Find The 90% Confidence Interval For The Mean For The Price Of An Adult Single-day Ski Lift Ticket. The

Find The 90% Confidence Interval For The Mean For The Price Of An Adult Single-day Ski Lift Ticket. The process of estimating the average price of adult single-day ski lift tickets is essential for skiers, resort owners, and industry analysts alike. Understanding the confidence interval associated with this mean provides a statistical range within which the true average price likely falls, with a specified level of confidence. In this article, we will explore how to calculate the 90% confidence interval for the mean price, why it's important, and how to interpret the results effectively.

Understanding Confidence Intervals and Their Importance

What Is a Confidence Interval?

A confidence interval (CI) is a statistical tool used to estimate the range within which a population parameter, such as the mean, is likely to lie. When we collect a sample of data—say, the prices of ski lift tickets at various resorts—we can calculate the sample mean and use it to estimate the population mean. However, due to sampling variability, this estimate alone isn't enough. The confidence interval accounts for this uncertainty, providing a range that, with a certain confidence level (e.g., 90%), contains the true mean.

Why Is a 90% Confidence Level Used?

The confidence level indicates the probability that the interval contains the true parameter. A 90% confidence level means that if we were to take many samples and compute the interval each time, approximately 90% of those intervals would contain the true mean price. This level strikes a balance between precision and certainty, offering a reasonably narrow interval while maintaining a high confidence in the estimate.

Gathering Data for the Calculation

Sample Data Collection

To compute a confidence interval, you need a representative sample of adult single-day ski lift ticket prices. This involves:

- Sampling multiple ski resorts across different regions and price ranges.

- Recording the prices of adult single-day tickets at each resort.
- Ensuring the sample size is sufficient to produce a reliable estimate.

Sample Size Considerations

The accuracy of the confidence interval depends on the sample size:

- Smaller samples lead to wider intervals due to increased uncertainty.
- Larger samples tend to produce narrower, more precise intervals.
- Typically, a sample size of at least 30 is recommended to leverage the Central Limit Theorem, which ensures the sampling distribution of the mean is approximately normal.

Calculating the 90% Confidence Interval

Step 1: Compute the Sample Mean and Standard Deviation

Given your data, calculate:

- **Sample Mean ($\bar{x}$):** The average price across all sampled resorts.
- **Sample Standard Deviation (s):** Measures the variability of the prices in your sample.

Step 2: Determine the Appropriate Critical Value

Since the sample size might be small or the population standard deviation unknown, the t-distribution is typically used:

- Find the degrees of freedom: $df = n - 1$, where n is your sample size.
- Identify the t-value corresponding to a 90% confidence level for your degrees of freedom from the t-distribution table.

Step 3: Calculate the Margin of Error (ME)

Use the formula:
\[

ME = t_{(1-\alpha/2, df)} \times \frac{s}{\sqrt{n}}
\]
where:
- \(t_{(1-\alpha/2, df)}\) is the critical t-value,
- \(s\) is the sample standard deviation,
- \(n\) is the sample size.

Step 4: Compute the Confidence Interval

Finally, the confidence interval is:
\[
\left(\bar{x} - ME, \quad \bar{x} + ME \right)
\]
This interval provides the range within which the true mean price of adult single-day ski lift tickets is likely to fall with 90% confidence.

Interpreting the Results

Understanding the Confidence Interval Range

Once calculated, the interval gives a practical estimate:

- If the interval is, for example, \$80 to \$100, we are 90% confident that the true average ticket price lies within this range.
- This information can assist skiers in budgeting and decision-making.

Implications for Ski Resorts and Industry Stakeholders

Resort owners and industry analysts can:

- Compare their prices against the average estimate.
- Assess pricing strategies in relation to regional or seasonal variations.
- Use the interval to inform marketing and promotional efforts.

Example Calculation

Suppose you collected data from 40 resorts with an average ticket price of \$85 and a standard deviation of \$10.

- Sample size (n) = 40
- Sample mean ($\bar{x}$) = \$85
- Sample standard deviation (s) = \$10
- Degrees of freedom (df) = 39

From a t-distribution table, the critical t-value for 90% confidence and 39 degrees of freedom is approximately 1.685.

Calculate Margin of Error (ME):

$$
ME = 1.685 \times \frac{10}{\sqrt{40}} = 1.685 \times \frac{10}{6.324} \approx 1.685 \times 1.58 \approx 2.66
$$

Thus, the 90% confidence interval is:

$$
(85 - 2.66, \quad 85 + 2.66) = (\$82.34, \quad \$87.66)
$$

This means you can be 90% confident that the true average price of adult single-day ski lift tickets falls between \$82.34 and \$87.66.

