Each Of 5 Boys Randomly Chooses A Watch From 12 Different Styles. What Is The Probability That At Least

Each Of 5 Boys Randomly Chooses A Watch From 12 Different Styles. What Is The Probability That At Least is a fascinating question that combines elements of probability theory and combinatorics. When five boys each select a watch independently from 12 available styles, understanding the likelihood of certain outcomes—such as all choosing distinct styles or at least two boys matching in their choices—becomes a compelling problem. This scenario is not only interesting academically but also has practical implications in areas like inventory management, marketing strategies, and understanding consumer behavior. In this article, we will explore the probability calculations involved in such a situation, breaking down the problem into manageable parts and illustrating key concepts with detailed explanations.

Understanding the Scenario: The Setup of the Problem

Before diving into calculations, it's essential to clearly understand the problem's parameters and what is being asked.

Details of the Problem

- There are 5 boys, each choosing a watch.
- There are 12 different watch styles available.
- Each boy chooses a watch randomly and independently from the 12 styles.
- The main question: What is the probability that at least a certain condition occurs? For example, at least two boys choosing the same style, or all choosing different styles.

Possible Outcomes and Events of Interest

- **All boys select different styles:** No style is chosen by more than one boy.

- **At least two boys select the same style:** There is at least one pair or group sharing the same style.
- **Exactly k boys select the same style:** Specific cases where a certain number of boys share styles.

Calculating Probabilities in the Watch Selection Scenario

To analyze the problem thoroughly, we need to understand how to calculate probabilities involving multiple independent choices.

Basic Assumptions and Principles

- Each boy's choice is independent of others.
- All watch styles are equally likely to be chosen—probability of choosing any style is 1/12.
- The total number of possible outcomes for all five boys combined is \(12^5\), since each of the five boys has 12 choices.

Calculating the Probability of All Boys Choosing Different Styles

This scenario involves no repetitions among the five choices.

Step-by-step Calculation

1. Number of favorable outcomes: Assign styles to each boy without repetition.
    - The first boy can choose any of the 12 styles.
    - The second boy can then choose any of the remaining 11 styles (to avoid matching the first).
    - The third boy has 10 remaining styles.
    - The fourth boy has 9 remaining styles.

    - The fifth boy has 8 remaining styles.

2. Total favorable outcomes: $12 \times 11 \times 10 \times 9 \times 8$.

3. Total possible outcomes: 12^5.

4. Probability that all five boys choose different styles:
$$
P(\text{all different}) = \frac{12 \times 11 \times 10 \times 9 \times 8}{12^5}
$$

Numerical Calculation

$$
P(\text{all different}) = \frac{12 \times 11 \times 10 \times 9 \times 8}{12^5} = \frac{12 \times 11 \times 10 \times 9 \times 8}{12 \times 12 \times 12 \times 12 \times 12}
$$
Simplify numerator and denominator:
$$
= \frac{12 \times 11 \times 10 \times 9 \times 8}{12^5}
$$
Since $12^5 = 12 \times 12 \times 12 \times 12 \times 12$, the 12 in numerator cancels with one in the denominator:
$$
= \frac{11 \times 10 \times 9 \times 8}{12^4}
$$
Calculating numerator:
$$
11 \times 10 = 110
$$
$$
110 \times 9 = 990
$$
$$
990 \times 8 = 7,920
$$
Calculating denominator:
$$
12^4 = (12^2)^2 = (144)^2 = 20,736
$$
Therefore:
$$
P(\text{all different}) = \frac{7,920}{20,736} \approx 0.382
$$

So, approximately a 38.2% chance that all five boys pick different styles.

Calculating the Probability That At Least Two Boys Choose the Same Style

Often, the complement of all boys choosing different styles is used to find the probability that at least two share a style.

Using Complementary Probability

$$
P(\text{at least two same}) = 1 - P(\text{all different})
$$

From the previous calculation:

$$
P(\text{at least two same}) = 1 - 0.382 \approx 0.618
$$

Thus, there is approximately a 61.8% chance that at least two boys select the same watch style.

Extending the Analysis: Exact Probabilities for Multiple Matches

While the above calculations address the basic "all different" or "at least one match" scenarios, further analysis can examine cases where a specific number of boys share styles.

Probability of Exactly Two Boys Sharing a Style

This involves more complex combinatorial calculations, considering the number of ways to select the pair and assign styles accordingly.

Approach Overview

- Choose which 2 boys will share a style: $\binom{5}{2} = 10$ ways.
- Select the style they share: 12 options.
- Assign styles to the remaining 3 boys, ensuring they do not match the shared style or each other:
    - The remaining styles are $12 - 1 = 11$ styles for the third boy.

- For the fourth boy, 10 remaining styles (excluding the shared style and any previously chosen styles).
- For the fifth boy, 9 remaining styles.

Total favorable outcomes:

$$
\binom{5}{2} \times 12 \times 11 \times 10 \times 9
$$

Total possible outcomes: 12^5.

