Find Both The Unit Tangent And Unit Normal To The Curve R(t) = (cost, Sint, T) At T = 1.Find The Length

Find Both The Unit Tangent And Unit Normal To The Curve R(t) = (cos t, sin t, t) At t = 1. Find The Length

Understanding how to analyze the properties of a space curve is fundamental in vector calculus. One of the essential aspects involves determining the tangent and normal vectors to a curve at a specific point, as well as calculating the curve's length, also known as the arc length. In this article, we will thoroughly explore how to find both the unit tangent vector and the unit normal vector for the curve \(R(t) = (\cos t, \sin t, t) \) at \(t = 1 \), and how to compute the length of the curve over a specified interval.

Introduction to the Curve \(R(t) = (\cos t, \sin t, t) \)

The given parametric curve describes a three-dimensional path where:

- The x-coordinate varies as \(\cos t \),
- The y-coordinate varies as \(\sin t \),
- The z-coordinate varies linearly with \(t \).

This type of curve is often called a helix, because as \(t \) increases, the point moves around a circle in the xy-plane while simultaneously ascending or descending along the z-axis.

Step-by-Step Approach to Find the Unit Tangent and Normal Vectors

To analyze the properties of the curve at a specific parameter $t = 1$, we follow a systematic process:

1. Find the derivative $R'(t)$

The derivative of the vector function gives the tangent vector (not necessarily of unit length):

$$
R'(t) = \frac{d}{dt} (\cos t, \sin t, t) = (-\sin t, \cos t, 1)
$$

2. Evaluate $R'(t)$ at $t = 1$

Substituting $t = 1$:

$$
R'(1) = (-\sin 1, \cos 1, 1)
$$

Since $\sin 1$ and $\cos 1$ are known constants (with 1 in radians), this gives a specific vector:

$$
R'(1) = (-\sin 1, \cos 1, 1)
$$

3. Find the unit tangent vector $T(t)$

The unit tangent vector $T(t)$ is obtained by normalizing $R'(t)$:

$$
T(t) = \frac{R'(t)}{\| R'(t) \|}
$$

where

$$
\| R'(t) \| = \sqrt{(-\sin t)^2 + (\cos t)^2 + 1^2} = \sqrt{\sin^2 t + \cos^2 t + 1}
$$

Since $\sin^2 t + \cos^2 t = 1$, the magnitude simplifies to:

$$
\| R'(t) \| = \sqrt{1 + 1} = \sqrt{2}
$$

Thus, at $t = 1$:

$$
\| R'(1) \| = \sqrt{2}
$$

and the unit tangent vector:

$$
T(1) = \frac{1}{\sqrt{2}} (-\sin 1, \cos 1, 1)
$$

Calculating the Unit Normal Vector

The unit normal vector \(N(t) \) points in the direction of the curve's principal curvature, perpendicular to \(T(t) \), and is obtained by differentiating \(T(t) \) with respect to \(t \) and normalizing.

1. Compute the derivative of \(T(t) \)

Given:

\[
T(t) = \frac{R'(t)}{\| R'(t) \|} = \frac{(-\sin t, \cos t, 1)}{\sqrt{2}}
\]

Differentiating \(T(t) \) with respect to \(t \):

\[
T'(t) = \frac{d}{dt} \left(\frac{-\sin t}{\sqrt{2}}, \frac{\cos t}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right)
\]

Since \(\frac{1}{\sqrt{2}} \) is constant:

\[
T'(t) = \frac{1}{\sqrt{2}} (-\cos t, -\sin t, 0)
\]

2. Evaluate \(T'(t) \) at \(t = 1 \):

\[

$$
T'(1) = \frac{1}{\sqrt{2}} (-\cos 1, -\sin 1, 0)
$$

3. Find the normal vector $N(t)$

The normal vector is in the direction of $T'(t)$, normalized:

$$
N(t) = \frac{T'(t)}{\| T'(t) \| }
$$

Calculate the magnitude of $T'(1)$:

$$
\| T'(1) \| = \frac{1}{\sqrt{2}} \sqrt{ (\cos 1)^2 + (\sin 1)^2 + 0 } = \frac{1}{\sqrt{2}} \times 1 = \frac{1}{\sqrt{2}}
$$

Thus,

$$
N(1) = \frac{T'(1)}{\| T'(1) \|} = \frac{\frac{1}{\sqrt{2}} (-\cos 1, -\sin 1, 0)}{\frac{1}{\sqrt{2}}} = (-\cos 1, -\sin 1, 0)
$$

Finally, the unit normal vector:

$$
\boxed{
N(1) = \frac{(-\cos 1, -\sin 1, 0)}{\sqrt{\cos^2 1 + \sin^2 1}} = (-\cos 1, -\sin 1, 0)
}
$$

\]

since this vector is already of unit length.

