Find An Equation Of The Tangent Plane To The Given Surface At The Specified Point. Z = = 2(x 1)^2 + 5(y

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Understanding how to find the tangent plane to a surface at a specific point is a fundamental concept in multivariable calculus. It allows us to approximate the surface locally and analyze its behavior in the vicinity of that point. Given a surface described by a function $z = f(x, y)$, the tangent plane at a point (x_0, y_0, z_0) provides a linear approximation of the surface near that point. This article will delve into the process of deriving the tangent plane equation for the surface $z = 2(x - 1)^2 + 5y$ at a specified point, explaining the underlying concepts, step-by-step procedures, and relevant formulas.

Understanding the Surface and the Problem

Before proceeding to find the tangent plane, it is crucial to understand the surface's structure and the information needed.

The Given Surface

The surface is defined by the function:

$$
z = 2(x - 1)^2 + 5y
$$

which is a quadratic in x and linear in y. This surface resembles a paraboloid opening along the z-axis, translated along the x-direction, and sloped along the y-direction.

The Goal

Our goal is to find the equation of the tangent plane to this surface at a particular point (x_0, y_0, z_0). This involves:

- Determining the point (x_0, y_0, z_0) on the surface.
- Computing the partial derivatives of z with respect to x and y at that point.
- Using these derivatives to form the tangent plane equation.

Steps to Find the Equation of the Tangent Plane

The general approach involves several key steps, which will be elaborated upon below.

1. Find the Point (x_0, y_0, z_0) on the Surface

To determine the tangent plane, we need a specific point on the surface. This point is usually provided. If not, we select one based on the problem context or given coordinates.

Example: Suppose the specified point is $(x_0, y_0) = (2, 3)$. Then, compute:

$$
z_0 = 2(2 - 1)^2 + 5 \times 3 = 2(1)^2 + 15 = 2 + 15 = 17
$$

Thus, the point of tangency is $(2, 3, 17)$.

Note: If a different point is provided, repeat this process accordingly.

2. Compute Partial Derivatives f_x and f_y

The tangent plane's slope in the x and y directions depends on the partial derivatives of $z = f(x, y)$.

Given:

$$
f(x, y) = 2(x - 1)^2 + 5 y
$$

Calculate:
- $f_x(x, y) = \frac{\partial f}{\partial x}$
- $f_y(x, y) = \frac{\partial f}{\partial y}$

Calculations:

$$
f_x(x, y) = \frac{\partial}{\partial x} \left[2(x - 1)^2 + 5 y \right] = 2 \times 2 (x - 1) = 4(x - 1)
$$

$$
f_y(x, y) = \frac{\partial}{\partial y} \left[2(x - 1)^2 + 5 y \right] = 5
$$

Evaluate these at (x_0, y_0):

$$
f_x(2, 3) = 4(2 - 1) = 4 \times 1 = 4
$$

$$
f_y(2, 3) = 5
$$

3. Formulate the Equation of the Tangent Plane

The general equation of the tangent plane to the surface $z = f(x, y)$ at (x_0, y_0, z_0) is:

$$
z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)
$$

Substituting the known values:

$$
z - 17 = 4(x - 2) + 5(y - 3)
$$

Simplify to obtain the tangent plane equation:

$$
z = 17 + 4(x - 2) + 5(y - 3)
$$

which can be expanded as:

$$
z = 17 + 4x - 8 + 5y - 15 = (4x + 5y) + (17 - 8 - 15) = 4x + 5y - 6
$$

Final equation:

$$
z = 4x + 5y - 6
$$

This is the equation of the tangent plane at the point $(2, 3, 17)$.

Additional Considerations and Generalizations

While the above example uses specific points, the methodology applies generally to any point on the surface. Here are some important notes:

Choosing the Point on the Surface

- If the point is given directly, proceed with the calculation.
- If only x_0 and y_0 are given, compute z_0 using the surface equation.
- Ensure the point lies on the surface; otherwise, the tangent plane is not well-defined at that point.

