Find All Points On Xy=exy Where The Tangent Line Is Horizontal.a.(0,1)b.(0,0)c.(1,0)d.None Of The Above

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Understanding how to find points on a given curve where the tangent line is horizontal is a fundamental skill in calculus. Such points, known as points of horizontal tangents, occur where the derivative of the function (or the slope of the tangent line) equals zero. In this article, we will explore how to identify all points on the curve defined by the equation $xy = e^{xy}$ where the tangent line is horizontal, and analyze the given options carefully.

Introduction to the Problem

The problem asks us to find all points (x, y) on the curve $xy = e^{xy}$ where the tangent line is horizontal. The options provided are specific points:

- a. (0, 1)
- b. (0, 0)
- c. (1, 0)
- d. None of the Above

Before delving into calculations, it's essential to understand the concepts involved:

- Horizontal Tangent Line: A tangent line is horizontal when its slope $dy/dx = 0$.
- Implicit Differentiation: Since the given relation involves both x and y implicitly, we will need to differentiate implicitly to find dy/dx.

Step 1: Understand the Equation $xy = e^{xy}$

The curve is given implicitly by:

$$
xy = e^{xy}
$$

This is a transcendental equation involving both algebraic and exponential components. Our goal is to find points where the tangent line to this curve is horizontal, which involves calculating the derivative \(dy/dx\).

Step 2: Differentiating Implicitly to Find \(dy/dx\)

To find the slope of the tangent line at any point \((x, y)\), differentiate both sides of the equation with respect to \(x\).

Starting with:

\[
xy = e^{xy}
\]

Differentiate both sides:

\[
\frac{d}{dx}(xy) = \frac{d}{dx}(e^{xy})
\]

Using the product rule on the left:

\[
x \frac{dy}{dx} + y = e^{xy} \cdot (x \frac{dy}{dx} + y \frac{dy}{dx}) \quad \text{(chain rule applied to RHS)}
\]

Note that:

\[
\frac{d}{dx}(xy) = x \frac{dy}{dx} + y
\]

and

\[
\frac{d}{dx}(e^{xy}) = e^{xy} \cdot \frac{d}{dx}(xy) = e^{xy} (x \frac{dy}{dx} + y)
\]

Thus, the derivative becomes:

\[
x \frac{dy}{dx} + y = e^{xy} (x \frac{dy}{dx} + y)
\]

Step 3: Solving for dy/dx

Rearranged, the expression is:

$$
x \frac{dy}{dx} + y = e^{xy} (x \frac{dy}{dx} + y)
$$

Bring all terms to one side:

$$
x \frac{dy}{dx} + y - e^{xy} (x \frac{dy}{dx} + y) = 0
$$

Factor out $\frac{dy}{dx}$:

$$
\left(x - e^{xy} x \right) \frac{dy}{dx} + \left(y - e^{xy} y \right) = 0
$$

Simplify:

$$
x (1 - e^{xy}) \frac{dy}{dx} + y (1 - e^{xy}) = 0
$$

Factor the common term $(1 - e^{xy})$:

$$
(1 - e^{xy}) \left(x \frac{dy}{dx} + y \right) = 0
$$

This gives us two cases:

1. $1 - e^{xy} = 0$

2. $x \frac{dy}{dx} + y = 0$

Step 4: Find Points Where the Tangent Line Is Horizontal

A horizontal tangent occurs when $\frac{dy}{dx} = 0$.

From the second case:

$$
x \frac{dy}{dx} + y = 0
$$

Set $\frac{dy}{dx} = 0$:

$$
x \times 0 + y = 0 \Rightarrow y = 0
$$

So, the points with horizontal tangents satisfy $y = 0$.

From the first case:

$$
1 - e^{xy} = 0 \Rightarrow e^{xy} = 1
$$

Since $e^{xy} = 1$, then:

$$
xy = 0
$$

This implies either:

- $x = 0$, or
- $y = 0$.

Now, we need to find the points (x, y) on the original curve $xy = e^{xy}$ satisfying these conditions.

Step 5: Find All Points on the Curve Where $y = 0$ or $xy=0$ and the Curve Holds

Let's analyze the points based on the conditions:

Case 1: $y = 0$

Plug into the original equation:

$$
x \times 0 = e^{x \times 0} \Rightarrow 0 = e^{0} = 1
$$

This is not true unless $0 = 1$, which is false. Therefore, no points with $y=0$ satisfy the original equation when $y=0$.

Conclusion: Points with $y=0$ are not on the curve.

Case 2: $x = 0$

Plug into the original:

$$
0 \times y = e^{0 \times y} \Rightarrow 0 = e^{0} \Rightarrow 0 = 1
$$

Again, false. So, points with $x=0$ are not on the curve either.

Step 6: Re-express the Conditions and Find Actual Points

Our earlier derivative analysis yielded that the tangent is horizontal where:

- Either $y=0$ (which does not lie on the curve),
- Or $xy=0$ (which leads to no points on the curve).

But, wait — we need to double-check the initial points in the options and verify whether they satisfy the original equation.

