

Estimate The Area Under The Graph Of $F(x) = x^2 + x + 1$ Over The Interval $[0, 4]$ Using Ten Approximating

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Estimating the area under a curve is a fundamental problem in calculus, often approached through techniques such as Riemann sums, trapezoidal rules, and Simpson's rule. In this comprehensive guide, we will explore how to estimate the area under the graph of the function $f(x) = x^2 + x + 1$ over the interval $[0, 4]$ using ten approximations. This process involves partitioning the interval into ten subintervals and calculating the sum of the areas of rectangles or trapezoids that approximate the region under the curve. Understanding this method not only enhances comprehension of integral calculus but also provides practical skills for numerical analysis.

Understanding the Function and the Problem

Function Definition

The given function is:

- $f(x) = x^2 + x + 1$

This is a quadratic function with a positive leading coefficient, which means the parabola opens upward.

Interval of Interest

The interval over which we need to estimate the area is:

- $[0, 4]$

This means our x -values range from 0 to 4.

Objective

Estimate the integral:

- $\int_0^4 (x^2 + x + 1) \, dx$

using ten approximations (subintervals).

Partitioning the Interval

Number of Subintervals

- We are dividing the interval $[0, 4]$ into 10 equal parts.
- Each subinterval has a width Δx .

Calculating Δx

$$\Delta x = \frac{b - a}{n} = \frac{4 - 0}{10} = 0.4$$

where:

- $a = 0$
- $b = 4$
- $n = 10$

Subintervals

- The subintervals are:
- $[0, 0.4], [0.4, 0.8], [0.8, 1.2], [1.2, 1.6], [1.6, 2.0],$
- $[2.0, 2.4], [2.4, 2.8], [2.8, 3.2], [3.2, 3.6], [3.6, 4.0]$

Choosing a Method for Approximation

There are several methods to estimate the area:

1. Left Riemann Sum

- Uses the left endpoints of each subinterval to calculate the height of rectangles.

2. Right Riemann Sum

- Uses the right endpoints.

3. Midpoint Rule

- Uses the midpoints of each subinterval for height.

4. Trapezoidal Rule

- Approximates the area using trapezoids instead of rectangles.

5. Simpson's Rule (Optional)

- Uses parabolic segments; more accurate but more complex.

In this guide, we'll focus on the Left Riemann Sum, Right Riemann Sum, and Trapezoidal Rule for a comprehensive understanding, as they are straightforward for beginners.

Calculating Function Values at Key Points

To perform the approximations, we need to evaluate $f(x)$ at specific points:

- Left endpoints: $(x_0, x_1, \dots, x_9)$
- Right endpoints: $(x_1, x_2, \dots, x_{10})$
- Midpoints: $(x_{i-1} + \frac{\Delta x}{2})$

Calculations:

$$f(x) = x^2 + x + 1$$

For each x :

x	$f(x) = x^2 + x + 1$
0.0	$(0 + 0 + 1 = 1)$
0.4	$(0.16 + 0.4 + 1 = 1.56)$
0.8	$(0.64 + 0.8 + 1 = 2.44)$
1.2	$(1.44 + 1.2 + 1 = 3.64)$
1.6	$(2.56 + 1.6 + 1 = 5.16)$
2.0	$(4 + 2 + 1 = 7)$
2.4	$(5.76 + 2.4 + 1 = 9.16)$
2.8	$(7.84 + 2.8 + 1 = 11.64)$
3.2	$(10.24 + 3.2 + 1 = 14.44)$
3.6	$(12.96 + 3.6 + 1 = 17.56)$
4.0	$(16 + 4 + 1 = 21)$

Applying the Riemann Sum Methods

Left Riemann Sum

- Uses (x_i) as the left point for each rectangle.
- Sum: $L = \sum_{i=0}^{n-1} f(x_i) \Delta x$

Calculations:

$$L = \Delta x \times (f(0) + f(0.4) + f(0.8) + f(1.2) + f(1.6) + f(2.0) + f(2.4) + f(2.8) + f(3.2) + f(3.6))$$

$$L = 0.4 \times (1 + 1.56 + 2.44 + 3.64 + 5.16 + 7 + 9.16 + 11.64 + 14.44 + 17.56)$$

Sum inside parentheses:

$$1 + 1.56 + 2.44 + 3.64 + 5.16 + 7 + 9.16 + 11.64 + 14.44 + 17.56 = 73.00$$

Therefore:

$$L = 0.4 \times 73.00 = 29.2$$

Right Riemann Sum

- Uses (x_i) as the right point:

$$R = \Delta x \times (f(0.4) + f(0.8) + f(1.2) + f(1.6) + f(2.0) + f(2.4) + f(2.8) + f(3.2) + f(3.6) + f(4.0))$$

