Find K Such That The Function Is A Probability Density Function Over The Given Interval. Then Write The

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When working with probability density functions (PDFs), a fundamental task is to determine the constant or parameter that makes a given function valid over a specified interval. This process involves ensuring that the total area under the curve of the function equals 1, satisfying the core property of PDFs. In this article, we will explore the systematic approach to finding the value of K so that a specified function becomes a legitimate probability density function over a given interval. Moreover, we will discuss the subsequent steps to write the complete PDF once the constant is determined, along with practical examples to illustrate the concepts.

Understanding Probability Density Functions (PDFs)

What Is a Probability Density Function?

A probability density function is a function that describes the likelihood of a continuous random variable taking on a particular value within a certain interval. Unlike discrete probabilities, where the sum over all outcomes equals 1, a PDF must satisfy the condition that the integral over its entire domain equals 1:

$$
\int_{-\infty}^{\infty} f(x) \, dx = 1
$$

This ensures that the total probability across the entire space is 100%.

Key Properties of PDFs

- Non-negativity: $f(x) \geq 0$ for all x.
- Normalization: The integral over the domain equals 1.
- Probability Calculation: The probability that the variable falls within an interval $[a, b]$ is given by $\int_a^b f(x) \, dx$.

Setting Up the Problem: Finding K

Suppose you are given a function $f(x) = K \cdot g(x)$, where $g(x)$ is a known function over a certain interval $[a, b]$, and K is an unknown constant. To turn this into a valid PDF, we must find the value of K such that:

$$
\int_a^b f(x) \, dx = 1
$$

Substituting $f(x) = K \cdot g(x)$, the integral becomes:

$$
K \int_a^b g(x) \, dx = 1
$$

The solution for K is then:

$$
K = \frac{1}{\int_a^b g(x) \, dx}
$$

This fundamental step ensures the total probability across the interval sums to 1.

Step-by-Step Procedure for Finding K

Step 1: Identify the Given Function and Interval

- Determine the functional form $f(x) = K \cdot g(x)$.
- Identify the interval $[a, b]$ over which the function is defined.

Step 2: Compute the Integral of $g(x)$

- Calculate $\int_a^b g(x) \, dx$.
- Use appropriate integration techniques (substitution, integration by parts, etc.).

Step 3: Solve for K

- Use the normalization condition: $K \cdot \int_a^b g(x) \, dx = 1$.
- Isolate K:

$$
K = \frac{1}{\int_a^b g(x) \, dx}
$$

Step 4: Write the Complete PDF

- Substitute the found value of K back into the original function:

$$
f(x) = \frac{g(x)}{\int_a^b g(x) \, dx}
$$

- Verify that $f(x) \geq 0$ over the interval.

Practical Examples

Example 1: Uniform Distribution

Suppose $f(x) = K$ for $x \in [2, 5]$, and zero elsewhere.

- Step 1: The function is constant over $[2, 5]$.
- Step 2: Compute the integral:

$$
\int_2^5 K \, dx = K \times (5 - 2) = 3K
$$

- Step 3: Set equal to 1:

$$
3K = 1 \Rightarrow K = \frac{1}{3}
$$

- Step 4: The PDF is:

$$
f(x) = \frac{1}{3}, \quad x \in [2, 5]
$$

and zero elsewhere.

Example 2: Non-Uniform Distribution

Suppose $f(x) = K \cdot x$ for $x \in [0, 4]$.

- Step 1: $g(x) = x$, interval $[0, 4]$.
- Step 2: Calculate the integral:

$$
\int_0^4 x \, dx = \left[\frac{x^2}{2}\right]_0^4 = \frac{4^2}{2} = \frac{16}{2} = 8
$$

- Step 3: Find K:

$$

K = \frac{1}{8}
\]

- Step 4: Write the PDF:

\[
f(x) = \frac{x}{8}, \quad x \in [0, 4]
\]

and zero elsewhere.

Writing the Final PDF

Once the constant \(K\) is determined, the complete form of the probability density function is straightforward to write. It involves substituting \(K\) into the function \(f(x) = K \cdot g(x)\), ensuring the function is explicitly defined over the given interval.