Step 7: Verify the Options

Let's see if the points provided satisfy $xy = e^{xy}$:

Option a: (0, 1)

$$
x y = 0 \times 1 = 0
$$
$$
e^{xy} = e^{0} = 1
$$

Equation:

$$
0 = 1

\]

False. So, (0, 1) is not on the curve.

Option b: (0, 0)

\[
0 \times 0 = 0
\]
\[
e^{0} = 1
\]

Equation:

\[
0 = 1
\]

False. So, (0, 0) is not on the curve.

Option c: (1, 0)

\[
1 \times 0 = 0
\]
\[
e^{0} = 1
\]

Equation:

\[
0 = 1
\]

False. So, (1, 0) is not on the curve.

Option d: None of the Above

Since none of the provided points satisfy the original relation \(xy = e^{xy} \), and the derivative analysis suggests no points of horizontal tangent exist on the given points, the conclusion is:

Answer: d. None of the Above

Summary and Final Remarks

In this detailed exploration, we:

- Differentiated implicitly to find the derivative dy/dx of the curve $xy = e^{xy}$.
- Identified that the tangent line is horizontal where $y=0$ or $xy=0$, but these points do not satisfy the original equation.
- Verified the provided options against the original curve and found none of them satisfy the defining relation.
- Concluded that none of the listed points are points on the curve where the tangent line is horizontal.

Conclusion

The problem demonstrates the importance of implicit differentiation and careful verification of points against the original equation. It also emphasizes that sometimes, the solutions to the derivative conditions do not correspond to points on the original curve, highlighting the need for thorough validation.

Final answer: d. None of the Above.

Understanding the principles behind such problems enhances your calculus skills and prepares you for more complex implicit differentiation and tangent line analyses in advanced mathematics.

Frequently Asked Questions

What is the equation of the curve for which we are finding points where the tangent line is horizontal?

The curve is given by the equation $xy = e^{xy}$.

How do we find points on the curve $xy = e^{xy}$ where the tangent line is horizontal?

We differentiate the equation implicitly to find dy/dx and set it equal to zero, since a horizontal tangent occurs when $dy/dx = 0$.

What is the first step in differentiating the equation $xy = e^{xy}$ implicitly?

Differentiate both sides with respect to x, applying the product rule to xy and the chain rule to e^{xy}.

After differentiation, what condition must be satisfied for the tangent to be horizontal?

Dy/dx must be equal to zero.

Which points among the options satisfy the condition for a horizontal tangent line?

Option b. (0,0) is the correct point where the tangent line is horizontal.

Why does the point (0,1) not satisfy the condition for a horizontal tangent?

Because at that point, the derivative Dy/dx does not equal zero when evaluated, so the tangent is not horizontal there.

Are the options (1,0) or 'None of the Above' correct for points with horizontal tangent lines?

No, only (0,0) satisfies the condition; (1,0) does not, and 'None of the Above' is incorrect.

What is the final answer to the question about points where xy = e^{xy} and the tangent is horizontal?

The correct point is (0,0).

Additional Resources

Find All Points On XY = e^{xy} Where The Tangent Line Is Horizontal is an intriguing calculus problem that combines implicit differentiation, the properties of exponential functions, and the geometric interpretation of derivatives. This problem asks us to identify all points on the curve defined by the equation XY = e^{XY} where the tangent line to the curve is horizontal. Understanding the behavior of the tangent line—specifically when its slope is zero—is fundamental in calculus, as it signals potential local maxima, minima, or points of inflection. This detailed review aims to explore the problem thoroughly, breaking down the concepts involved, solving the problem step-by-step, and analyzing the solutions relative to the options provided.

Understanding the Problem

Equation and Its Implications

The given equation:

$$XY = e^{XY}$$

is a form of an implicit relationship between x and y. It implicitly defines y as a function of x, but not explicitly. The key task is to find all points (x, y) on this curve where the tangent line is horizontal.

A horizontal tangent line occurs where the derivative of y with respect to x (denoted as dy/dx) equals zero. Therefore, the first step is to understand how to find dy/dx given the implicit equation, and then determine where it equals zero.

Why Horizontal Tangents Matter

Identifying points with horizontal tangents is crucial because:

- They often correspond to local maxima or minima.
- They can be points of inflection or saddle points.
- They reveal the shape and behavior of the curve.

In the context of the equation $XY = e^{XY}$, understanding these points helps visualize the curve's geometry and its critical points.

Mathematical Approach to the Problem

Implicit Differentiation

Given the equation:

$$XY = e^{XY}$$

we differentiate both sides with respect to x, treating y as a function of x ($y = y(x)$). Since both X and Y depend on x, we need to apply the product rule to XY and the chain rule to e^{XY}.

Differentiating the left side:

$$\frac{d}{dx}(XY) = X \frac{dy}{dx} + y \frac{dX}{dx}$$

But since $X = x$, $dX/dx = 1$, so:

$$\frac{d}{dx}(XY) = x \frac{dy}{dx} + y$$

Differentiating the right side:

$$ \frac{d}{dx} (e^{XY}) = e^{XY} \frac{d}{dx}(XY) $$

Again, using the product rule:

$$ \frac{d}{dx}(XY) = x \frac{dy}{dx} + y $$

which is the same as above.

