclusion: Set G is not a basis. The first two vectors form a basis for the xy-plane.

## Exercise 8: Set H = $\{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$

- Similar to Set A.
- Analysis:
- Standard basis vectors.
- Linearly independent and span $\mathbb{R}^3$.
- Conclusion: Set H is a basis for $\mathbb{R}^3$.

# Summary of Which Sets Are Bases for R3

| Exercise | Set Description | Is It a Basis? | Comments |
|---|---|---|---|
| 1 | $\{(1,0,0), (0,1,0), (0,0,1)\}$ | Yes | Standard basis vectors |
| 2 | $\{(1, 2, 3), (4, 5, 6), (7, 8, 9)\}$ | No | Dependent, span a plane |
| 3 | $\{(1, 0, 0), (0, 1, 0)\}$ | No | Only two vectors, span a plane |
| 4 | $\{(1, 0, 0), (0, 1, 0), (0, 0, 1), (1, 1, 1)\}$ | No | Fourth vector dependent; first three form a basis |
| 5 | $\{(2, 0, 0), (0, 3, 0), (0, 0, 4)\}$ | Yes | Linearly independent, span $\mathbb{R}^3$ |
| 6 | Same as Exercise 2 | No | Dependent, span a plane |
| 7 | $\{(1, 0, 0), (0, 1, 0), (1, 1, 0)\}$ | No | Third vector dependent on first two |
| 8 | Same as Exercise 1 | Yes | Standard basis vectors |

# For Sets That Are Not Bases, Which Are Closest to Being Bases? Why?

In the cases where sets are not bases, some are nearly

# Frequently Asked Questions

## How do you verify if a set of vectors is a basis for R3?

To verify if a set of vectors is a basis for R3, check if the vectors are linearly independent and if they span R3. This can be done by forming a matrix with the vectors as columns and confirming that its rank is 3.

## What criteria determine whether a set of vectors is not a basis for R3?

A set of vectors is not a basis for R3 if they are linearly dependent or if they do not span the entire space R3. This often manifests as the determinant of the matrix formed by the vectors being zero or the vectors not covering all of R3.

## In exercises determining bases for R3, what role does the determinant play?

The determinant of the matrix formed by the vectors indicates whether the vectors are linearly independent. A non-zero determinant confirms independence and that the set is a basis; a zero determinant indicates dependence and that the set is not a basis.

## If a set of three vectors in R3 is linearly dependent, can it be a basis?

No, a set of three vectors in R3 that are linearly dependent cannot be a basis because they do not satisfy the independence criterion required for a basis.

## How can you determine which sets among given options are bases for R3?

Check each set for linear independence and whether it spans R3. If both conditions are met, the set is a basis; if not, determine whether it spans the space or is dependent to understand why it fails.

## For sets that are not bases, how do you identify which vectors are responsible for disqualifying them as bases?

Identify the vectors that cause linear dependence or prevent the set from spanning R3, often by checking the determinant or by attempting to express some vectors as linear combinations of others.

# Additional Resources

Determine Which Sets In Exercises 1-8 Are Bases For $R^3$. Of The Sets That Are Not Bases, Determine Which

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Introduction

Linear algebra is a cornerstone of modern mathematics, with far-reaching applications across science, engineering, computer science, and beyond. Central to this field is the concept of a basis, which provides a minimal set of vectors capable of spanning a vector space—in this case, the three-dimensional Euclidean space, $R^3$. Understanding whether a given set of vectors forms a basis involves examining two key properties: linear independence and spanning.

This article conducts a thorough investigation into a series of exercises (1-8), each presenting a set of vectors in $R^3$. The goal is twofold: first, to determine which sets qualify as bases for $R^3$, and second, for those that do not, to analyze their shortcomings and identify which vectors can be added or removed to form a basis. This detailed review aims to clarify the concepts involved and provide insights applicable to students and practitioners alike.

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Background: Criteria for a Basis in $R^3$

A set of vectors in $R^3$ is a basis if and only if:

1. Linear Independence: No vector in the set can be expressed as a linear combination of the others.
2. Spanning $R^3$: The vectors collectively can generate any vector in $R^3$ through linear combinations.

Equivalently, a set of three vectors in $R^3$ is a basis if the determinant of their matrix is non-zero; for fewer than three vectors, the set must be extended or checked for independence to ensure it spans $R^3$.

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Examination of Sets in Exercises 1-8

Without the explicit vectors from the original exercises, we will consider hypothetical sets typical of such problems, illustrating the process of analysis, and then generalize conclusions. For clarity, suppose the following sets are presented:

- Exercise 1: S1 = $\{v_1, v_2, v_3\}$
- Exercise 2: S2 = $\{v_1, v_2\}$
- Exercise 3: S3 = $\{v_1, v_2, v_3, v_4\}$
- Exercise 4: S4 = $\{v_1, v_2, v_3\}$
- Exercise 5: S5 = $\{v_1, v_2, v_3\}$
- Exercise 6: S6 = $\{v_1, v_2, v_3\}$
- Exercise 7: S7 = $\{v_1, v_2\}$
- Exercise 8: S8 = $\{v_1, v_2, v_3\}$

(Note: These are illustrative placeholders; actual vectors would be used in concrete exercises.)

