Find All Solutions Of The Equation In The Interval [0, 2). (Enter Your Answers As A Comma-sepa 4 Cos(x)

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Introduction

Understanding how to find solutions to trigonometric equations within specified intervals is a fundamental skill in mathematics, especially in calculus and algebra. The interval [0, 2) refers to all real numbers starting from 0 up to, but not including, 2. When dealing with trigonometric functions such as cosine, solutions often involve analyzing periodic behaviors, symmetry, and properties of the functions.

In this article, we will explore a generic approach to solving a typical trigonometric equation involving cosine within the interval [0, 2). Since the specific equation was not provided, we will assume a common form such as:

$$\cos(x) = k$$

where k is a constant within the range of cosine, i.e., $-1 \leq k \leq 1$.

Throughout the discussion, the goal is to find all solutions x within [0, 2) and express the solutions as a sequence involving cosine values, formatted as "4 Cos(x)". We will clarify what this notation signifies and how to derive the solutions systematically.

Understanding the Cosine Function and Its Properties

Periodicity and Symmetry

- The cosine function, $\cos(x)$, is periodic with a period of 2π. This means:

$$ \cos(x + 2\pi) = \cos(x) \quad \text{for all } x $$

- It is an even function, satisfying:

$$ \cos(-x) = \cos(x) $$

- The maximum value of cosine is 1, and the minimum is -1.

Range of Cosine

- The set of all possible cosine values is:

$$ \text{Range}(\cos x) = [-1, 1] $$

- When solving equations such as $\cos x = k$, solutions exist only if $-1 \leq k \leq 1$.

General Approach to Solving $\cos x = k$ in [0, 2)

Step 1: Find the Reference Angle

- For a given k, find the reference angle θ in [0, π] such that:

$$ \cos \theta = |k| $$

- The reference angle is the acute angle between the positive x-axis and the terminal side of the angle in the unit circle.

Step 2: Determine the Solutions Based on the Sign of k

- If $k \geq 0$:

$$ x = \theta + 2\pi n \quad \text{or} \quad x = 2\pi - \theta + 2\pi n $$

- If $k < 0$:

$$ x = \pi - \theta + 2\pi n \quad \text{or} \quad x = \pi + \theta + 2\pi n $$

where n is an integer such that solutions lie within [0, 2).

Step 3: Find All Solutions in [0, 2)

- Since the interval is [0, 2), which is less than 2π, the solutions will be limited to the first period.
- For each candidate solution, verify whether it falls within [0, 2).

Example: Solving $\cos x = \frac{1}{2}$ in [0, 2)

Step 1: Find the Reference Angle

- $\cos \theta = \frac{1}{2}$
- $\theta = \frac{\pi}{3}$, since $\cos(\pi/3) = 1/2$

Step 2: Determine Solutions

- For $k = 1/2 \geq 0$:

$$ x = \frac{\pi}{3} + 2\pi n \quad \text{or} \quad x = 2\pi - \frac{\pi}{3} + 2\pi n $$

- Simplify:

$$ x = \frac{\pi}{3} + 2\pi n \quad \text{or} \quad x = \frac{5\pi}{3} + 2\pi n $$

Step 3: Find Solutions in [0, 2)

- For $n=0$:

$$ x = \frac{\pi}{3} \approx 1.0472 $$

$$ x = \frac{5\pi}{3} \approx 5.23599 $$

- Since 5.236 > 2, discard the second.
- The only solution in [0, 2) is:

\[x = \frac{\pi}{3} \]

Expressing the solution as per the required format:

Note: The problem mentions "Enter Your Answers As A Comma-sepa 4 Cos(x)". This seems to suggest the solutions should be expressed in a particular notation involving cosine, perhaps as a sequence of cosine values or as a formula involving Cos(x). Since the instructions are somewhat ambiguous, we'll interpret this as providing solutions in the form of cosine expressions.

Solution:

\[
x = \frac{\pi}{3}
\]

and the corresponding cosine value:

\[
\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}
\]

Thus, the answer could be expressed as:

Answer:

\[
\frac{\pi}{3}
\]

or, if following the format "4 Cos(x)", perhaps:

$$
\text{Cos}(x) = \frac{1}{2}
$$

which, in a list form, could be written as:

$$
\frac{1}{2}
$$

or, if explicitly asked to enter solutions as "4 Cos(x)", perhaps:

$$
4 \times \cos(\frac{\pi}{3}) = 4 \times \frac{1}{2} = 2
$$

Analyzing Additional Examples and Common Trigonometric Equations

Equation: $\cos x = -\frac{1}{2}$

- Reference angle: $\theta = \frac{\pi}{3}$
- Solutions:

$$ x = \pi - \frac{\pi}{3} = \frac{2\pi}{3} $$

$$ x = \pi + \frac{\pi}{3} = \frac{4\pi}{3} $$

- Check whether these fall within [0, 2):
- $\frac{2\pi}{3} \approx 2.094$, which is greater than 2, so discard.

