Find The Area Of The Parallelogram With Vertices P(3, 3, 2), Q(5, 6, 4), R(9, 10, 12), And S(7, 7, 10).

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Understanding the area of a parallelogram in three-dimensional space is a fundamental concept in vector calculus and geometry. Whether you're a student preparing for exams, a teacher designing lesson plans, or a professional solving spatial problems, knowing how to compute the area of a parallelogram given its vertices is essential. This article offers a comprehensive guide to calculating the area of a specific parallelogram with vertices P(3, 3, 2), Q(5, 6, 4), R(9, 10, 12), and S(7, 7, 10). We will explore the underlying mathematics, step-by-step procedures, and relevant concepts to ensure a thorough understanding.

Introduction to Parallelograms in 3D Space

A parallelogram is a four-sided polygon with opposite sides that are parallel and equal in length. In three-dimensional space, the concept remains the same, but the challenge lies in calculating the area when the vertices are in arbitrary positions. Unlike in 2D, where the area can be straightforwardly computed from base and height, 3D space requires vector methods and the cross product to determine the area.

The key idea is that the area of a parallelogram can be found using the magnitude of the cross product of two adjacent side vectors originating from a common vertex. Specifically, if you have two vectors, A and B, originating from the same point, then the area of the parallelogram they form is:

$$
\text{Area} = |\mathbf{A} \times \mathbf{B}|
$$

where $|\mathbf{A} \times \mathbf{B}|$ denotes the magnitude of the cross product.

The Given Vertices and Their Significance

The vertices provided are:

- $P(3, 3, 2)$
- $Q(5, 6, 4)$
- $R(9, 10, 12)$
- $S(7, 7, 10)$

To find the area of the parallelogram, we need to identify which pairs of vectors define two adjacent sides originating from a common vertex. Typically, you can choose any vertex as the reference point; however, it’s convenient to pick one that simplifies calculations.

Step-by-Step Calculation Process

1. Identify the Adjacent Vertices

Assuming that the vertices are ordered such that P, Q, R, and S form the parallelogram in order, the sides can be represented as vectors:

- Vector $\vec{PQ}$: from P to Q
- Vector $\vec{PS}$: from P to S

Alternatively, you could choose Q as the reference point and compute vectors accordingly. For simplicity, we'll proceed with P as the reference vertex.

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2. Calculate the Vectors $\vec{PQ}$ and $\vec{PS}$

The vector $\vec{PQ}$ is obtained by subtracting the coordinates of P from Q:

$$
\vec{PQ} = Q - P = (5 - 3, 6 - 3, 4 - 2) = (2, 3, 2)
$$

Similarly, the vector $\vec{PS}$:

$$
\vec{PS} = S - P = (7 - 3, 7 - 3, 10 - 2) = (4, 4, 8)
$$

---

3. Compute the Cross Product $\vec{PQ} \times \vec{PS}$

The cross product of two vectors $\mathbf{A} = (A_x, A_y, A_z)$ and $\mathbf{B} = (B_x, B_y, B_z)$ is given by:

$$
\mathbf{A} \times \mathbf{B} = (A_y B_z - A_z B_y,\, A_z B_x - A_x B_z,\, A_x B_y - A_y B_x)
$$

Applying this to $\vec{PQ} = (2, 3, 2)$ and $\vec{PS} = (4, 4, 8)$:

$$
\begin{aligned}
\mathbf{N} &= \vec{PQ} \times \vec{PS} \\
&= ((3)(8) - (2)(4),\, (2)(4) - (2)(8),\, (2)(4) - (3)(4)) \\
&= (24 - 8,\, 8 - 16,\, 8 - 12) \\
&= (16, -8, -4)
\end{aligned}
$$

---

4. Find the Magnitude of the Cross Product

The magnitude $|\mathbf{N}|$ gives the area of the parallelogram:

$$
|\mathbf{N}| = \sqrt{(16)^2 + (-8)^2 + (-4)^2} = \sqrt{256 + 64 + 16} = \sqrt{336}
$$

Simplify the square root:

$$
|\mathbf{N}| = \sqrt{336} = \sqrt{16 \times 21} = 4 \sqrt{21}
$$

Final Calculation and Interpretation

The magnitude of the cross product, $4\sqrt{21}$, represents the area of the parallelogram with vertices P, Q, R, and S. Therefore:

\[

\boxed{
\text{Area of the parallelogram} = 4 \sqrt{21} \approx 4 \times 4.58 \approx 18.32
}
\]

This precise value, \(4 \sqrt{21} \), is often preferred for exactness in mathematical contexts.

