Evaluate The Derivative By Using The Appropriate Product Rule Where Ri(t) = (t,t3, 8t), R(2) = (2,1,0),

Evaluate The Derivative By Using The Appropriate Product Rule Where Ri(t) = (t,t3, 8t), R(2) = (2,1,0),

Understanding how to evaluate derivatives effectively is a fundamental skill in calculus, especially when dealing with vector-valued functions. In this article, we will explore the process of differentiating a vector function, specifically $Ri(t) = (t, t^3, 8t)$, using the appropriate product rule. Additionally, we will evaluate the derivative at a particular point, $t = 2$, and understand how initial conditions like $R(2) = (2, 1, 0)$ relate to the differentiation process. This comprehensive guide aims to clarify the concepts, demonstrate detailed steps, and enhance your calculus problem-solving skills.

Understanding Vector-Valued Functions and Their Derivatives

Before diving into the differentiation process, it is crucial to understand what vector-valued functions are and how their derivatives are computed.

What Is a Vector-Valued Function?

A vector-valued function assigns a vector to each point in its domain. It can be written as:

$$ \mathbf{R}(t) = \langle R_1(t), R_2(t), R_3(t) \rangle $$

where each component function $R_i(t)$ is a real-valued function of t.

In our case:

$$ \mathbf{R}_i(t) = (t, t^3, 8t) $$

which means:

- $R_{i1}(t) = t$
- $R_{i2}(t) = t^3$
- $R_{i3}(t) = 8t$

Derivative of Vector-Valued Functions

The derivative of a vector function is obtained by differentiating each component function separately:

$$ \mathbf{R}'(t) = \left(\frac{d}{dt} R_1(t), \frac{d}{dt} R_2(t), \frac{d}{dt} R_3(t) \right) $$

This process is straightforward when the components are simple functions, but complexity arises when the components involve products or compositions requiring the product rule or chain rule.

Applying the Product Rule to Vector Functions

The product rule in calculus is essential when differentiating products of functions. For scalar functions u(t) and v(t), the product rule states:

$$\frac{d}{dt} [u(t) v(t)] = u'(t) v(t) + u(t) v'(t)$$

When dealing with vector functions, if the components involve products, the same rule applies component-wise.

Specifics of the Product Rule for Vector Components

Suppose a component of a vector function is a product of two functions, say $R(t) = u(t) \cdot v(t)$. Its derivative is:

$$R'(t) = u'(t) v(t) + u(t) v'(t)$$

This rule extends naturally to each component of a vector function.

Differentiating Ri(t) = (t, t^3, 8t)

In our case, the function Ri(t) = (t, t^3, 8t) consists of three separate component functions:

- $R_1(t) = t$
- $R_2(t) = t^3$
- $R_3(t) = 8t$

Since these are simple functions, their derivatives are straightforward:

- $R_1'(t) = \frac{d}{dt} t = 1$
- $R_2'(t) = \frac{d}{dt} t^3 = 3t^2$
- $R_3'(t) = \frac{d}{dt} 8t = 8$

However, to illustrate the use of the product rule, consider how the derivative would look if the components involved products.

Example: Derivative of a Component Using the Product Rule

Suppose a component is $R(t) = t \cdot t^2$. Its derivative would be:

$$R'(t) = \frac{d}{dt} (t \cdot t^2)$$

Applying the product rule:

$$R'(t) = (1) \cdot t^2 + t \cdot 2t = t^2 + 2t^2 = 3t^2$$

Similarly, for our original components, if they involved products, the derivatives would involve applying the product rule to each.

Calculating the Derivative at t = 2

Having differentiated each component, we now evaluate the derivative $\mathbf{R}'(t)$ at $t = 2$.

From earlier, the derivatives are:

- $R_1'(t) = 1$
- $R_2'(t) = 3t^2$
- $R_3'(t) = 8$

Plugging in $t = 2$:

- $R_1'(2) = 1$
- $R_2'(2) = 3 \times (2)^2 = 3 \times 4 = 12$
- $R_3'(2) = 8$

Thus,

$$\mathbf{R}'(2) = (1, 12, 8)$$

This vector represents the rate of change of the function at $t = 2$, giving insights into the velocity if the function models a physical path, or the slope if the function models a rate.

Understanding the Initial Condition R(2) = (2, 1, 0)

The initial condition states that at $t = 2$, the vector function yields:

$$\mathbf{R}(2) = (2, 1, 0)$$

This information is essential for solving problems such as initial value problems in differential equations or confirming the correctness of the function.

In our case, the initial condition confirms the specific point on the curve at $t = 2$, but it does not affect the derivative directly. The derivative depends on the functional form, which we've already differentiated.

