Find All Solutions Of The Equation 2 Sin 2 X Cos X = 1 2sin2x-cosx=1 In The Interval [0 , 2) . [0,2).

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Understanding how to solve trigonometric equations is fundamental in mathematics, especially in fields such as calculus, physics, and engineering. The given equation involves multiple trigonometric functions—sine and cosine—and requires an analytical approach to find all solutions within a specified interval. This article provides a comprehensive guide to solving the equation:

$$ 2 \sin 2x \cos x = 1 \quad \text{or equivalently} \quad 2 \sin^2 x - \cos x = 1 $$

over the interval $[0, 2)$.

Understanding the Given Equation

Before diving into the solution process, it's crucial to analyze and understand the structure of the equation.

The Original Equation

The equation is presented as:

$$ 2 \sin 2x \cos x = 1 $$

which can be rewritten as:

$$ 2 \sin^2 x - \cos x = 1 $$

This indicates two forms are considered equivalent, and solving either form will lead to the solutions within the interval $[0, 2)$.

Key Trigonometric Identities

- Double-Angle Identity for sine:

$$ \sin 2x = 2 \sin x \cos x $$

- Expressing $\sin 2x$ in the original equation:

$$ 2 \sin 2x \cos x = 2 \cdot (2 \sin x \cos x) \cdot \cos x = 4 \sin x \cos^2 x $$

However, notice that in the initial form, the equation is:

$$ 2 \sin 2x \cos x = 1 $$

or

$$ 2 \sin^2 x - \cos x = 1 $$

which suggests that perhaps the equation has been manipulated into different forms for ease of solving.

Step-by-Step Solution Approach

To find all solutions in $[0, 2)$, we'll analyze each form of the equation separately.

1. Solving the Equation: $2 \sin 2x \cos x = 1$

This form involves the double angle and product of sine and cosine.

Step 1: Rewrite the equation using identities

Recall:

$$ \sin 2x = 2 \sin x \cos x $$

Substitute into the equation:

$$ 2 \times 2 \sin x \cos x \times \cos x = 1 $$
$$ 4 \sin x \cos^2 x = 1 $$

which simplifies the equation to:

$$ 4 \sin x \cos^2 x = 1 $$

or

$$\sin x \cos^2 x = \frac{1}{4}$$

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2. Solving the Transformed Equation: $\sin x \cos^2 x = \frac{1}{4}$

This is a transcendental equation involving both $\sin x$ and $\cos x$, which can be expressed in terms of a single trigonometric function using identities.

Since $\cos^2 x = 1 - \sin^2 x$, rewrite as:

$$\sin x (1 - \sin^2 x) = \frac{1}{4}$$

Let $y = \sin x$, then:

$$y (1 - y^2) = \frac{1}{4}$$

which expands to:

$$y - y^3 = \frac{1}{4}$$

Rearranged as a cubic:

$$y^3 - y + \left(-\frac{1}{4}\right) = 0$$

or

$$y^3 - y = \frac{1}{4}$$

---

3. Solving the Cubic Equation for y

The cubic equation:

$$y^3 - y = \frac{1}{4}$$

can be written as:

$$y^3 - y - \frac{1}{4} = 0$$

This cubic can be solved analytically using methods such as Cardano's formula, or numerically approximated.

Applying Cardano's Method

The depressed cubic form is:

$$ y^3 + p y + q = 0 $$

with:

- $p = -1$
- $q = -\frac{1}{4}$

Calculate the discriminant:

$$ D = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3 $$

Compute:

$$ \frac{q}{2} = -\frac{1}{8} $$
$$ \left(\frac{q}{2}\right)^2 = \frac{1}{64} $$

and

$$ \frac{p}{3} = -\frac{1}{3} $$
$$ \left(\frac{p}{3}\right)^3 = -\frac{1}{27} $$

Thus:

$$ D = \frac{1}{64} + \left(-\frac{1}{27}\right) = \frac{1}{64} - \frac{1}{27} $$

Find common denominator:

$$ \frac{1}{64} - \frac{1}{27} = \frac{27}{1728} - \frac{64}{1728} = -\frac{37}{1728} $$

Since $D < 0$, the cubic has three real roots.

---

4. Finding the Roots y

When $D < 0$, the solutions can be expressed using trigonometric functions:

$$y_k = 2 \sqrt{-\frac{p}{3}} \cos \left(\frac{\theta + 2\pi k}{3} \right), \quad k=0,1,2$$

where:

$$\theta = \arccos \left(\frac{3q}{2p} \sqrt{-\frac{3}{p}} \right)$$

Calculate:

$$-\frac{p}{3} = -\frac{-1}{3} = \frac{1}{3}$$

$$\sqrt{-\frac{p}{3}} = \sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}}$$

and

$$\frac{3q}{2p} = \frac{3 \times -\frac{1}{4}}{2 \times -1} = \frac{-\frac{3}{4}}{-2} = \frac{\frac{3}{4}}{2} = \frac{3}{8}$$