Handling Different Surface Forms

- For more complex surfaces, partial derivatives may involve more intricate calculations.
- For surfaces defined implicitly or parametrically, different techniques such as implicit differentiation or parametric derivatives are used.

Geometric Interpretation

- The gradient vector $\nabla f = \left(f_x, f_y, -1 \right)$ at the point provides a normal vector to the surface.
- The tangent plane is perpendicular to this gradient vector.

Summary

To find the tangent plane to a surface at a specific point, follow these steps:

1. Identify the point (x_0, y_0, z_0) on the surface.
2. Compute the partial derivatives f_x and f_y at (x_0, y_0).
3. Use the tangent plane formula:
$$
z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)
$$
4. Simplify to obtain the explicit equation of the tangent plane.

This method provides a powerful tool for approximating surfaces locally and analyzing their behavior in multivariable calculus.

Note: Always verify your calculated point lies on the surface and that derivatives are correctly evaluated. The tangent plane's accuracy depends on the smoothness and differentiability of the surface at that point.

Frequently Asked Questions

How do I find the equation of the tangent plane to the surface Z = 2(x - 1)^2 + 5y at a given point?

First, compute the partial derivatives of Z with respect to x and y at the point, then use the point-slope form for the tangent plane: $Z - Z_0 = (\partial Z/\partial x)_0 (x - x_0) + (\partial Z/\partial y)_0 (y - y_0)$.

What is the importance of calculating partial derivatives when finding a tangent plane?

Partial derivatives give the slopes of the surface in the x and y directions at the point, which are essential for constructing the tangent plane equation.

Can you demonstrate the step-by-step process to find the tangent plane at point (x_0, y_0, z_0) for Z = 2(x - 1)^2 + 5y?

Yes. First, find Z at (x_0, y_0). Then, calculate $\partial Z/\partial x = 4(x - 1)$ and $\partial Z/\partial y = 5$, evaluate both at ($x_0$, y_0). Finally, substitute into the tangent plane formula: $Z - Z_0 = (\partial Z/\partial x)(x - x_0) + (\partial Z/\partial y)(y - y_0)$.

What is the significance of the point at which the tangent plane is found?

The point specifies where on the surface the tangent plane is tangent, ensuring the plane accurately approximates the surface near that point.

How does the surface Z = 2(x - 1)^2 + 5y differ from a plane, and why is finding the tangent plane useful?

The surface is quadratic in x and linear in y, making it curved. The tangent plane provides the best linear approximation of the surface near a specific point for analysis or modeling.

If the point given is (x_0, y_0) = (3, 2), what is the equation of the tangent plane to Z = 2(x - 1)^2 + 5y at this point?

First, compute Z at (3, 2): Z = 2(3 - 1)^2 + 5(2) = 2(2)^2 + 10 = 2(4) + 10 = 8 + 10 = 18. Next, find derivatives: $\partial Z/\partial x = 4(x - 1) = 4(3 - 1) = 8$; $\partial Z/\partial y = 5$. The tangent plane equation is: Z - 18 = 8(x - 3) + 5(y - 2).

What are common mistakes to avoid when deriving the tangent plane for this surface?

Common mistakes include mixing up the point coordinates with the variables, forgetting to evaluate derivatives at the point, and not substituting correctly into the tangent plane formula.

Additional Resources

Find An Equation Of The Tangent Plane To The Given Surface At The Specified Point is a fundamental problem in multivariable calculus, often encountered in advanced mathematics, engineering, and physics. Understanding how to determine the tangent plane provides crucial insights into the local behavior of surfaces, optimization, and modeling real-world phenomena. This guide will walk you through the process step-by-step, using the specific surface $z = 2(x - 1)^2 + 5(y)$ as our example, and demonstrate how to find the tangent plane at a given point.

Understanding the Problem

Before diving into calculations, it's essential to clarify what the problem entails. Given a surface defined by a function $z = f(x, y)$, and a specific point (x_0, y_0, z_0) on that surface, the goal is to find the equation of the tangent plane at that point. The tangent plane is a flat, tangent surface that just touches the original surface at the point, providing a linear approximation of the surface near that point.