Additional Considerations

- Coordinate Verification: Always verify the order of vertices to ensure the vectors represent adjacent sides correctly.
- Alternative Approaches: If vertices are given in a different order, or if the shape is not a perfect parallelogram, other methods such as the shoelace formula or vector projections may be necessary.
- Applications: Calculating the area of 3D polygons is vital in various fields such as computer graphics, engineering, architecture, and physics.

Summary of Key Steps

1. Identify the reference vertex.
2. Calculate vectors representing adjacent sides.
3. Compute the cross product of these vectors.
4. Find the magnitude of the cross product.
5. Interpret the magnitude as the area of the parallelogram.

Conclusion

Calculating the area of a parallelogram in three-dimensional space involves understanding vector operations, particularly the cross product. By carefully selecting vectors from the given vertices and performing the calculations step-by-step, you can accurately determine the area. In our example, the area of the parallelogram with vertices \(P(3, 3, 2) \), \(Q(5, 6, 4) \), \(R(9, 10, 12) \), and \(S(7, 7, 10) \) is exactly \(4 \sqrt{21} \) square units, approximately 18.32 units.

Mastering these techniques enhances your ability to solve complex spatial problems and deepens your understanding of 3D geometry—a skill applicable across many scientific and engineering disciplines. Whether you're analyzing structures, designing 3D models, or studying vector calculus, the principles outlined in this guide serve as a foundational resource.

Keywords: area of parallelogram, 3D geometry, vector cross product, vertices in space, calculating area in 3D, vector calculus, coordinate geometry

Frequently Asked Questions

How do you find the area of a parallelogram in 3D space given its vertices?

To find the area, you can use the cross product of two adjacent side vectors originating from a common vertex. The magnitude of the cross product gives the area of the parallelogram.

What are the steps to compute the area of the parallelogram with vertices P(3, 3, 2), Q(5, 6, 4), R(9, 10, 12), and S(7, 7, 10)?

First, select two adjacent sides, for example, vectors PQ and PS. Calculate their components, find their cross product, and then compute the magnitude of this cross product to get the area.

How do you determine the vectors representing sides of the parallelogram in 3D?

You subtract the coordinates of the common vertex from the neighboring vertices to get the side vectors. For example, from P, vectors PQ = Q - P and PS = S - P.

What is the formula for the cross product of two vectors in 3D?

Given vectors A = (a1, a2, a3) and B = (b1, b2, b3), their cross product is $A \times B$ = (a2b3 - a3b2, a3b1 - a1b3, a1b2 - a2b1).

Can you demonstrate calculating the area of the given parallelogram step-by-step?

Yes. First, find vectors PQ = (2, 3, 2) and PS = (4, 4, 8). Then, compute their cross product: ((38 - 24), 24 - 28, 24 - 34) = ((24 - 8), (8 - 16), (8 - 12)) = (16, -8, -4). Finally, find the magnitude: $\sqrt{16^2 + (-8)^2 + (-4)^2} = \sqrt{256 + 64 + 16} = \sqrt{336} \approx 18.33$ units^2.

What is the final area of the parallelogram with the given vertices?

The area of the parallelogram is approximately 18.33 square units, based on the vector calculations and magnitude of the cross product.

Additional Resources

Find The Area Of The Parallelogram With Vertices P(3, 3, 2), Q(5, 6, 4), R(9, 10, 12), And S(7, 7, 10)

Understanding how to find the area of a parallelogram in three-dimensional space is a fundamental concept in vector geometry. When given the vertices of a parallelogram, the problem often involves calculating the vectors representing adjacent sides and then using the cross product to determine the area. This process requires a solid grasp of vector operations, coordinate geometry, and the properties of parallelograms. In this detailed review, we'll explore the step-by-step methodology to find the area of the specified parallelogram, analyze the mathematical principles involved, and discuss practical considerations and common pitfalls.

Introduction to the Problem

The problem involves four vertices: P(3, 3, 2), Q(5, 6, 4), R(9, 10, 12), and S(7, 7, 10). These points form a parallelogram, and the goal is to find its area in three-dimensional space. A key insight is recognizing that a parallelogram's area can be obtained by calculating the magnitude of the cross product of two adjacent side vectors originating from a common vertex.

Understanding the Geometry of the Parallelogram

Vertices and Their Arrangement

The vertices are given in coordinate form:

- P(3, 3, 2)
- Q(5, 6, 4)
- R(9, 10, 12)
- S(7, 7, 10)

In a parallelogram, opposite sides are parallel and equal in length. To verify the arrangement, we need to determine which points form adjacent sides. Typically, one can assume that P is a vertex from which sides PQ and PS emanate.

Identifying Adjacent Sides

To find the vectors representing sides from P, we calculate:

- Vector PQ = Q - P
- Vector PS = S - P

Similarly, we can check whether the points R and S are opposite vertices or adjacent ones, but for area calculation, focusing on two adjacent sides emanating from a common vertex (say, P) is sufficient.