Conclusion: Making Data-Driven Decisions with Confidence Intervals

Calculating the 90% confidence interval for the mean price of adult single-day ski lift tickets offers valuable insights for consumers, resort owners, and industry analysts. It provides a statistically sound range that reflects the typical cost, accounting for variability in data. By understanding how to gather representative data, perform the calculations, and interpret the results, stakeholders can make informed decisions, optimize pricing strategies, and better understand market dynamics.

Whether you are planning a ski trip and want to budget effectively or a resort owner seeking competitive pricing insights, mastering the concept of confidence intervals is a powerful tool in the realm of sports and recreation economics. Always remember that confidence intervals are about probability and estimation—they give you a range, not an exact figure, but that range is a critical piece of information for sound decision-making in the ski industry.

Frequently Asked Questions

What is a 90% confidence interval for the mean price of an adult single-day ski lift ticket?

A 90% confidence interval estimates the range within which the true average price of an adult single-day ski lift ticket lies, with 90% certainty based on sample data.

How do I calculate the 90% confidence interval for the mean price of a ski lift ticket?

You calculate it by taking the sample mean, adding and subtracting the margin of error (which involves the standard deviation, sample size, and the critical t-value for 90% confidence).

What information do I need to find the 90% confidence interval for the ticket price?

You need the sample mean, sample standard deviation, sample size, and the t-distribution critical value corresponding to 90% confidence and degrees of freedom.

How does the sample size affect the width of the confidence interval?

A larger sample size decreases the standard error, leading to a narrower confidence interval, which indicates a more precise estimate of the mean.

Why is the t-distribution used for calculating the confidence interval in this context?

The t-distribution is used because the population standard deviation is unknown and the sample size is typically small, making it appropriate for estimating the mean from sample data.

What assumptions are made when constructing a 90% confidence interval for the mean price?

Assumptions include that the sample data are randomly selected, the data are approximately normally distributed, and the observations are independent.

Can I interpret the 90% confidence interval as a 90% probability that the true mean lies within this interval?

No, the correct interpretation is that if many samples are taken and their confidence intervals computed, about 90% of those intervals will contain the true population mean.

What is the impact of outliers on calculating the confidence interval for ski lift ticket prices?

Outliers can inflate the standard deviation and skew the sample mean, potentially widening the confidence interval and reducing the accuracy of the estimate.

How can I improve the accuracy of the confidence interval

estimate for the mean price?

Increasing the sample size, ensuring random sampling, and checking for outliers can improve the accuracy and precision of the confidence interval estimate.

What are common mistakes to avoid when calculating a 90% confidence interval for the mean?

Common mistakes include using the wrong critical t-value, not checking assumptions, mixing up sample and population parameters, and ignoring the effect of outliers or non-normal data distributions.

Additional Resources

Confidence Interval

Introduction

When planning a skiing trip or evaluating the affordability of a ski resort, understanding the typical cost of an adult single-day ski lift ticket is essential. But prices can fluctuate due to seasonality, demand, and other economic factors. Instead of relying on a single price point, statistical analysis offers a more reliable approach to estimate the average price within a certain confidence level. One such powerful tool is the confidence interval—a range of values that likely contains the true average price with a specified level of certainty.

In this article, we delve into the process of calculating a 90% confidence interval for the mean price of an adult single-day ski lift ticket. Whether you're a data analyst, a resort manager, or a savvy skier, understanding this statistical concept will help you make more informed decisions based on data.

Understanding Confidence Intervals

What Is a Confidence Interval?

A confidence interval (CI) is a statistical range, derived from sample data, that is believed to contain the true population parameter—in this context, the mean price—with a certain level of confidence. The confidence level, such as 90%, indicates the probability that the interval computed from repeated samples will contain the true mean.

Key points:
- Sample Data: Since collecting data on all prices at every resort is impractical, we gather a sample.
- Interval Estimation: The CI provides an estimated range based on the sample.
- Confidence Level: The probability (e.g., 90%) that the interval contains the actual population mean if the sampling process is repeated many times.

Why Use a Confidence Interval?

- Practical Insight: Instead of a single point estimate, a CI portrays the uncertainty around the mean price.
- Decision-Making: Helps consumers compare prices confidently.
- Data Variability: Accounts for natural fluctuations in prices across different resorts or seasons.

Gathering and Preparing Data

Before calculating a confidence interval, it’s essential to have a reliable sample of price data. Suppose we have collected prices from a variety of ski resorts across different regions and seasons, resulting in a dataset of adult single-day lift ticket prices.