Putting it all together:

$$ x \frac{dy}{dx} + y = e^{XY} (x \frac{dy}{dx} + y) $$

Solving for dy/dx

Rearranged:

$$ x \frac{dy}{dx} + y = e^{XY} (x \frac{dy}{dx} + y) $$

Subtract $e^{XY} (x \frac{dy}{dx} + y)$ from both sides:

$$ x \frac{dy}{dx} + y - e^{XY} (x \frac{dy}{dx} + y) = 0 $$

Factoring:

$$ (x \frac{dy}{dx} + y)(1 - e^{XY}) = 0 $$

This product equals zero when either:

1. $x \frac{dy}{dx} + y = 0$, or
2. $1 - e^{XY} = 0$

Case 1: $x \frac{dy}{dx} + y = 0$

Solve for dy/dx:

$$ x \frac{dy}{dx} = - y $$
$$ \frac{dy}{dx} = - \frac{y}{x} $$

The horizontal tangent occurs when:

$$ \frac{dy}{dx} = 0 \Rightarrow - \frac{y}{x} = 0 \Rightarrow y = 0 $$

Thus, the points with $y=0$ on the curve could have horizontal tangents, provided they satisfy the original equation.

Case 2: $1 - e^{XY} = 0$

$$ e^{XY} = 1 $$

Since the exponential function equals 1 only when its exponent is zero:

$$ XY = 0 $$

This gives two possible subcases:

- When $X = x = 0$
- When $Y = y = 0$

Note that in the original equation, $XY = e^{XY}$, substituting $XY=0$:

$$ 0 = e^{0} = 1 $$

But this is a contradiction unless the original equation is satisfied with $XY=0$.

Let's verify whether points with $XY=0$ satisfy the original:

- If $XY=0$, then the original equation is:

$$ 0 = e^{0} = 1 $$

which does not satisfy the equation unless the right side is also zero, but $e^{0} = 1$. Therefore, the points where $XY=0$ do not satisfy the original equation unless the original equation is inconsistent, which it isn't.

Hence, the only feasible solution from the second case is when $XY=0$, but it does not satisfy the original equation unless there's a special context. The better approach is to check whether points with $Y=0$ or $X=0$ satisfy the original.

Finding the Points with $Y=0$

Suppose $Y=0$:

$$ XY = 0 \Rightarrow 0 = e^{0} = 1 $$

which is inconsistent, so $Y=0$ does not correspond to points on the original curve.

Similarly, for $X=0$:

$$0 \times y = e^{0 \times y} \Rightarrow 0 = 1$$

which is inconsistent as well.

So, points where $XY=0$ do not satisfy the original equation.

Summary of Findings

- From the derivative analysis, the points where the tangent line is horizontal occur at $y=0$ (from Case 1).
- However, we must verify whether such points lie on the curve $XY = e^{XY}$.

Checking the Candidate Points: $y=0$

Substitute $y=0$ into the original equation:

$$x \times 0 = e^{x \times 0} \Rightarrow 0 = e^{0} = 1$$

which is false unless $0=1$. Therefore, points with $y=0$ are not on the original curve.

Reassessing the Solution: When does $dy/dx=0$ occur?

Recall that the derivative dy/dx is:

$$\frac{dy}{dx} = - \frac{y}{x}$$

which is zero only when $y=0$.

But since $y=0$ does not satisfy the original equation (as shown), the only potential points where the tangent is horizontal are those where the derivative formally equals zero but are not on the curve.

Thus, the only remaining candidate is the case when:

$$e^{XY} = 1 \Rightarrow XY=0$$

and the points satisfy the original equation.

Given the earlier contradiction, revisit the original curve:

$$ XY = e^{XY} $$

Suppose $XY = c$, then:

$$ c = e^{c} $$

which is a well-known equation. The solutions to:

$$ c = e^{c} $$

are called the Lambert W function solutions. The key point is, the only real solution to:

$$ c = e^{c} $$

is at $c = W(1)$, where W is Lambert W function.

The Lambert W function satisfies:

$$ W(z) e^{W(z)} = z $$

For $z=1$:

$$ W(1) e^{W(1)} = 1 $$

which has a real solution at $W(1) \approx 0.567$.

Therefore, the solutions to $c = e^{c}$ are $c \approx 0.567$.

Hence, the equation $XY = c$ with $c \approx 0.567$ satisfies:

$$ c = e^{c} $$

which means the points (x, y) on the curve satisfy:

$$ xy = c $$

and at these points, the derivative:

$$ \frac{dy}{dx} = - \frac{y}{x} $$

which can be zero only if $y=0$, but as shown earlier, $XY=c \neq 0$, so $y \neq 0$ in these cases.

Conclusion:

- The points where the tangent line is horizontal occur where the

Find All Points On Xy Exy Where The Tangent Line Is Horizontal A 0 1 B 0 0 C 1 0 D None Of The Above

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