Calculate θ:

$$\theta = \arccos \left(\frac{3}{8} \times \sqrt{3} \right) = \arccos \left(\frac{3 \sqrt{3}}{8} \right)$$

Numerically,

$$\frac{3 \sqrt{3}}{8} \approx \frac{3 \times 1.732}{8} \approx \frac{5.196}{8} \approx 0.6495$$

So,

$$
\theta \approx \arccos(0.6495) \approx 0.865 \text{ radians}
$$

Now, compute y_k:

$$
y_k = 2 \times \sqrt{\frac{1}{3}} \times \cos \left(\frac{0.865 + 2\pi k}{3} \right)
$$

$$
\sqrt{\frac{1}{3}} \approx 0.577
$$

Therefore,

$$
y_k \approx 2 \times 0.577 \times \cos \left(\frac{0.865 + 2\pi k}{3} \right) \approx 1.1547 \times \cos \left(\frac{0.865 + 2\pi k}{3} \right)
$$

for $k = 0, 1, 2$.

Finding the Corresponding x Values

Recall that:

$$
y = \sin x
$$

and the solutions for y are real numbers within $[-1, 1]$. We need to check whether each root y_k lies within this interval.

1. Check the Roots y_k

Calculate approximate values:

- For $k=0$:

\[
\cos \left(\frac{0.865}{3} \right) \approx \cos(0.288) \approx 0.959
\]
\[
y_0 \approx 1.1547 \times 0.959 \approx 1.107
\]

which is greater than 1, invalid as \(\sin x\) must be \(\leq 1\). So discard.

- For \(k=1 \):

\[
\cos \left(\frac{0.865 + 2\pi}{3} \right) = \

Frequently Asked Questions

How do I find all solutions to the equation 2 sin 2x cos x = 1 in the interval [0, 2π)?

Start by simplifying the equation using identities: 2 sin 2x cos x = 2 2 sin x cos x cos x = 4 sin x cos^2 x. Set this equal to 1 and solve for sin x and cos x within the interval [0, 2π).

What identities can I use to simplify the equation 2 sin 2x cos x = 1?

You can use the double angle identity sin 2x = 2 sin x cos x to rewrite the equation as 2 (2 sin x cos x) cos x = 4 sin x cos^2 x = 1, which simplifies the problem.

Are there special solutions or values of x where the equation simplifies significantly?

Yes, solutions where sin x or cos x are 0 or 1 often simplify the equation. For example, at x = 0, π/2, π, 3π/2, check whether the original equation holds to find potential solutions.

How do I check for extraneous solutions when solving the equation in [0, 2π)?

After finding potential solutions algebraically, substitute them back into the original equation 2 sin 2x cos x = 1 to verify they satisfy the equation within the interval. Discard any that do not.

What is the final set of solutions for the equation 2 sin 2x cos x = 1 in [0, 2π)?

The solutions are $x = \arccos(\sqrt{2}/2)$ and $x = 2\pi - \arccos(\sqrt{2}/2)$, corresponding to angles where the simplified form holds true; exact solutions depend on solving the resulting equations numerically or algebraically.

Additional Resources

Finding all solutions to the trigonometric equation $2 \sin^2 x \cos x = \frac{1}{2} \sin^2 x - \cos x = 1$ within the interval $[0, 2)$ is a classic problem that combines algebraic manipulation, fundamental trigonometric identities, and analytical reasoning. This problem not only tests one's understanding of basic trigonometric functions but also highlights the importance of systematic problem-solving techniques in mathematics. In this comprehensive review, we will explore the step-by-step process of solving this equation, analyze the nature of its solutions, and discuss the broader implications of such problems in mathematical studies.

Understanding the Problem and Its Components

Before diving into the solution, it is essential to thoroughly understand the problem statement, the given equation, and the domain over which solutions are sought.

The Given Equation

The problem presents an equation involving sine and cosine functions:

$$
2 \sin^2 x \cos x = \frac{1}{2} \sin^2 x - \cos x = 1
$$

At first glance, this appears to be a compound or possibly ambiguous expression. To clarify, it seems to combine two equations:

1. $2 \sin^2 x \cos x$
2. $\frac{1}{2} \sin^2 x - \cos x$

and states that both are equal to 1, or perhaps that the entire expression simplifies to 1. However, the most common interpretation is that the equation is:

\[

2 \sin^2 x \cos x = \frac{1}{2} \sin^2 x - \cos x = 1
\]

which suggests the two expressions are equal to each other and both equal to 1.

Clarifying the Equation:

- The proper interpretation is likely that the equation involves two parts:

\[
2 \sin^2 x \cos x = 1
\]
and
\[
\frac{1}{2} \sin^2 x - \cos x = 1
\]

and the problem asks to find solutions where these expressions are satisfied within the interval \([0, 2)\).

Alternatively, it could be read as a single combined equation involving these expressions, but the most logical assumption—based on standard problem statements—is that the original is:

\[
2 \sin^2 x \cos x = 1
\]
and
\[
\frac{1}{2} \sin^2 x - \cos x = 1
\]

and solutions should satisfy these equations within \([0, 2)\).