Example Data Set (Hypothetical)

Resort	Price ($)
Resort A	75.00
Resort B	82.50
Resort C	78.00
Resort D	85.00
Resort E	80.00
Resort F	77.50
Resort G	83.00
Resort H	79.00
Resort I	81.00
Resort J	76.50

Sample size (n): 10

Sample mean ($\bar{x}$):

$$
\bar{x} = \frac{75 + 82.5 + 78 + 85 + 80 + 77.5 + 83 + 79 + 81 + 76.5}{10} = \frac{798}{10} = 79.80
$$

Sample standard deviation (s):

Calculating s involves the following steps:

1. Find each price's deviation from the mean.
2. Square each deviation.
3. Sum all squared deviations.
4. Divide by $n - 1$ (degrees of freedom).
5. Take the square root.

Calculations:

Price	Deviation ($x_i - \bar{x}$)	Squared Deviation

75.00	-4.80	23.04
82.50	2.70	7.29
78.00	-1.80	3.24
85.00	5.20	27.04
80.00	0.20	0.04
77.50	-2.30	5.29
83.00	3.20	10.24
79.00	-0.80	0.64
81.00	1.20	1.44
76.50	-3.30	10.89

Sum of squared deviations:

$$
23.04 + 7.29 + 3.24 + 27.04 + 0.04 + 5.29 + 10.24 + 0.64 + 1.44 + 10.89 = 89.09
$$

Variance:

$$
s^2 = \frac{89.09}{10 - 1} = \frac{89.09}{9} \approx 9.90
$$

Standard deviation:

$$
s = \sqrt{9.90} \approx 3.15
$$

Calculating the 90% Confidence Interval

Step 1: Identify the Correct Distribution

Since the sample size is small ($n < 30$) and the population standard deviation is unknown, we use the t-distribution to compute the confidence interval.

Step 2: Find the t-Score

The t-score depends on:

- The confidence level (90%)
- The degrees of freedom ($df = n - 1 = 9$)

From the t-distribution table or calculator:

$$
t_{0.05, 9} \approx 1.833
$$

(Note: For a 90% CI, the significance level $\alpha = 1 - 0.9 = 0.10$). Since the CI is two-tailed, each tail has 0.05, so the t-score corresponds to 0.05 in the upper tail.)

Step 3: Compute the Standard Error (SE)

$$
SE = \frac{s}{\sqrt{n}} = \frac{3.15}{\sqrt{10}} \approx \frac{3.15}{3.162} \approx 1.00
$$

Step 4: Calculate Margin of Error (ME)

$$
ME = t \times SE = 1.833 \times 1.00 \approx 1.83
$$

Step 5: Determine the Confidence Interval

$$
\text{Lower bound} = \bar{x} - ME = 79.80 - 1.83 = 77.97
$$
$$
\text{Upper bound} = \bar{x} + ME = 79.80 + 1.83 = 81.63
$$

Result:

The 90% confidence interval for the mean adult single-day ski lift ticket price is approximately \$77.97 to \$81.63.

Interpreting the Confidence Interval

This interval suggests that based on our sample data, we can be 90% confident that the true average price of an adult single-day ski lift ticket at all ski resorts falls between \$77.97 and \$81.63.

Implications:

- If you're a skier, this range gives a realistic expectation of what you might pay.
- For resort managers, it offers a benchmark to evaluate whether their pricing aligns with the industry average.
- For analysts, it quantifies the uncertainty and variability inherent in price data.

Factors Affecting Confidence Interval Accuracy

While the above method provides a rigorous statistical estimate, several factors can influence the precision and validity of the confidence interval:

1. Sample Size

- Larger samples tend to produce narrower, more precise intervals.
- Small samples, like the example, increase uncertainty.

2. Data Variability

- High variability in prices widens the interval.
- Consistent pricing across resorts results in tighter estimates.

3. Sampling Method

- Random, representative sampling ensures the interval accurately reflects the population.
- Biased samples can distort the estimate.

4. Distribution Assumptions

- The t-distribution assumes the underlying data is approximately normally distributed, especially important for small samples.
- For larger samples, the Central Limit Theorem mitigates this concern.

Practical Applications and Limitations

Applications:

- Consumer Planning: Knowing the typical price range aids in budgeting.
- Market Analysis: Resorts can compare their prices against industry averages.
- Economic Studies: Researchers can analyze how prices fluctuate over seasons or regions.

Limitations:

- Small sample sizes may not capture all price variations.
- Price data may be outdated or not representative of current market conditions.
- External factors (e.g., inflation, seasonal discounts) can affect prices but may not be reflected in the sample.

Enhancing Confidence in Your Estimates

To improve the reliability of the confidence interval:

- Increase Sample Size: Collect more data points across different resorts and times.
- Ensure Randomness: Use random sampling to avoid bias.
- Segment Data: Analyze separate intervals for peak vs. off-peak seasons.
- Update Data Regularly: Use recent prices to reflect current market conditions.

Conclusion

Calculating a 90% confidence interval for the mean price of an adult single-day ski lift ticket provides valuable insight into the typical cost and its variability. Using statistical methods, such as the t-distribution, allows both consumers and industry stakeholders

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