Calculating the Length of the Curve $R(t)$

The length of a parametric curve between $t = a$ and $t = b$ is given by:

$$
L = \int_a^b \| R'(t) \| dt
$$

1. Understanding the length integral for our curve

In our case, we are asked to find the length at $t = 1$. Typically, this refers to the arc length from $t = 0$ (or some initial point) to $t = 1$. Assuming the initial point is at $t = 0$, the length L from $t=0$ to $t=1$ is:

$$
L = \int_0^1 \| R'(t) \| dt
$$

Recall that:

$$
\| R'(t) \| = \sqrt{\sin^2 t + \cos^2 t + 1} = \sqrt{2}
$$

since $\sin^2 t + \cos^2 t = 1$.

2. Compute the integral

Since $\| R'(t) \| = \sqrt{2}$ is constant:

$$
L = \int_0^1 \sqrt{2} \, dt = \sqrt{2} \times (1 - 0) = \sqrt{2}
$$

Thus, the length of the curve from $t=0$ to $t=1$ is:

$$
\boxed{
L = \sqrt{2}
}
$$

If the problem specifies a different interval, adjust the limits accordingly.

Summary of Results

Quantity	Expression	At $t=1$
Derivative $R'(t)$	$(-\sin t, \cos t, 1)$	$(-\sin 1, \cos 1, 1)$
Magnitude $\| R'(t) \|$	$\sqrt{2}$	$\sqrt{2}$
Unit tangent $T(t)$	$\frac{1}{\sqrt{2}} (-\sin t, \cos t, 1)$	$\frac{1}{\sqrt{2}} (-\sin 1, \cos 1, 1)$
Derivative of $T(t)$	$\frac{1}{\sqrt{2}} (-\cos t, -\sin t, 0)$	$\frac{1}{\sqrt{2}} (-\cos 1, -\sin 1, 0)$

|

| Unit normal \(N(t) \) | \((-\cos t, -\sin t, 0) \) | \((-\cos 1, -\sin 1, 0) \) |

| Curve length from 0 to 1 | \(\sqrt{2} \) | – |

Conclusion

In

Frequently Asked Questions

How do you find the unit tangent vector to the curve R(t) = (cos t, sin t, t) at t = 1?

First, compute the derivative R'(t) = (-sin t, cos t, 1). Then evaluate at t = 1: R'(1) = (-sin 1, cos 1, 1). Next, find its magnitude and divide R'(1) by that magnitude to obtain the unit tangent vector T(1).

What is the process to determine the unit normal vector to the curve at t = 1?

Calculate the derivative of the unit tangent vector T(t), then evaluate at t = 1. Normalize this derivative to get the unit normal vector N(1), which is perpendicular to T(1).

How do you compute the length of the curve R(t) = (cos t, sin t, t) from t = 0 to t = 1?

Calculate the integral of the magnitude of R'(t) from 0 to 1: Length = $\int_0^1$ |R'(t)| dt. Since R'(t) = (-sin t, cos t, 1), its magnitude is $\sqrt{(\sin^2 t + \cos^2 t + 1)} = \sqrt{(1 + 1)} = \sqrt{2}$. Therefore, length = $\int_0^1 \sqrt{2}\, dt = \sqrt{2} \times (1 - 0) = \sqrt{2}$.

What is the value of the unit tangent vector to R(t) at t = 1?

The unit tangent vector T(1) is obtained by dividing R'(1) = (-sin 1, cos 1, 1) by its magnitude, which is $\sqrt{(\sin^2 1 + \cos^2 1 + 1)} = \sqrt{(1 + 1)} = \sqrt{2}$. So, T(1) = (-sin 1 / $\sqrt{2}$, cos 1 / $\sqrt{2}$, 1 / $\sqrt{2}$).