For clarity, we'll treat the problem as solving the system:

\[
\boxed{
\begin{cases}
2 \sin^2 x \cos x = 1 \\
\frac{1}{2} \sin^2 x - \cos x = 1
\end{cases}
}
\]

within the interval $x \in [0, 2)$.

Step-by-Step Solution Approach

To find all solutions, we need to analyze both equations systematically.

Step 1: Analyze the First Equation $2 \sin^2 x \cos x = 1$

This is a trigonometric equation involving the product of $\sin^2 x$ and $\cos x$.

Key strategies:

- Use identities to rewrite the equation in terms of a single trigonometric function.
- Express $\sin^2 x$ in terms of $\cos x$ using the Pythagorean identity:

$$
\sin^2 x = 1 - \cos^2 x
$$

Rewriting the first equation:

$$
2 (1 - \cos^2 x) \cos x = 1
$$

$$
2 \cos x - 2 \cos^3 x = 1
$$

which simplifies to:

$$
2 \cos x - 2 \cos^3 x = 1
$$

or:

$$
2 \cos x - 2 \cos^3 x - 1 = 0

Dividing through by 2:

$$
\cos x - \cos^3 x - \frac{1}{2} = 0
$$

Let $y = \cos x$. The equation becomes:

$$
y - y^3 = \frac{1}{2}
$$

or:

$$
y^3 - y + \frac{1}{2} = 0
$$

This is a cubic equation in y:

$$
y^3 - y + \frac{1}{2} = 0
$$

Step 2: Solve the Cubic Equation $y^3 - y + \frac{1}{2} = 0$

Cubic solutions:

- The general cubic formula applies, but it's often more practical to analyze the roots graphically or via rational root testing.

- Rational root test: possible roots are factors of the constant term $\pm 1/2$, i.e., $\pm 1, \pm 1/2$.

Test $y = 1/2$:

$$
(1/2)^3 - (1/2) + 1/2 = \frac{1}{8} - \frac{1}{2} + \frac{1}{2} = \frac{1}{8} \neq 0
$$

Test $y = -1/2$:

$$
(-1/2)^3 - (-1/2) + 1/2 = -\frac{1}{8} + \frac{1}{2} + \frac{1}{2} = -\frac{1}{8} + 1 = \frac{7}{8} \neq 0
$$

Test $y=1$:

$$
1 - 1 + 1/2 = 1/2 \neq 0
$$

Test $y = -1$:

$$
-1 + 1 + 1/2 = 1/2 \neq 0
$$

No rational roots are evident; thus, roots are irrational or complex. Given the cubic's shape, it's easier to analyze the number of real roots by examining the function:

$$
f(y) = y^3 - y + \frac{1}{2}
$$

Behavior of $f(y)$:

- $f(y)$ is continuous and differentiable.
- Derivative:

$$
f'(y) = 3 y^2 - 1
$$

- Critical points at:

$$
3 y^2 - 1 = 0 \Rightarrow y^2 = \frac{1}{3} \Rightarrow y = \pm \frac{1}{\sqrt{3}}
$$

- At these points, evaluate $f(y)$:

1. For $y = \frac{1}{\sqrt{3}}$:

$$
f\left(\frac{1}{\sqrt{3}} \right) = \left(\frac{1}{\sqrt{3}} \right)^3 - \frac{1}{\sqrt{3}} + \frac{1}{2} = \frac{1}{3 \sqrt{3}} - \frac{1}{\sqrt{3}} + \frac{1}{2}
$$

Approximate:

$$
\frac{1}{3 \sqrt{3}} \approx \frac{1}{3 \times 1.732} \approx 0.192
$$
$$
\frac{1}{\sqrt{3}} \approx 0.577
$$

Thus:

$$
f \approx 0.192 - 0.577 + 0.5 \approx 0.115
$$

Positive.

2. For $y = -\frac{1}{\sqrt{3}}$:

$$
f\left(-\frac{1}{\sqrt{3}} \right) = -\frac{1}{3 \sqrt{3}} + \frac{1}{\sqrt{3}} + \frac{1}{2} \approx -0.192 + 0.577 + 0.5 \approx 0.885
$$

Positive as well.

End behavior:

- As $y \to \pm \infty$, y^3 dominates, so $f(y) \to \pm \infty$.

- Since $f(y)$ crosses zero between critical points, and the function is continuous, there must be three real roots (by the Intermediate Value Theorem).

Approximate roots:

- The cubic has one root less than -1, one between 0 and 1, and one around $y \approx 0.6$.

However, because $\cos x$ must be in $[-1, 1]$, only roots within this interval are relevant.

Step 3: Find the Valid \(\cos x\) Values and Corresponding \(x\)

Suppose the roots of the cubic within \([-1, 1]\) are:

- \(y_1 \approx -0.8 \)
- \(y_2 \approx 0.5 \)
- \(

Find All Solutions Of The Equation 2 Sin 2 X Cos X 1 2sin2x Cosx 1 In The Interval 0 2 0 2

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