Calculations:

$$0.4 \times (1.56 + 2.44 + 3.64 + 5.16 + 7 + 9.16 + 11.64 + 14.44 + 17.56 + 21)$$

Sum:

$$\backslash[1.56 + 2.44 + 3.64 + 5.16 + 7 + 9.16 + 11.64 + 14.44 + 17.56 + 21 = 93.6 \backslash]$$

Estimate:

$$\backslash[R = 0.4 \times 93.6 = 37.44 \backslash]$$

Trapezoidal Rule

- Approximates the area by trapezoids:

$$\backslash[T = \frac{\Delta x}{2} \times \left[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n) \right] \backslash]$$

Calculations:

$$\backslash[T = \frac{0.4}{2} \times \left[1 + 2(1.56 + 2.44 + 3.64 + 5.16 + 7 + 9.16 + 11.64 + 14.44 + 17.56) + 21 \right] \backslash]$$

Sum inside:

$$\backslash[2 \times (1.56 + 2.44 + 3.64 + 5.16 + 7 + 9.16 + 11.64 + 14.44 + 17.56) = 2 \times 72.6 = 145.2 \backslash]$$

Now, sum total:

$$\backslash[1 + 145.2 + 21 = 167.2 \backslash]$$

Finally:

Frequently Asked Questions

What is the primary goal when estimating the area under the graph of $f(x) = x^2 + x + 1$ over $[0, 4]$ using ten approximations?

The primary goal is to approximate the definite integral of the function over the interval, providing an estimated area under the curve using ten subdivisions or rectangles.

Which numerical method can be used to estimate the area under the curve with ten approximations?

Methods such as the Riemann sum (left, right, or midpoint) or the Trapezoidal rule can be used to estimate the area with ten subdivisions.

How do you determine the width of each subinterval when dividing $[0, 4]$ into ten parts?

The width of each subinterval, Δx , is calculated as $(4 - 0) / 10 = 0.4$.

What is the difference between using left, right, and midpoint Riemann sums in this context?

Left sums use the function value at the start of each subinterval, right sums at the end, and midpoint sums at the center, each providing different approximations of the area.

How can the Trapezoidal rule improve the accuracy of the area estimate over Riemann sums?

The Trapezoidal rule approximates the area under the curve by summing the areas of trapezoids, which generally provides a more accurate estimate than using rectangles alone.

What is the approximate value of the area under $f(x) = x^2 + x + 1$ over $[0, 4]$ using ten trapezoidal approximations?

Using the trapezoidal rule with ten subdivisions, the estimated area is approximately 22.4 units squared.

Why is it important to choose an appropriate number of subintervals when estimating the area?

Choosing an appropriate number of subintervals balances computational effort and accuracy; more subintervals typically yield a more precise estimate of the integral.

Can increasing the number of approximations beyond ten improve the accuracy? How?

Yes, increasing the number of subintervals reduces the width of each, leading to a closer

approximation of the true area and thus increasing accuracy.

Additional Resources

Estimate The Area Under The Graph Of $F(x) = x^2 + x + 1$ Over The Interval $[0, 4]$ Using Ten Approximating is a classic problem in calculus that illustrates how we can approximate the area under a curve using simple methods before delving into more advanced techniques like definite integrals. This process not only solidifies our understanding of the geometric interpretation of integrals but also provides a practical approach to estimating areas when exact calculations are complex or unnecessary. In this article, we'll explore step-by-step how to estimate this area using ten approximating rectangles, discuss the reasoning behind each step, and analyze the accuracy of our approximation.

Understanding the Problem

The goal is to estimate the area under the graph of the function $f(x) = x^2 + x + 1$ over the interval $[0, 4]$. Visualizing this, the graph is a parabola opening upward, shifted vertically, and the area under the curve from 0 to 4 can be thought of as the total space between the x-axis and the curve within these bounds.

Why approximate?

Exact calculation involves definite integrals, which are straightforward for polynomial functions like this. However, the purpose of estimation is often to understand the process of approximation techniques, such as Riemann sums, which are foundational to integral calculus. Using ten rectangles (or subintervals) provides a manageable yet reasonably accurate approximation.

Setting Up the Approximation

Step 1: Divide the Interval

Since we're dividing the interval $[0, 4]$ into ten equal parts, each subinterval's width, (Δx) , is:

$$\Delta x = \frac{b - a}{n} = \frac{4 - 0}{10} = 0.4$$

This means each rectangle spans an x-interval of length 0.4.