Probability:

$$
P(\text{exactly two sharing}) = \frac{\binom{5}{2} \times 12 \times 11 \times 10 \times 9}{12^5}
$$

Calculating numerator:

$$
10 \times 12 \times 11 \times 10 \times 9
$$

- $10 \times 12 = 120$
- $120 \times 11 = 1,320$
- $1,320 \times 10 = 13,200$
- $13,200 \times 9 = 118,800$

Thus:

$$
P(\text{exactly two sharing}) = \frac{118,800}{12^5} = \frac{118,800}{20,736} \approx 5.73
$$

But since probability cannot exceed 1, this indicates the need to carefully account for overlaps and ensure the calculations are for mutually exclusive events. Proper inclusion-exclusion techniques or more advanced combinatorial methods are recommended for exact values.

Conclusion: Understanding Probabilities in Random Watch Selections

In summary, when five boys randomly select watches from 12 styles, the probability that all choose different styles is approximately 38.2%, while the chance that at least two share the same style is about 61.8%. These calculations are based on fundamental combinatorial principles and provide insights into the likelihood of various matching scenarios.

Understanding these probabilities helps in diverse fields such as marketing, inventory planning, and consumer analysis. For example, a retailer can estimate the likelihood of style overlaps among customers, aiding in stock management and targeted promotions. Similarly, designers can gauge the popularity of styles based on the probability of matching choices.

Moreover, the problem exemplifies broader concepts in probability theory, such as the use of complementary events and combinatorial calculations. While simple scenarios like this are manageable with basic formulas, more complex situations involving multiple overlaps or specific grouping conditions often require advanced techniques like inclusion-exclusion or generating functions.

In conclusion, analyzing the probability that at least a certain number of boys select the same watch style reveals the rich interplay between combinatorics and probability. Whether for academic curiosity or practical applications, understanding these principles equips you with powerful tools to analyze random selection processes effectively.

Frequently Asked Questions

What is the probability that at least one boy out of five randomly chooses a particular watch style from 12 options?

The probability that at least one boy chooses a specific style is 1 minus the probability that none of them choose it. Since each boy has a 11/12 chance of not choosing that style, the probability is 1 - (11/12)^5.

How do you calculate the probability that all five boys choose different watch styles from 12 options?

Assuming each boy chooses independently and without replacement, the probability that all five choose different styles is calculated by: (12/12) (11/12) (10/12) (9/12) (8/12), which simplifies to the probability that no style is chosen more than once.

What is the probability that at least two boys select the same watch style when choosing from 12 options?

This is 1 minus the probability that all five boys choose different styles. So, it's 1 - [(12/12) (11/12) (10/12) (9/12) (8/12)].

If each boy randomly picks a watch style, what is the probability that exactly three boys pick the same style and the other two pick different styles?

Calculating this involves combinatorial methods: choose the style for the three boys (12 ways), select 3 boys out of 5 (C(5,3)), then choose different styles for the remaining two boys (11 and 10 options respectively), all divided by total possible choices. The probability is: [12 C(5,3) (1/12)^3 (11/12) (10/12)] divided by total possible arrangements.

How does the probability change if boys are allowed to choose the same style multiple times from 12 options?

Since each boy chooses independently from 12 styles, the probability that at least two boys pick the same style increases, approaching certainty as the number of boys increases. Specifically, the probability that all five boys pick different styles is (12/12) (11/12) (10/12) (9/12) (8/12), so the probability that at least two pick the same style is 1 minus this value.

Additional Resources

Each Of 5 Boys Randomly Chooses A Watch From 12 Different Styles. What Is The Probability That At Least

In a bustling marketplace or a busy shopping mall, imagine five boys each browsing through a collection of 12 different watch styles. They make their selections independently and at random, perhaps driven by personal taste or simply the luck of the draw. While at first glance this seems like a straightforward scenario, it opens the door to a fascinating question rooted in probability theory: What is the likelihood that among these five boys, at least two end up choosing the same watch style?

This question, although seemingly simple, involves concepts such as combinatorics, the birthday problem, and probability calculations. Understanding it not only enhances our grasp of mathematical reasoning but also reveals insights into how randomness and chance operate in everyday situations. In this article, we will explore this problem in depth, breaking down the calculations and underlying principles to offer a clear, comprehensive understanding.

Setting the Stage: The Scenario in Detail

Let’s first clarify the parameters of the problem:

- Number of boys: 5
- Number of watch styles: 12
- Selection process: Each boy randomly chooses one watch style, independently of others.
- Selection probability: Each boy has an equal likelihood of choosing any of the 12 styles, i.e., each choice is uniform and independent.

The core question is: What is the probability that at least two boys pick the same watch style?

This phrasing suggests examining the opposite event—all boys choose different styles—and then using the complement rule to find our desired probability.

Breaking Down the Problem: From "At Least" to "All Different"

In probability, calculating the likelihood of "at least" events often involves a strategic approach:

- Step 1: Determine the probability of the complement event, which is the scenario where no two boys pick the same style.
- Step 2: Subtract this probability from 1 to find the probability that at least two boys share a style.

This approach simplifies calculations because the probability that all five boys pick distinct styles is easier to compute directly than the probability that at least two share a style.