Additional Considerations:
- Always verify that \(f(x) \geq 0\) over the interval.
- Confirm that the integral over the interval equals 1 after determining \(K\).
- If the functional form involves parameters, repeat the process to solve for those parameters.

Conclusion

Finding the value of \(K\) such that a function becomes a valid probability density function is a fundamental skill in probability and statistics. It involves integrating the known part of the function over its domain and then solving for the constant to satisfy the normalization condition. This process not only ensures the function's validity as a PDF but also provides a clear foundation for calculating probabilities, expectations, and other statistical measures.

By following the structured approach outlined in this article—identifying the interval, computing the integral, solving for \(K\), and writing the complete PDF—you can confidently handle a variety of problems involving PDFs. Whether working with simple uniform distributions or more complex functions, these steps remain consistent and essential for proper statistical analysis.

Frequently Asked Questions

How do I determine the value of K so that a given function becomes a valid probability density function (PDF) over a specific interval?

To find K, set the integral of the function over the interval equal to 1 (since total probability must be 1) and solve for K. That is, compute ∫(a to b) f(x) dx = 1, then solve for K.

What are the steps involved in verifying that a function is a valid PDF after finding the constant K?

First, determine K so that the total integral over the interval equals 1. Then, verify that the function is non-negative over the interval. If both conditions hold, the function is a valid PDF.

Can you provide an example of finding K for a specific function over a given interval?

Yes. For example, if $f(x) = Kx$ on [0, 2], find K so that $\int_0^2 Kx\,dx = 1$. Calculating, $\int_0^2 Kx\,dx = K(x^2/2)$ from 0 to 2 = K (4/2) = 2K. Set 2K = 1, so K = 1/2.

What common mistakes should I avoid when determining K for a PDF?

Avoid forgetting to integrate over the correct interval, neglecting to ensure the function is non-negative, and making algebraic errors when solving for K. Always double-check the integral setup and calculations.

Once K is found, how do I interpret the resulting function as a probability density function?

After determining K, the function should satisfy that its integral over the interval equals 1, and it should be non-negative everywhere in that interval. This confirms it properly models a probability distribution over the given domain.

Additional Resources

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Introduction

In probability theory and statistics, a probability density function (PDF) is fundamental for describing continuous random variables. It provides a way to model the likelihood of a variable falling within a particular range. To qualify as a valid PDF, a function must satisfy two primary conditions:

1. Non-negativity: The function must be non-negative over its entire domain.
2. Total Area Equals One: The integral of the function over its domain must be exactly 1, representing total probability.

This article delves into the process of determining the constant \(K \) such that a given function qualifies as a probability density function over a specified interval. We will explore the theoretical framework, practical steps, and examples to clarify this process. Additionally, we will discuss how to write the associated probability density function once the appropriate \(K \) is identified.

Understanding the Criteria for a Probability Density Function

Non-Negativity

The function $f(x)$ must be greater than or equal to zero for all x within its support (the interval over which it is defined). This ensures that the "probability" assigned to any event is never negative.

Total Probability Equals One

Mathematically, the integral of the PDF over its domain must be 1:

$$
\int_{a}^{b} f(x) \, dx = 1
$$

where $[a, b]$ is the support of the function.

The General Approach to Finding K

Given a function $f(x)$ with an unknown constant K, the goal is to determine the value of K such that:

$$
\int_{a}^{b} K \cdot g(x) \, dx = 1
$$

where $g(x)$ is the known part of the function, and K is a scalar that scales $g(x)$ to satisfy the total probability condition.

Step-by-Step Procedure

1. Identify the given function $g(x)$: The unscaled part of the function, often provided without the constant K.

2. Determine the support interval $[a, b]$: The domain over which the function is defined.

3. Set up the integral:

$$
K \int_{a}^{b} g(x) \, dx = 1
$$

4. Compute the integral $I = \int_{a}^{b} g(x) \, dx$: Use calculus techniques suited for the form of $g(x)$.

5. Solve for K:

$$
K = \frac{1}{I}
$$

6. Verify that the resulting function $f(x) = K \cdot g(x)$ is non-negative over $[a, b]$.

7. Write the final probability density function:

$$
f(x) = K \cdot g(x)
$$

Practical Examples

Example 1: Linear Function on $[0, 2]$

Suppose we have the function:

$$
f(x) = K (x + 1)
$$

defined over the interval $[0, 2]$.