Step 2: Determine the Subintervals

The subintervals are:

Subinterval	Start (x_i)	End (x_{i+1})	Midpoint (x_i^*)
1	0.0	0.4	0.2
2	0.4	0.8	0.6

3	0.8	1.2	1.0
4	1.2	1.6	1.4
5	1.6	2.0	1.8
6	2.0	2.4	2.2
7	2.4	2.8	2.6
8	2.8	3.2	3.0
9	3.2	3.6	3.4
10	3.6	4.0	3.8

Approximating the Area: Riemann Sum Approach

To estimate the area, we can use Riemann sums, which involve summing up the areas of rectangles that approximate the region under $f(x)$.

Step 3: Choosing Sample Points

There are different ways to choose the height of each rectangle:

- Left Riemann Sum: Use the value at the left endpoint of each subinterval.
- Right Riemann Sum: Use the value at the right endpoint.
- Midpoint Riemann Sum: Use the value at the midpoint.

For a better approximation, the midpoint method often yields the most accurate estimate among these, especially with a small number of subintervals. Let's proceed with midpoints.

Step 4: Calculate the Function Values at Midpoints

Using the midpoints x_i^* :

$$f(x) = x^2 + x + 1$$

Calculate $f(x_i^*)$ for each midpoint:

x_i^*	$f(x_i^*)$
0.2	$(0.2^2 + 0.2 + 1 = 0.04 + 0.2 + 1 = 1.24)$
0.6	$(0.36 + 0.6 + 1 = 1.96)$
1.0	$(1 + 1 + 1 = 3)$
1.4	$(1.96 + 1.4 + 1 = 4.36)$
1.8	$(3.24 + 1.8 + 1 = 6.04)$
2.2	$(4.84 + 2.2 + 1 = 8.04)$
2.6	$(6.76 + 2.6 + 1 = 10.36)$
3.0	$(9 + 3 + 1 = 13)$
3.4	$(11.56 + 3.4 + 1 = 15.96)$
3.8	$(14.44 + 3.8 + 1 = 19.24)$

Calculating the Approximate Area

Step 5: Summing Up

Using the midpoint method, the approximate area is:

$$\text{Area} \approx \sum_{i=1}^{10} f(x_i^*) \Delta x$$

Where $(\Delta x = 0.4)$.

Let's compute:

$$\text{Sum of } f(x_i^*) = 1.24 + 1.96 + 3 + 4.36 + 6.04 + 8.04 + 10.36 + 13 + 15.96 + 19.24$$

Adding these:

- $(1.24 + 1.96 = 3.2)$
- $(3.2 + 3 = 6.2)$
- $(6.2 + 4.36 = 10.56)$
- $(10.56 + 6.04 = 16.6)$
- $(16.6 + 8.04 = 24.64)$
- $(24.64 + 10.36 = 35)$
- $(35 + 13 = 48)$
- $(48 + 15.96 = 63.96)$
- $(63.96 + 19.24 = 83.2)$

Therefore,

$$\text{Approximate area} = 83.2 \Delta x = 33.28$$

Estimated area under the curve $(f(x) = x^2 + x + 1)$ over $[0, 4]$: approximately 33.28 square units.

Analyzing the Accuracy of the Approximation

Step 6: Consider the Error

While this estimate provides a reasonable approximation, it's important to recognize potential sources of error:

- Number of rectangles: Using only ten rectangles introduces some approximation error.
- Method of approximation: Midpoints generally give a better estimate than left or right endpoints for smooth functions like this parabola.
- Function behavior: Since $(f(x))$ is convex (curving upwards), the midpoint sum tends to

underestimate the true area, although for convex functions, it often provides a close estimate.

Step 7: Comparing with the Exact Integral

The exact area under $f(x)$ from 0 to 4 is given by the definite integral:

$$\int_0^4 (x^2 + x + 1) \, dx$$

Calculating:

$$\int x^2 \, dx = \frac{x^3}{3}$$

$$\int x \, dx = \frac{x^2}{2}$$

$$\int 1 \, dx = x$$

Evaluating from 0 to 4:

$$\left[\frac{4^3}{3} + \frac{4^2}{2} + 4 \right] - \left[0 + 0 + 0 \right] = \left[\frac{64}{3} + \frac{16}{2} + 4 \right] = \frac{64}{3} + 8 + 4$$

Simplify:

$$\frac{64}{3} + 12 = \frac{64 + 36}{3} = \frac{100}{3} \approx 33.33$$

Comparison: Our approximation of 33.28 is remarkably close to the exact area of approximately 33.33, demonstrating the effectiveness of the midpoint Riemann sum with ten rectangles.

Extending the Analysis

Using Other Approximation Methods

- Left Riemann Sum: Slightly underestimates the area because the function is increasing over $[0, 4]$.
- Right

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