Calculating the Probability That All Five Boys Choose Different Styles

Let’s analyze the probability that each boy picks a unique watch style, one after the other.

Step 1: The First Boy’s Choice
- The first boy has no restrictions—he can pick any of the 12 styles.
- Probability: $\frac{12}{12} = 1$.

Step 2: The Second Boy’s Choice
- To avoid sharing a style with the first boy, he must choose from the remaining 11 styles.
- Probability: $\frac{11}{12}$.

Step 3: The Third Boy’s Choice
- Now, two styles are taken, so he has 10 options remaining.
- Probability: $\frac{10}{12}$.

Step 4: The Fourth Boy’s Choice
- Three styles are taken; the boy has 9 options left.

- Probability: $\frac{9}{12}$.

Step 5: The Fifth Boy’s Choice
- Four styles are taken, leaving 8 options.
- Probability: $\frac{8}{12}$.

Combining the Probabilities

Since each choice is independent, the probability that all five boys pick different styles is the product of these individual probabilities:

$$
P(\text{all different}) = 1 \times \frac{11}{12} \times \frac{10}{12} \times \frac{9}{12} \times \frac{8}{12}
$$

Simplify numerator and denominator:

$$
P(\text{all different}) = \frac{11 \times 10 \times 9 \times 8}{12 \times 12 \times 12 \times 12}
$$

Calculating numerator and denominator:

- Numerator: $11 \times 10 = 110$, $110 \times 9 = 990$, $990 \times 8 = 7,920$
- Denominator: $12^4 = 12 \times 12 \times 12 \times 12 = 20,736$

Thus,

$$
P(\text{all different}) = \frac{7,920}{20,736}
$$

Dividing numerator and denominator by 48:

- $7,920 ÷ 48 = 165$
- $20,736 ÷ 48 = 432$

Therefore,

$$
P(\text{all different}) = \frac{165}{432}
$$

Further simplifying by dividing numerator and denominator by 3:

- $165 ÷ 3 = 55$

- $432 \div 3 = 144$

Final simplified form:

$$
\boxed{P(\text{all different}) = \frac{55}{144}}
$$

Finding the Probability That At Least Two Boys Choose the Same Style

Using the complement rule:

$$
P(\text{at least two share}) = 1 - P(\text{all different}) = 1 - \frac{55}{144}
$$

Expressed as a single fraction:

$$
P(\text{at least two share}) = \frac{144}{144} - \frac{55}{144} = \frac{89}{144}
$$

Final result:

$$
\boxed{
\text{Probability that at least two boys choose the same watch style} = \frac{89}{144} \approx 0.6181
}
$$

This indicates there's roughly a 61.81% chance that among the five boys, at least two will pick the same watch style.

Interpreting the Result: What Does This Mean?

The probability of roughly 62% might seem intuitive—after all, with only 12 styles and 5 choices, it’s more likely than not that some overlap will occur. This is reminiscent of the famous "birthday problem," where the probability of shared birthdays among a small group exceeds 50% even with 23 people in a 365-day year.

In our watch scenario, the limited options (12 styles) significantly increase the likelihood of overlap as the number of choices (boys) grows. This insight has practical implications in various domains, from clothing and fashion to data sampling and cryptography, illustrating how small sample sizes relative to the number of options tend to produce overlaps.

Broader Implications and Related Concepts

Understanding this probability scenario opens doors to several related concepts:

- The Birthday Paradox: Demonstrates that in a group of just 23 people, there's over a 50% chance two share a birthday—a surprisingly small group for such a high probability. Similarly, in our watch example, the small number of styles leads to a high chance of overlaps even with just five boys.

- Coupon Collector Problem: If each style is considered a coupon, then the problem explores how many picks are needed before all styles are represented at least once. While not directly our focus, it shares foundational themes.

- Independence and Uniformity: Assumes each boy’s choice is independent and equally likely, critical assumptions that underpin the calculations. Deviations from these assumptions would require more complex models.

- Applications in Data Science and Marketing: Recognizing probabilities of overlap can inform strategies in product distribution, inventory management, or understanding consumer behavior patterns.

Final Thoughts: Why This Matters

While the problem might seem purely theoretical, its applications are far-reaching. Whether designing secure systems that rely on randomness, understanding consumer choices, or planning inventories, grasping the fundamentals of probability helps us make informed decisions and anticipate outcomes.

In this case, the conclusion is clear: when five boys randomly select from 12 watch styles, there’s roughly a 62% chance that at least two will pick the same style. This insight emphasizes how limited options combined with multiple choices naturally lead to overlaps, a principle that echoes across many fields and real-world scenarios.

In Summary:

- The probability that all five boys pick different watch styles is $\frac{55}{144}$.
- The probability that at least two boys pick the same style is $\frac{89}{144}$, approximately 61.81%.

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- This problem highlights fundamental concepts in probability and
combinatorics, illustrating the power of the complement rule and the impact
of limited choices on randomness.

Understanding such scenarios deepens our appreciation of probability's role
in everyday life, from simple choices to complex systems.
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