Key points to remember:

- The surface is given explicitly as $z = f(x, y)$.
- The point must lie on the surface, i.e., $z_0 = f(x_0, y_0)$.
- The tangent plane's equation involves the partial derivatives f_x and f_y.

Step 1: Confirm the Point Lies on the Surface

Suppose you're given the surface:

$$
z = 2(x - 1)^2 + 5y
$$

and a point (x_0, y_0). To proceed, verify that the point actually lies on the surface by calculating:

$$
z_0 = 2(x_0 - 1)^2 + 5 y_0
$$

For example, if the point is $(x_0, y_0, z_0) = (2, 3, 2(2 - 1)^2 + 5 \times 3)$:

$$
z_0 = 2(1)^2 + 15 = 2 + 15 = 17
$$

Thus, the point on the surface is $(2, 3, 17)$.

Step 2: Find the Partial Derivatives

The partial derivatives of $z = f(x, y)$ are essential for constructing the tangent plane. They describe how the surface changes in the x- and y-directions near the point.

Given:

$$
f(x, y) = 2(x - 1)^2 + 5 y
$$

Calculate:

- Partial derivative with respect to x:

$$
f_x(x, y) = \frac{\partial}{\partial x} [2(x - 1)^2 + 5 y] = 2 \times 2 (x - 1) = 4(x - 1)
$$

- Partial derivative with respect to y:

$$
f_y(x, y) = \frac{\partial}{\partial y} [2(x - 1)^2 + 5 y] = 5
$$

Evaluate these derivatives at the point $(x_0, y_0) = (2, 3)$:

$$
f_x(2, 3) = 4(2 - 1) = 4 \times 1 = 4
$$
$$
f_y(2, 3) = 5
$$

Step 3: Write the Equation of the Tangent Plane

The general equation of the tangent plane to the surface $z = f(x, y)$ at the point (x_0, y_0, z_0) is:

$$
z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)
$$

Substituting the known values:

$$
z - 17 = 4(x - 2) + 5(y - 3)
$$

This is the equation of the tangent plane in point-slope form.

Step 4: Simplify the Equation

Expand and rearrange to get the standard form:

$$
z - 17 = 4x - 8 + 5 y - 15
$$
$$
z = 4x + 5 y - 8 - 15 + 17
$$
$$
z = 4x + 5 y - 6
$$

Thus, the equation of the tangent plane is:

$$
z = 4x + 5 y - 6
$$

This linear equation approximates the surface near the point $(2, 3, 17)$.

Additional Tips and Common Pitfalls

- Always verify the point is on the surface: Confirm that the point satisfies the surface equation before proceeding.
- Partial derivatives are evaluated at the point: Make sure to substitute the

point's coordinates into the derivatives.
- Maintain clarity in notation: Be consistent with variables and their derivatives to avoid confusion.
- Interpretation of the tangent plane: It provides a linear approximation of the surface near the point, useful in optimization and analysis.

Practical Applications of Finding Tangent Planes

Understanding how to find tangent planes is more than an academic exercise; it has practical significance in various fields:

- Engineering: Stress analysis on surfaces and material deformation.
- Physics: Approximating potential energy surfaces and field interactions.
- Economics: Local linear approximations in multivariable optimization.
- Computer Graphics: Rendering surfaces and understanding local surface behavior.

Summary

To find an equation of the tangent plane to the surface $z = 2(x - 1)^2 + 5y$ at a given point:

1. Verify the point lies on the surface.
2. Compute the partial derivatives f_x and f_y at the point.
3. Apply the tangent plane formula:

$$
z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)
$$

4. Simplify to obtain the linear equation representing the tangent plane.

By following these steps, you can efficiently determine the tangent plane for any surface defined explicitly as $z = f(x, y)$. Mastering this process is a crucial skill in multivariable calculus, enabling deeper understanding of surface behavior and supporting a wide array of scientific and engineering applications.

Remember: Practice with various surfaces and points to solidify your understanding and develop intuition about how tangent planes approximate complex surfaces locally.

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