- $\frac{4\pi}{3} \approx 4.188$, also greater than 2, discard.

- No solutions in [0, 2) for this case.

Equation: $\cos x = 1$

- $\cos x = 1$ at $x= 0 + 2\pi n$

- For $n=0$:

$$ x= 0 $$

- Within [0, 2), solution:

$$ x= 0 $$

Equation: $\cos x = 0$

- $\cos x = 0$ at:

$$ x= \frac{\pi}{2} + \pi n $$

- For $n=0$:

$$ x= \frac{\pi}{2} \approx 1.5708 $$

- For $n=1$:

$$ x= \frac{\pi}{2} + \pi \approx 1.5708 + 3.1416 \approx 4.7124 $$

- Only $x= \frac{\pi}{2}$ falls within [0, 2).

Summary of Solution Strategies

- Always identify the reference angle for the given cosine value.

- Use symmetry properties to find all solutions within a single period.

- Limit solutions to the given interval [0, 2).

- Express solutions in terms of π or decimal approximations, depending on the context.

- When asked to enter solutions as "4 Cos(x)", interpret accordingly, possibly converting the solutions into cosine values or the expression involving cosine.

Conclusion

Finding all solutions to trigonometric equations within a specific interval requires understanding the periodic nature of functions like cosine, their symmetry, and their range. By systematically applying these principles, one can derive all solutions efficiently and accurately. Remember to verify each solution's inclusion within the interval and express the answers clearly, whether as angles, cosine values, or in the specified format.

Note: Since the original problem statement was somewhat ambiguous regarding the exact form of the solutions ("Enter Your Answers As A Comma-sepa 4 Cos(x)"), the most precise interpretation involves

Frequently Asked Questions

What are the solutions of the equation 4 cos(x) = 2 in the interval [0, 2)?

x = arccos(0.5) = □/3; since cosine is positive in [0, 2), the solutions are x = □/3.

How do I find all solutions of 4 cos(x) = 1 in the interval [0, 2)?

x = arccos(1/4); since cos(x) is positive in [0, 2), the solutions are x = arccos(1/4). Since arccos(1/4) □ 1.318, and cosine is decreasing in [0, □], the only solution in [0, 2) is x □ 1.318.

Are there multiple solutions for 4 cos(x) = 0 in [0, 2)?

Yes, since cos(x) = 0 at x = □/2 within [0, 2). The solution is x = □/2.

What solutions exist for 4 cos(x) = -2 in the interval [0, 2)?

x = arccos(-0.5) = 2π/3; since cosine is decreasing in [0, π], the solution in [0, 2) is x = 2π/3 ≈ 2.094.

How do I determine all solutions of 4 cos(x) = 3 in the interval [0, 2)?

x = arccos(3/4); since 3/4 ≈ 0.75 and cos(x) is positive in [0, 2), the solution is x = arccos(0.75) ≈ 0.722 radians.

Additional Resources

Comprehensive Guide to Finding All Solutions of the Equation in the Interval [0, 2π)

Understanding how to solve trigonometric equations within specific intervals is a fundamental skill in calculus and algebra. This guide will explore the process of finding all solutions of a general trigonometric equation within the interval [0, 2π), emphasizing the importance of understanding the behavior of trigonometric functions, solving systematically, and verifying solutions.

Introduction to Trigonometric Equations

Trigonometric equations involve functions such as sine, cosine, tangent, cotangent, secant, and cosecant. They often appear in physics, engineering, and mathematics, modeling periodic phenomena such as waves, oscillations, and circular motion.

Key points:

- Equations can be algebraic or involve multiple functions.

- Solutions are typically angles within specified domains.
- The interval $[0, 2\pi)$ covers one full cycle of the sine and cosine functions, making it a common domain for solutions.

Understanding the Interval $[0, 2\pi)$

The interval $[0, 2\pi)$ includes all real numbers starting from 0 up to but not including 2π, covering a complete revolution in radians.

Why this interval?
- It represents a full period of sine and cosine functions.
- It simplifies the analysis as solutions repeat every 2π.
- It ensures all solutions within one period are captured.

Important notes:
- The endpoints: 0 and 2π are often included or excluded depending on the context. Here, 0 is included, and 2π is excluded.
- The periodicity of functions: For cosine, solutions repeat every 2π; for sine, similarly.

Approach to Solving Trigonometric Equations

To find solutions systematically:

1. Simplify the Equation: Use identities to reduce the equation to a basic form.

2. Identify the Type of Equation: Is it linear, quadratic, or involves multiple functions?

3. Solve for the Basic Trigonometric Function: Isolate sine or cosine.

4. Determine General Solutions: Find solutions over the entire real line.

5. Restrict to the Given Interval: Extract solutions lying within $[0, 2\pi)$.

6. Verify Solutions: Substitute back into the original equation where necessary.

Deep Dive into Solving Specific Types of Equations

2.1 Linear Equations Involving Cosine

Equations like:

$$ \cos x = a $$

Where a is a constant between -1 and 1.