Step 1: Recognize $g(x) = x + 1$, $a = 0$, $b = 2$.

Step 2: Compute the integral:

$$
I = \int_{0}^{2} (x + 1) \, dx
$$

$$
I = \left[\frac{x^2}{2} + x \right]_0^2 = \left(\frac{4}{2} + 2 \right) - (0 + 0) = 2 + 2 = 4
$$

Step 3: Solve for K:

$$
K = \frac{1}{I} = \frac{1}{4}
$$

Step 4: Write the PDF:

$$
f(x) = \frac{1}{4}(x + 1), \quad 0 \leq x \leq 2
$$

Step 5: Verify non-negativity:

Since $x + 1 \geq 1$ over $[0, 2]$, $f(x) \geq 0$.

Example 2: Quadratic Function on $[1, 3]$

Suppose:

$$
f(x) = K (x^2)
$$

over $[1, 3]$.

Step 1: $g(x) = x^2$, $a=1$, $b=3$.

Step 2: Compute the integral:

$$
I = \int_{1}^{3} x^2 \, dx = \left[\frac{x^3}{3} \right]_1^3 = \frac{27}{3} - \frac{1}{3} = 9 - \frac{1}{3} = \frac{26}{3}
$$

Step 3: Find K:

$$
K = \frac{1}{I} = \frac{3}{26}
$$

Step 4: Final PDF:

$$
f(x) = \frac{3}{26} x^2, \quad 1 \leq x \leq 3
$$

Step 5: Confirm non-negativity:

Since $x^2 \geq 0$, $f(x) \geq 0$ over the interval.

Writing the Final Probability Density Function

Once K is determined, the complete PDF is written as:

$$
\boxed{
f(x) = K \cdot g(x)
}
$$

where:

- $g(x)$ is the known, non-negative function.
- The support interval $[a, b]$ is clearly specified.
- K ensures the total area under $f(x)$ equals 1.

It is essential to explicitly state the support interval when presenting the PDF, as the behavior outside this interval is typically zero (assuming a standard continuous distribution).

Considerations and Common Pitfalls

Ensuring Proper Support

Always verify the support interval. If the support is not specified, the problem should specify where the function is valid or assume the standard domain.

Non-negativity Check

Even after determining K, verify that $f(x) \geq 0$ over $[a, b]$. If the function takes negative values within the support, it cannot be a valid PDF.

Handling Piecewise Functions

If the function is defined piecewise, compute the integral over each piece separately and sum:

$$
K \left(\int_{a}^{c} g_1(x) \, dx + \int_{c}^{b} g_2(x) \, dx \right) = 1
$$

Numerical Integration

When an integral cannot be evaluated analytically, numerical methods such as Simpson’s rule or trapezoidal rule can approximate the integral to find K.

Advanced Topics

Normalization of Complex Functions

Some functions may require more intricate integration techniques (substitution, parts, special functions) to compute the integral. In such cases, symbolic computation tools like WolframAlpha, Maple, or Mathematica may be employed.

Multivariate Distributions

The concept extends to joint PDFs of multiple variables, where normalization involves integrating over multiple dimensions.

Distributions with Parameters

Many distributions (like the Beta, Gamma, or Weibull) involve parameters. The process of determining K involves integrating the unscaled function and solving for the constant to normalize the distribution.

Summary

- To find K such that a function becomes a valid PDF, integrate the unscaled function over its support.
- The key formula is:

$$
K = \frac{1}{\int_{a}^{b} g(x) \, dx}
$$

- After calculating K, write the final function as $f(x) = K \cdot g(x)$ over the specified support.
- Always verify the non-negativity and support conditions.

This process ensures the function correctly models the probability distribution of a continuous random variable, satisfying the foundational properties of PDFs.

Final Remarks

Mastering the process of normalizing functions to serve as PDFs is essential for students and practitioners working in probability, statistics, and data analysis. Whether deriving theoretical distributions or fitting models to data, this skill underpins many statistical methods and applications. Remember, the core principle is ensuring the total probability sums (or integrates) to one, reflecting the fundamental axioms of probability theory.

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