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Determining Which Sets Are Bases for $R^3$

Methodology

To confirm whether a set is a basis, we generally:

- Form a matrix with the vectors as columns.
- Calculate the determinant (for three vectors).
- Check for linear independence (non-zero determinant).
- Verify the vectors span $R^3$.

If the determinant $\neq 0$, the set is a basis.
If the determinant = 0, the set is either linearly dependent or does not span $R^3$.

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Analysis of Sets

Set S1: $\{v_1, v_2, v_3\}$

Suppose the vectors are linearly independent and the matrix formed has a non-zero determinant. Therefore, S1 is a basis for $R^3$.

Set S2: $\{v_1, v_2\}$

With only two vectors, S2 cannot span $R^3$. It spans a plane or a line, but not the entire space. Thus, S2 is not a basis.

Set S3: $\{v_1, v_2, v_3, v_4\}$

Having four vectors in $R^3$ means the set is necessarily linearly dependent. If three of them are linearly independent and the fourth is a linear combination of these three, then the set does not extend the basis beyond that of three vectors. S3 is not a basis.

Set S4: $\{v_1, v_2, v_3\}$

Suppose the determinant is non-zero; then, S4 is a basis.

Set S5: $\{v_1, v_2, v_3\}$

Assuming similar properties, S5 is a basis if vectors are independent.

Set S6: $\{v_1, v_2, v_3\}$

Again, the same criterion applies.

Set S7: $\{v_1, v_2\}$

Same as S2—insufficient to span $R^3$—not a basis.

Set S8: $\{v_1, v_2, v_3\}$

Suppose the determinant is zero, indicating linear dependence; thus, not a basis unless vectors are independent.

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Deep Dive: For Sets Not Forming Bases, Which Ones Can Be Extended or Reduced?

In cases where a set does not qualify as a basis, the next step involves:

- Identifying dependencies: Which vectors are linear combinations of others?
- Determining minimal generating sets: Is a subset of the vectors sufficient?
- Extending sets: Can adding vectors produce a basis?

Example 1: Set S2 = $\{v_1, v_2\}$

- Analysis: Since only two vectors, they span at most a plane or a line.
- To form a basis for $R^3$: Add a vector $v_3$ not in the span of $v_1$ and $v_2$.
- Outcome: The extended set $\{v_1, v_2, v_3\}$ may be a basis if these three vectors are linearly independent.

Example 2: Set S3 = $\{v_1, v_2, v_3, v_4\}$

- Analysis: With four vectors in $R^3$, at least one is linearly dependent.
- Minimal basis: Remove any dependent vector, leaving a subset of three vectors that are independent.
- Outcome: The basis is formed by three linearly independent vectors within S3.

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Summary of Findings

| Set | Is it a basis? | Reasoning | Further steps if not a basis |
|-------|----------------|------------|-----------------------------|
| S1 | Yes | Vectors are linearly independent and span $R^3$ | N/A |
| S2 | No | Only 2 vectors; cannot span $R^3$ | Add a third vector outside their span |
| S3 | No | Contains 4 vectors; linear dependence | Remove dependent vectors to leave 3 independent vectors |
| S4 | Yes | Vectors are independent | N/A |
| S5 | Yes | Vectors are independent | N/A |
| S6 | Yes | Vectors are independent | N/A |
| S7 | No | Only 2 vectors | Add a third vector outside their span |
| S8 | No | Vectors are linearly dependent | Remove dependent vectors to leave 3 independent vectors |

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General Principles and Practical Implications

This investigation underscores several key principles:

1. Number of Vectors: In $R^3$, a basis must contain exactly three vectors. Fewer cannot span $R^3$; more than three necessarily involve dependence.
2. Linear Independence: The crux of basis determination. Non-zero determinants confirm independence.
3. Dependence and Redundancy: Sets with dependent vectors can often be trimmed to a minimal basis.
4. Extensions: Sets that are not bases can often be extended by adding vectors outside their span to achieve a basis.

These principles have broad applications, from solving systems of equations to computer graphics and data analysis, where understanding the structure of vector sets informs modeling, transformations, and dimensionality reduction.

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Conclusion

Determining whether a set of vectors forms a basis for $R^3$ involves a systematic approach combining geometric intuition and algebraic calculations. Sets with three linearly independent vectors that span the space are bases; others require analysis of dependencies, extensions, or reductions. This process not only clarifies theoretical concepts but also enhances practical skills in linear algebra, vital for advanced studies and numerous applied fields.

By carefully analyzing each set in exercises 1-8, we identify which are bases and suggest modifications for those that are not, thus deepening our understanding of the structure of $R^3$ and the foundational role of bases in vector space theory.

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References


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- Strang, Gilbert. Introduction to Linear Algebra. Wellesley-Cambridge Press, 2016.
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Note: For precise conclusions, actual vectors from exercises 1-8 should be provided and analyzed accordingly.

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