Solution steps:

- Use the inverse cosine function:

$$ x = \arccos a + 2k\pi \quad \text{or} \quad x = -\arccos a + 2k\pi $$

- Find all solutions with k such that x lies in $[0, 2\pi)$.

Example:

Solve $\cos x = \frac{1}{2}$ in $[0, 2\pi)$:

- $\arccos \frac{1}{2} = \frac{\pi}{3}$

- Solutions:

$$ x = \frac{\pi}{3} + 2k\pi $$

$$ x = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3} + 2k\pi $$

- Within $[0, 2\pi)$:

$$ x = \frac{\pi}{3}, \frac{5\pi}{3} $$

2.2 Quadratic Equations in Cosine

Equations like:

$$ a \cos^2 x + b \cos x + c = 0 $$

Solution steps:

- Set $y = \cos x$.

- Solve the quadratic:

$$ a y^2 + b y + c = 0 $$

- Find roots y_1, y_2.

- For each root, check if $y_i \in [-1, 1]$. If yes, find x via:

$$ x = \arccos y_i + 2k\pi \quad \text{or} \quad x = -\arccos y_i + 2k\pi $$

- Select solutions within $[0, 2\pi)$.

2.3 Equations Involving Both Sine and Cosine

Some equations involve both functions, such as:

$$ \sin x + \cos x = 1 $$

Solution approach:

- Use identities or substitution:

$$ \sin x + \cos x = \sqrt{2} \sin \left(x + \frac{\pi}{4} \right) $$

- Then solve:

$$ \sqrt{2} \sin \left(x + \frac{\pi}{4} \right) = 1 $$

- Find:

$$ \sin \left(x + \frac{\pi}{4} \right) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} $$

- Solutions:

$$ x + \frac{\pi}{4} = \arcsin \frac{\sqrt{2}}{2} + 2k\pi = \frac{\pi}{4} + 2k\pi $$
$$ x + \frac{\pi}{4} = \pi - \frac{\pi}{4} + 2k\pi = \frac{3\pi}{4} + 2k\pi $$

- Therefore:

$$ x = 0 + 2k\pi $$
$$ x = \frac{\pi}{2} + 2k\pi $$
$$ x = \frac{\pi}{2} + 2k\pi $$
$$ x = \frac{\pi}{2} + 2k\pi $$

- Selecting solutions within $[0, 2\pi)$:

$$ x = 0, \frac{\pi}{2} $$

Specific Example: Find All Solutions of $\cos x = \frac{1}{2}$ in $[0, 2\pi)$

Let's walk through an example to illustrate the entire process.

Equation:

$$ \cos x = \frac{1}{2} $$

Step 1: Recognize the function and range

- Cosine ranges from -1 to 1, so solutions exist.
- The value $\frac{1}{2}$ is within the range.

Step 2: Find the reference angle

$$ \arccos \left(\frac{1}{2} \right) = \frac{\pi}{3} $$

Step 3: Find all solutions within $[0, 2\pi)$

- Cosine is positive in the first and fourth quadrants, solutions are:

$$ x = \frac{\pi}{3} + 2k\pi $$

$$ x = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3} + 2k\pi $$

- For $k=0$:

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

- Both solutions are within $[0, 2\pi)$.

Step 4: Final solutions

$$\boxed{\frac{\pi}{3}, \frac{5\pi}{3}}$$

Handling Multiple Solutions and Special Cases

While many equations are straightforward, some may involve multiple solutions, extraneous roots, or special angles.

2.1 Equations with Multiple Solutions

- When solving equations with multiple solutions, always consider the periodicity.
- For sine and cosine, solutions repeat every 2π.
- For tangent, solutions repeat every π.

2.2 Extraneous Roots and Domain Restrictions

- Not all roots from the algebraic solutions are valid; always verify that they satisfy the original equation.
- Check solutions against the original equation, especially when dealing with identities or transformations.

2.3 Special Angles and Known Values

- Familiar angles: 0, $\pi/6$, $\pi/4$, $\pi/3$, $\pi/2$, π, $3\pi/2$, 2π.
- Use these to quickly identify solutions.

Practical Tips and Strategies

- Use identities liberally: Double angle, half-angle, sum and difference formulas.
- Graphically interpret the solutions: Plotting functions helps visualize solutions.
- Check the principal value: When using inverse functions, ensure solutions are within the principal range.
- Systematic approach: Break down complex equations into simpler parts.
- Note the periodicity: Remember solutions repeat every 2π for sine and cosine.

Summary of Steps to Find All Solutions in $[0, 2\pi)$

1. Simplify the equation using identities if needed.
2. Isolate the trigonometric function.
3. Find the reference solution(s) in the principal range.
4. Use periodicity to find all solutions within $[0, 2\pi)$.
5. Verify solutions satisfy the original equation.
6. List solutions in ascending order, separated by commas.

Example: Find All Solutions of \(2 \cos^2 x - 3 \

[Find All Solutions Of The Equation In The Interval 0 2 Enter Your Answers As A Comma Sepa 4 Cos X]

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