How is the unit normal vector related to the curvature of the curve?

The unit normal vector points in the direction of the curve's principal curvature and is perpendicular to the unit tangent vector. It is found by differentiating the unit tangent vector with respect to t and normalizing the result.

Can you confirm the length of the curve R(t) = (cos t, sin t, t) from t=0 to t=1?

Yes. Since |R'(t)| = $\sqrt{2}$, the length from t=0 to t=1 is $\sqrt{2} \times (1 - 0) = \sqrt{2}$.

What is the significance of the unit tangent and normal vectors in curve analysis?

They describe the direction of the curve at a point (tangent) and the direction in which the curve is turning (normal). Together, they help analyze the curvature and geometric behavior of the curve.

How do you verify that the vectors obtained are indeed unit vectors?

Calculate their magnitudes. If the magnitude equals 1, the vectors are unit vectors. If not, normalize by dividing each component by the magnitude.

What is the practical application of finding both the unit tangent and

unit normal vectors?

They are essential in physics and engineering for analyzing motion along a path, determining forces, and understanding the curvature and torsion of paths in space.

Additional Resources

Find Both The Unit Tangent And Unit Normal To The Curve R(t) = (cos t, sin t, t) At t = 1. Find The Length

Understanding how to find both the unit tangent and unit normal vectors of a curve at a specific point is fundamental in vector calculus and has wide applications across physics, engineering, and computer graphics. This process provides insights into the behavior of a curve, such as its direction, curvature, and how it changes with respect to a parameter t. In this article, we will thoroughly analyze the curve $R(t) = (\cos t, \sin t, t)$, particularly at $t = 1$, and explore the steps needed to compute its unit tangent, unit normal, and the length of the curve over an interval.

Understanding the Curve $R(t) = (\cos t, \sin t, t)$

Before diving into the calculations, it's essential to understand the nature of the given curve. The curve $R(t)$ combines circular motion in the xy-plane with a linear increase in the z-direction. Specifically:

- The x and y components, $\cos t$ and $\sin t$, trace a circle in the xy-plane.
- The z component, t, causes the curve to ascend (or descend) along the z-axis.

This combination results in a helix, a common curve in three-dimensional space characterized by a circular base and a consistent upward or downward progression.

Step-by-Step Approach to Find the Unit Tangent and Normal Vectors

The process involves several key steps:

1. Compute the derivative $R'(t)$.
2. Calculate the magnitude of $R'(t)$.
3. Determine the unit tangent vector $T(t)$.
4. Find the derivative of the unit tangent $T'(t)$.
5. Calculate the unit normal vector $N(t)$.
6. Optional: Compute the length of the curve over a specified interval.

Let's explore each step in detail.

---

1. Computing $R'(t)$: The Velocity Vector

The derivative of the position vector $R(t)$ with respect to t gives the velocity vector:

$$
R'(t) = \frac{d}{dt} (\cos t, \sin t, t) = (-\sin t, \cos t, 1)
$$

This vector indicates the direction and rate at which the point moves along the curve at any parameter t.

---

2. Magnitude of $R'(t)$: Speed Along the Curve

The magnitude (or norm) of $R'(t)$ is:

$$
| R'(t) | = \sqrt{(-\sin t)^2 + (\cos t)^2 + 1^2} = \sqrt{\sin^2 t + \cos^2 t + 1}
$$

Since $\sin^2 t + \cos^2 t = 1$, this simplifies to:

$$
| R'(t) | = \sqrt{1 + 1} = \sqrt{2}
$$

This is a constant magnitude, which simplifies subsequent calculations.

---

3. Determining the Unit Tangent Vector $T(t)$

The unit tangent vector points in the direction of the curve's motion and is given by:

$$
T(t) = \frac{R'(t)}{| R'(t) |}
$$

Substituting the computed derivatives:

$$
T(t) = \frac{(-\sin t, \cos t, 1)}{\sqrt{2}} = \left(-\frac{\sin t}{\sqrt{2}}, \frac{\cos t}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right)
$$

At $t = 1$:

$$
T(1) = \left(-\frac{\sin 1}{\sqrt{2}}, \frac{\cos 1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right)
$$

This vector indicates the direction of the curve at $t = 1$.