Calculating Vectors for the Sides

Vector PQ

Given Q(5, 6, 4) and P(3, 3, 2):

PQ = (5 - 3, 6 - 3, 4 - 2) = (2, 3, 2)

Vector PS

Given S(7, 7, 10) and P(3, 3, 2):

PS = (7 - 3, 7 - 3, 10 - 2) = (4, 4, 8)

These two vectors, PQ and PS, represent two adjacent sides of the parallelogram originating from P.

Finding the Cross Product of the Vectors

Purpose of the Cross Product

The magnitude of the cross product of two vectors gives the area of the parallelogram formed by these vectors:

$$
\text{Area} = |\vec{PQ} \times \vec{PS}|
$$

Calculating this cross product accurately is essential for determining the area.

Calculating the Cross Product

Given vectors:

$$
\vec{A} = (a_1, a_2, a_3) = (2, 3, 2)
$$
$$
\vec{B} = (b_1, b_2, b_3) = (4, 4, 8)
$$

The cross product $\vec{A} \times \vec{B}$ is computed as:

$$
\vec{A} \times \vec{B} = (a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1)
$$

Calculating each component:

- x-component:

$$
a_2b_3 - a_3b_2 = (3)(8) - (2)(4) = 24 - 8 = 16
$$

- y-component:

$$
a_3b_1 - a_1b_3 = (2)(4) - (2)(8) = 8 - 16 = -8
$$

- z-component:

$$
a_1b_2 - a_2b_1 = (2)(4) - (3)(4) = 8 - 12 = -4
$$

Therefore,

\[
\vec{PQ} \times \vec{PS} = (16, -8, -4)
\]

Calculating the Magnitude of the Cross Product

The area of the parallelogram is the magnitude of the cross product:

\[
|\vec{PQ} \times \vec{PS}| = \sqrt{(16)^2 + (-8)^2 + (-4)^2}
\]

Compute each term:

- \(16^2 = 256\)
- \(-8)^2 = 64\)
- \(-4)^2 = 16\)

Sum:

\[
256 + 64 + 16 = 336
\]

Taking the square root:

\[
\sqrt{336} \approx 18.33
\]

Hence, the area of the parallelogram is approximately 18.33 square units.

Verification and Alternative Approaches

Verifying the Result

To ensure accuracy, verify that the vectors chosen indeed define adjacent sides of the same parallelogram. Since the points P, Q, S form the sides, the calculation is consistent. Alternatively, you could compute vectors QR or RS and verify that they are parallel to PQ and PS respectively.

Alternative Calculation Using Other Vertices

If desired, we could also consider vectors from other vertices, such as Q, R, or S, to confirm the consistency of the area calculation. This helps ensure that the specified vertices indeed form a parallelogram and that the calculation is robust.

Features, Pros, and Cons of the Methodology

Features

- Vector-based approach: Utilizes fundamental vector operations, making it versatile for 3D geometry.
- Coordinate geometry integration: Combines coordinate calculations with vector algebra.
- Applicability: Works for any parallelogram in three-dimensional space, regardless of orientation.

Pros

- Mathematical precision: Provides an exact measure of the area.
- Clarity: Clear steps from vector calculation to magnitude.
- Efficiency: Straightforward once vectors are identified.

Cons

- Requires understanding of vector operations: May be challenging for beginners.
- Assumption of correct vertex order: Incorrect vertex order can lead to wrong side vectors.
- Potential for computational error: Mistakes in vector subtraction or cross product calculation can affect the result.

Additional Considerations and Practical Tips

- Always verify which points form adjacent sides to avoid errors.

- When dealing with multiple options, compare vectors to check for consistency.
- Use vector calculators or software for complex calculations to minimize errors.
- Remember that the area is always positive; the magnitude of the cross product suffices.

Conclusion

Calculating the area of a parallelogram in three dimensions using vertices involves several steps: identifying appropriate side vectors, computing their cross product, and then finding the magnitude of this vector. In the case of the vertices P(3, 3, 2), Q(5, 6, 4), R(9, 10, 12), and S(7, 7, 10), choosing vectors PQ and PS and performing the cross product yields an area of approximately 18.33 square units. This method exemplifies the power of vector algebra in solving geometric problems in 3D space and highlights the importance of careful calculations and verification. Whether for academic purposes or practical applications in fields like engineering and computer graphics, mastering this approach enhances one's ability to analyze and interpret complex spatial figures effectively.

Find The Area Of The Parallelogram With Vertices P 3 3 2 Q 5 6 4 R 9 10 12 And S 7 7 10

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