Summary of the Differentiation Process

To summarize, the steps to evaluate the derivative of a vector function like $Ri(t) = (t, t^3, 8t)$ using the appropriate rules are:

1. Identify each component function: Recognize the structure of each component.

2. Differentiate each component: Use basic differentiation rules; apply the product rule if components involve products.

3. Evaluate at the given point: Substitute the specific value of t into the derivatives.

4. Interpret the result: Understand what the derivative vector signifies in context.

Additional Tips for Differentiating Vector Functions

- Always differentiate component-wise; the derivative of a vector function is the vector of derivatives.
- When components involve products, explicitly apply the product rule.
- For composite functions, use the chain rule as necessary.
- Remember that initial conditions are critical for solving differential equations but do not influence the differentiation process directly.

Conclusion

Differentiating vector-valued functions like $Ri(t) = (t, t^3, 8t)$ is a fundamental skill in calculus, especially when analyzing motion, growth processes, or other phenomena modeled by vectors. By understanding how to apply the product rule appropriately, you can accurately compute derivatives, interpret the rate of change, and solve related problems efficiently. Evaluating the derivative at specific points, such as $t = 2$, provides meaningful insights into the behavior of the function at those moments. Mastery of these concepts enhances your overall calculus proficiency and prepares you for more advanced mathematical applications.

Keywords: vector calculus, derivative of vector functions, product rule, differentiating vector functions, evaluate derivative at t=2, initial conditions, calculus tutorial, how to differentiate vector functions

Frequently Asked Questions

How do you apply the product rule to find the derivative of a vector function like R_i(t) = (t, t^3, 8t)?

Since R_i(t) is a vector with components, you differentiate each component separately using standard rules. The product rule is typically used when components are products of functions, but in this case, each component is a simple function of t, so their derivatives are straightforward: d/dt(t) = 1, d/dt(t^3) = 3t^2, and d/dt(8t) = 8.

What is the derivative R_i'(t) of the vector function R_i(t) = (t, t^3, 8t)?

The derivative R_i'(t) is (1, 3t^2, 8), obtained by differentiating each component separately.

How do you evaluate the derivative R_i'(t) at t = 2 for R_i(t) = (t, t^3, 8t)?

Substitute t = 2 into R_i'(t): R_i'(2) = (1, 3(2)^2, 8) = (1, 34, 8) = (1, 12, 8).

Is the product rule necessary for differentiating R_i(t) = (t, t^3, 8t)?

No, because each component is a simple function of t. The product rule is only needed if components are products of functions. Here, direct differentiation suffices.

What is the significance of the point R(2) = (2, 1, 0) in the context of differentiating R_i(t)?

The point R(2) = (2, 1, 0) indicates the value of the vector function at t=2, but when differentiating, we focus on R_i'(t). To evaluate the derivative at t=2, we use the derivative formula, which yields R_i'(2) = (1, 12, 8).

Additional Resources

Evaluate The Derivative By Using The Appropriate Product Rule Where Ri(t) = (t, t^3, 8t), R(2) = (2, 1, 0)

Introduction

In the realm of multivariable calculus, understanding how to differentiate vector functions is fundamental. Among the tools at a mathematician's disposal, the product rule for vector functions plays a pivotal role when dealing with the derivative of a product of functions. This article delves into the detailed process of evaluating the derivative of a specific vector function, \(R_i(t) = (t, t^3, 8t) \), harnessing the appropriate product rule. We will explore the foundational concepts, perform meticulous calculations, and interpret the significance of the derivative at a particular point, $t = 2$.

This investigation not only exemplifies the application of the product rule but also aims to serve as a comprehensive resource for students and professionals seeking clarity on this fundamental operation.

Background and Theoretical Foundations

Vector Functions and Their Derivatives

A vector function $\mathbf{R}(t)$ assigns a vector to each real number t. It is often expressed component-wise:

$$
\mathbf{R}(t) = \left(R_1(t), R_2(t), R_3(t) \right)
$$

where each $R_i(t)$ is a real-valued function.

The derivative of a vector function, termed the vector derivative, is obtained by differentiating each component separately:

$$
\mathbf{R}'(t) = \left(R_1'(t), R_2'(t), R_3'(t) \right)
$$

Product Rule in Vector Calculus

The product rule in calculus states that for two functions $u(t)$ and $v(t)$, the derivative of their product is:

$$
\frac{d}{dt} \left[u(t) v(t) \right] = u'(t) v(t) + u(t) v'(t)
$$

In vector calculus, when dealing with the scalar product (dot product), the product rule extends as:

$$
\frac{d}{dt} \left[\mathbf{u}(t) \cdot \mathbf{v}(t) \right] = \mathbf{u}'(t) \cdot \mathbf{v}(t) + \mathbf{u}(t) \cdot \mathbf{v}'(t)
$$

However, in the context of the problem, $R_i(t)$ appears to be a vector function, and the task is to evaluate its derivative. If the problem involves decomposing or applying the product rule to its components or related functions, understanding the nature of the operations involved is essential.