---

4. Computing the Derivative of the Unit Tangent $T'(t)$

The next step involves differentiating $T(t)$:

$$
T'(t) = \frac{d}{dt} \left(-\frac{\sin t}{\sqrt{2}}, \frac{\cos t}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right)
$$

Note that the third component is constant, so its derivative is zero. For the first two components:

$$
T'(t) = \left(-\frac{\cos t}{\sqrt{2}}, -\frac{\sin t}{\sqrt{2}}, 0 \right)
$$

At $t = 1$:

$$
T'(1) = \left(-\frac{\cos 1}{\sqrt{2}}, -\frac{\sin 1}{\sqrt{2}}, 0 \right)
$$

---

5. Finding the Unit Normal Vector $N(t)$

The unit normal vector points in the direction of the curve's principal normal, which indicates how the curve bends. It is obtained by normalizing $T'(t)$:

$$
N(t) = \frac{T'(t)}{| T'(t) |}
$$

First, compute the magnitude of $T'(t)$:

$$
| T'(t) | = \sqrt{\left(-\frac{\cos t}{\sqrt{2}}\right)^2 + \left(-\frac{\sin t}{\sqrt{2}}\right)^2 + 0^2}
= \frac{1}{\sqrt{2}} \sqrt{\cos^2 t + \sin^2 t} = \frac{1}{\sqrt{2}} \times 1 = \frac{1}{\sqrt{2}}
$$

Thus, at $t = 1$:

$$
| T'(1) | = \frac{1}{\sqrt{2}}
$$

Now, normalize $T'(1)$:

\[

N(1) = \frac{T'(1)}{| T'(1) |} = \frac{\left(-\frac{\cos 1}{\sqrt{2}}, -\frac{\sin 1}{\sqrt{2}}, 0

\right)}{\frac{1}{\sqrt{2}}} = (-\cos 1, -\sin 1, 0)

\]

So, the unit normal vector at \(t=1 \) is:

\[

\boxed{

N(1) = (-\cos 1, -\sin 1, 0)

}

\]

Calculating the Length of the Curve

The length \(L \) of the curve over an interval \([a, b] \) is given by:

\[

L = \int_a^b | R'(t) | \, dt

\]

Since \(| R'(t) | = \sqrt{2} \) is constant, the length over the interval \([a, b] \) simplifies to:

\[

L = \sqrt{2} \times (b - a)

\]

Example: To find the length of the curve from \(t=0 \) to \(t=1 \):

$$
L = \sqrt{2} \times (1 - 0) = \sqrt{2}
$$

This straightforward calculation highlights the significance of the constant speed in the curve's parametrization.

Features and Applications of the Calculations

Understanding the tangent and normal vectors at a point on a curve offers several advantages:

Features:

- Unit Tangent Vector $T(t)$: Indicates the direction of the curve at a point, essential for understanding motion and velocity.
- Unit Normal Vector $N(t)$: Shows the direction in which the curve bends, related to curvature.
- Curvature and Torsion: These vectors serve as foundational components in calculating the curvature and torsion of the curve, which measure how sharply the curve bends and twists.
- Simplified Calculations: The constant magnitude of $R'(t)$ simplifies many computations, making the curve's analysis more straightforward.

Applications:

- Physics: Analyzing particle motion, especially in electromagnetism and mechanics.
- Engineering: Designing paths and trajectories, such as in robotics or aerospace.
- Computer Graphics: Rendering smooth curves and understanding how objects move along paths.
- Mathematical Modeling: Describing natural phenomena like DNA helices, springs, or spiral galaxies.

Pros and Cons of the Approach

Pros:

- Simplicity: The constant magnitude of $R'(t)$ simplifies calculations.
- Clarity: Step-by-step derivation enhances understanding.
- Applicability: The method applies broadly to similar parametric curves.
- Insightful: Provides geometric intuition about the curve's behavior.

Cons:

- Limited to Smooth Curves: The approach assumes differentiability and smoothness.

Find Both The Unit Tangent And Unit Normal To The Curve R T Cost Sint T At T 1 Find The Length

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