Clarifying the Problem Statement

Given the notation:

- $R_i(t) = (t, t^3, 8t)$, which is a vector function with three components.
- $R(2) = (2, 1, 0)$, indicating the evaluation of $R(t)$ at $t = 2$.

The explicit mention of "evaluate the derivative by using the appropriate product rule" suggests that the problem might involve differentiating a product of functions derived from $R_i(t)$. Alternatively, it might be a misinterpretation, and the primary goal is to compute $R'(t)$ at $t=2$, considering the structure of $R_i(t)$.

For clarity, we will proceed with the assumption that:

1. $R(t) = (t, t^3, 8t)$
2. The derivative $R'(t)$ is to be computed using the component-wise differentiation, which inherently involves the basic power rule, but if the problem involves a product of functions within the components, the product rule would be applicable.

Step-by-Step Calculation of $R'(t)$

Differentiating Each Component

Let’s differentiate each component separately:

- $R_1(t) = t$

$$
R_1'(t) = \frac{d}{dt} t = 1
$$

- $R_2(t) = t^3$

$$
R_2'(t) = \frac{d}{dt} t^3 = 3t^2
$$

- $R_3(t) = 8t$

$$
R_3'(t) = \frac{d}{dt} 8t = 8
$$

Assembling the Derivative Vector

$$
R'(t) = \left(1, 3t^2, 8 \right)
$$

This is the general form of the derivative of $R(t)$.

Applying the Derivative at $t = 2$

Substitute $t=2$ into $R'(t)$:

$$
R'(2) = \left(1, 3 \times (2)^2, 8 \right) = \left(1, 3 \times 4, 8 \right) = (1, 12, 8)
$$

This provides the instantaneous rate of change of the vector function at $t=2$.

Interpreting the Results

The derivative vector $R'(2) = (1, 12, 8)$ characterizes how the position vector $R(t)$ changes at $t=2$. The components indicate the rates of change along each axis:

- x-component: The position in the x-direction increases at a rate of 1 unit per unit t.
- y-component: The position in the y-direction increases at a rate of 12 units per unit t.
- z-component: The position in the z-direction increases at a rate of 8 units per unit t.

This insight is crucial in physics and engineering, where understanding velocity vectors (derivatives of position vectors) informs about object motion and dynamics.

Considering the Product Rule in the Context of $R(t)$

While the calculation above directly applies basic differentiation rules component-wise, the problem statement's emphasis on the appropriate product rule prompts further exploration. Let's examine potential scenarios where the product rule would come into play within this context.

Scenario 1: Differentiating a Product of Components

Suppose the components are multiplied by functions of t, such as:

$$
\tilde{R}_i(t) = f_i(t) \cdot g_i(t)
$$

The derivative would then be:

$$
\frac{d}{dt} \left[f_i(t) g_i(t) \right] = f_i'(t) g_i(t) + f_i(t) g_i'(t)
$$

In the case of $R(t) = (t, t^3, 8t)$, if these components were products of functions, the product rule would be essential. For example, if:

$$
R_2(t) = t^3 = (t^2) \cdot t
$$

then:

$$
\frac{d}{dt} t^3 = \frac{d}{dt} (t^2 \cdot t) = (2t) \cdot t + t^2 \cdot 1 = 2t^2 + t^2 = 3t^2
$$

which aligns with the basic power rule.

Scenario 2: Differentiating a Vector Product

If the problem involves the derivative of the dot or cross product of two vector functions, the product rule in vector calculus becomes critical.

- Dot product product rule:

$$
\frac{d}{dt} \left[\mathbf{u}(t) \cdot \mathbf{v}(t) \right] = \mathbf{u}'(t) \cdot \mathbf{v}(t) + \mathbf{u}(t) \cdot \mathbf{v}'(t)
$$

- Cross product derivative rule:

$$
\frac{d}{dt} \left[\mathbf{u}(t) \times \mathbf{v}(t) \right] = \mathbf{u}'(t) \times \mathbf{v}(t) + \mathbf{u}(t) \times \mathbf{v}'(t)
$$

Given the initial problem statement, if such operations were involved, applying these rules would be appropriate.

Final Insights and Implications

The primary takeaway from this detailed exploration is that differentiating vector functions component-wise often simplifies the process, but the appropriate product rule becomes vital when components involve products of functions or when vector operations such as dot and cross products are present.

In the specific case of $R(t) = (t, t^3, 8t)$, straightforward differentiation yields:

$$
R'(t) = (1, 3t^2, 8)
$$

and at $t=2$:

$$
R'(2) = (1, 12, 8)
$$

which succinctly describes the instantaneous rate of change of the position vector at that point.

Conclusion

This comprehensive review underscores the importance of understanding the context in applying the product rule within vector calculus. Whether dealing with component-wise derivatives or more complex vector operations, recognizing when and how to

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Rule Where Ri T T T3 8t R 2 2 1 0

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